Ressources numériques
Optique pour l'instrumentation

Sensors for photonics and imaging systems

Colorimetry

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Color is a major criterion for a consumer when he selects and purchases a commercial product; more generally it is a signature associated to each object which makes it recognizable. The notion of color is highly dependent on the nature of the light source, which can be either natural (sun) or artificial (light bulbs). As the perception of color involves a complex processing by the brain, a straightforward definition of color is hard to find, and could not be restricted to something like "the spectrum of the light captured by the human eye". A more precise definition should include:

1.the properties of the object surface;

2.the quality of visual perception;

3.the nature of the light source;

4.the geometry of observation.

All these parameters are summarized in figure 1.

Figure 1

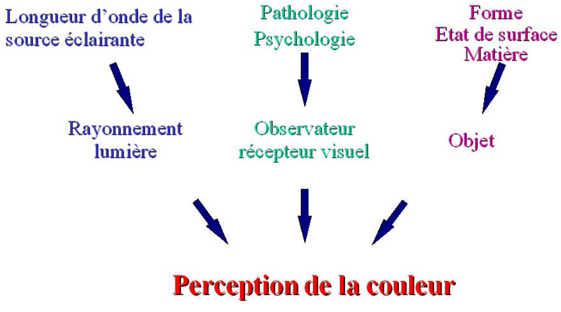

Figure 1 : Parameters playing a role in color perception.

The visual sensation experienced by an observer exposed to a colored visual stimulus is complex: as our knowledge of human vision mechanisms is still improving, putting color vision into equations is necessarily a "work in progress", likely to be subjected to constant revision.

Historically, the main objective of colorimetry (and of related measurement systems) had been to define visual equivalents of colored stimuli. More recently, its objective has been to supply methods that enable quantifying color matches and color differences, since it is of foremost importance, in a quality procedure, to ensure that the color of a commercial product will be perfectly reproducible.

A. Basic mechanisms of color vision

1. Introduction

The eye (figure 2) has an almost spherical shape. Its function is threefold: collecting light, forming an image of the observed scene, and processing the information. These tasks are realized thanks to only a few key elements:

Definition: the cornea

A transparent and resistant membrane acting as a protective barrier between the ocular globe and the outside world. The greatest refraction of light occurs at the corneal surface because at the transition between air and tissue the difference in refractive index is the greatest.

Definition: The iris

The colored ring around the pupil. The pupil regulates the flux of light entering the eye.

Definition: The retina

A thin membrane on the back of the eye, on which images are created. It contains two types of photoreceptors: cones (*photopic* or daily vision) and rods (*scotopic* or night vision).

Definition: The macula

Also called the yellow spot, it contains a small pit in its center called the *fovea,* of about 1,5 mm in diameter, corresponding to a vision angle of 5°. At the center of the fovea, a 400-µm-in-diameter zone (corresponding to a vision angle of 1.3°), the *fovea centralis,* contains only cones which are thinner and closer to each other. This latter zone corresponds to the area where the visual acuity is maximum.

Definition: the optic nerve

The optic nerve conveys the information to the brain, through the *lateral geniculate nucleus,* located in the thalamus, which plays the role of a signal amplifier.

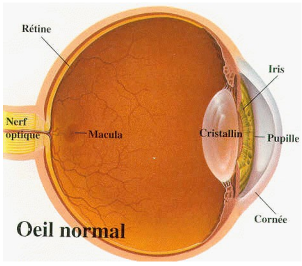

Figure 2 : a cross section through the human eye.

2. Anatomy of the retina

The retina contains light-sensitive cells and other nerve cells, devoted to information processing and transmission of this information up to the brain. The thickness of the retina is about 250µm except around the *fovea* where it is thinner., Approximately 150 millions of cells are present on the retina, spread over a total area of about 1100 mm^2 , divided schematically into three main layers (see figure 3):

1. The layer containing **retinal photoreceptors** (about 100 million rods and 5 million cones). Rods are highly stretched in one dimension (rod-like shape), and lie perpendicular to the retina surface;

they form a dense and regular mosaic.

2. The granular layer containing **bipolar cells**.

3. The layer containing **ganglion cells,** which are spiking nerve cells (neurons). They all recombine to form the optic nerve which transmits nerve impulses up to the **lateral geniculate nucleus.**

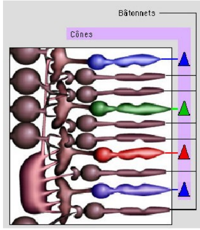

Figure 3 : The different nerve cells forming the retina.

a) Photoreceptors: cones and rods

There are 4 different kinds of retinal photoreceptors, three kinds of cones and only one kind of rods. They all have broad sensitivity spectra but with clearly distinct maxima that allows them to be discriminated. These maxima of sensitivity occur at:

- 495 nm for rods
- 420 nm for S cones (Short wavelength or B (blue))
- 530 nm for M cones (Medium wavelength or G (green))
- 560 nm for L cones (Long wavelength or R (red))

The spectral sensitivity curves for cones are given in figure 4. They have similar shapes and overlap significantly.

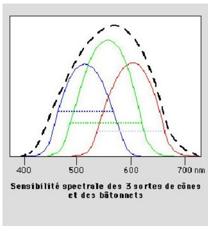

Figure 4 : Spectral sensitivity curves of cones (dotted line: the averaged spectral sensitivity of the eye)

At low light levels (scotopic vision: luminance$<10^{-3}$cd/m^2), rods work alone. At high levels (photopic vision; luminance>10cd/m^2) only cones are active. For intermediate levels (mesopic vision; 10^{-3} to 10cd/m^2) the two types of photoreceptors are involved.

b) Trichromatic encoding

Since the three types of cones have specific absorption spectra, a given radiation incident on the eye will yield three different and simultaneous physiological responses. This is called **trichromatic encoding** of color vision: this very important property of human vision is the foundation of the whole theory of colorimetry. The trichromatic theory was first proposed by Young in 1801, further explored by Helmholtz in 1852, and was later confirmed by micro-spectrophotometric experiments [[1]] as well as by electrophysiological measurements [[2]].

However, the existence of three different types of cones does not allow explaining all the features of color vision. There are many physio-psychological aspects that play a role in the whole process of color vision, which are still subject of intensive research. Colorimetry deals only with the interaction of light with the photosensitive cells of the retina.

3. Color vision

One can attempt to draw an analogy between color vision and signal (or information) processing. In this case one considers a color stimulus as a "signal" made of three inputs (the three cones) and yielding three outputs, corresponding to three channels [3]:

1. The **blue-yellow** channel: the nerves response results from the antagonistic excitation coming from the addition of R and V cones versus the B cone excitation (output is B-(R+V))

2. The **green-red channel** yielding a response which opposes the R cones to the V cones (output is V-R);

3. The **achromatic (or contrast) channel** yielding an additive response coming from the excitation of V and R cones: this signal carries the information about the light level (luminance), notwithstanding the spectral content of the incoming light.

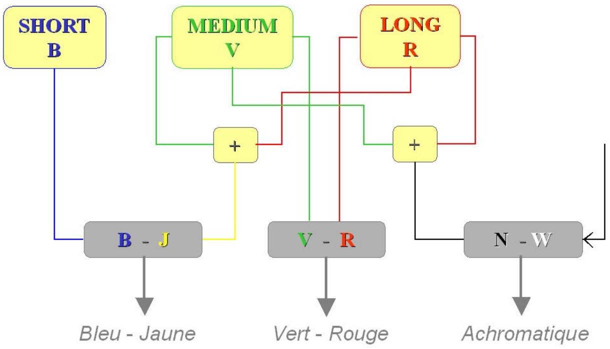

Figure 5 : Representation of retinal "signal processing" for color vision.

The analogy presented here does not describe the whole physiological process of color vision but is just a simplified result of these processes. In particular the notion of channel is certainly too simple and has to be refined. Color information processing is today described by the **opponent color code theory** based on many experimental results [4] [5]

B. Perception of color mixtures

As already stated before, the aim of colorimetry is primarily to quantify the color of light sources or objects from **visual color matches**, meaning that the eye of the observer is used as a tool able to estimate whether two colored stimuli are identical or not.

1. Metamerism

Metamerism

Take two different colored surfaces: they can appear to have exactly the same "color" under daylight natural illumination for instance, and appear to have different colors when illuminated by an incandescent or fluorescent light bulb. This means that distinct spectral compositions incident onto the eye can induce the same color perception: this is called **metamerism**.

Two different diffuse reflection spectra may then appear to be visually identical under a given condition of illumination, and be visually different under other illumination conditions. This may occur when pigments or dyes used to give the color to an object are not the same. This may also occur when a given pigment is coated onto different substrate materials, as it is the case for instance in colored car bodies when metallic, plastic or synthetic resin pieces are used.

Metamerism is mostly seen for dark blue or green, as well as for brown.

2. Distinction between color mixtures

One has to make a difference between **additive color mixing** , which is the mixing of different colored light beams, and **subtractive color mixing**, which deals with the removal of a given part of the incident light spectrum by, for instance, colored pigments.

a) Additive color mixing

Addition of colored light beams is performed onto the retina. It can be obtained by three different ways:

1. By **superposition** over the same retinal area of colored light beams, scattered onto the observed object towards the eye of the observer (think for instance of lighting spots in a theater stage)

2. By **juxtaposition** of colored zones on very close retinal areas, separated by less than the spatial resolution of the eye (visual acuity ≈ 1.5 arcminute). This is the case of TV or computer screens pixels, made up of three sub-pixels (Red, Green, Blue) which are not distinguishable at normal viewing distance.

3. By **successive display** of colored beams with a frequency higher than the characteristic cutoff frequency of the human "sensor" (eye+brain). It is for instance the case of the spinning color wheel, composed of different colored segments: it appears white when the spinning speed is high enough.

An additive mixture of (see figure 6) :

- Green and Red gives Yellow (Green + Blue = Cyan ; Red + Blue = Magenta)
- Yellow and Blue gives White (as well as Cyan + Red or Green + Magenta)
- Red, Green and Blue gives White.

Figure 6

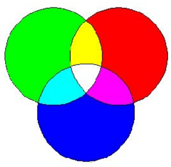

Figure 6 : Additive color mixing

b) Subtractive color mixing

In this case the spectrum of the light is modified by its propagation in a colored medium before entering the eye. One can distinguish two types of subtractive processes:

1. **Absorption and selective transmission of light.** This is the case of a colored filter which absorbs selectively one part of the incident spectrum. This principle is used in color CCD sensors, for instance.

2. **Absorption et selective scattering of light**. It is the case of painting pigments, which absorb selectively one part of the incident spectrum and reflect the other part (a yellow pigment will absorb blue, for instance).

A subtractive mixture of (see figure 7):

- Green pigments (reflecting only intermediate wavelengths, absorbing blue and red) and red pigments (reflecting only long wavelengths) will give black (or in practice, dark brown).
- Yellow pigments (absorbing only short wavelengths, *i.e.* blue) and cyan pigments (absorbing only long wavelengths, *i.e.* red) will give green. Yellow and Magenta will give Red, Cyan and Magenta will give Blue.
- Magenta, Cyan and Yellow pigments will give black.

Figure 7

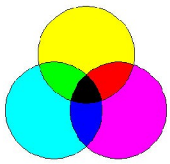

Figure 7 : Subtractive color mixing.

c) Autotypical mixing

In the printing industry, color images are produced using a four-color process with cyan, magenta, yellow and black inks: halftone dots are sized depending on the desired color tone. When overprinted, some of the dots corresponding to the individual colors are adjacent to one another, while others partially or entirely overlap. In this case the resulting color is the result of both subtractive mixing and additive mixing, since when looking at an offset-printed item at a normal viewing distance, our eyes are unable to distinguish the individual dots. The combination of additive and subtractive mixing is called autotypical mixing (figure 8).

Figure 8

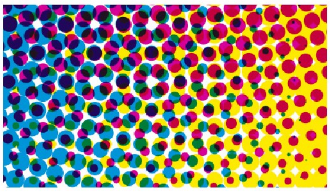

Figure 8 : Autotypical mixing used for offset printing

C. Color order systems

Our eyes are able to see 350.000 different colors. In order to classify in a simple way all these colors, it is necessary to find an objective chart in which the eye itself – with its defaults or imperfections which may vary from one person to another – is not involved.

There are three different ways of classifying colors:

- A pure **visual approach**: Chevreul, Munsell, Ostwald color sytems;
- A pure **physical approach**: RVB, CIE XYZ systems;
- A **physical approach**, but corrected by **psychometric data** : CIELAB, CIELUV color

systems.

1. The Munsell system

In this color system, colors are sorted on a cylindrical chart based on three visual attributes:

- **Hue :** associated to the common sense of "color" : tells whether an object is rather red, green, blue...
- **Chroma** : tells whether the color is pure or conversely close to white ("washed out" or pastel).
- **Value (lightness):** an intensity parameter, tells whether the color is dark or bright.

The Munsell system can be represented under the form of a 3-D atlas made up of chips with given values of hue, chroma and value (cf. figure 9)

Figure 9

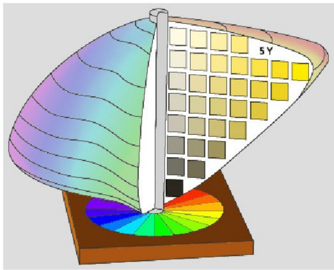

Figure 9 : The 3D arrangement of color chips in the Munsell system.

In addition...

The system is based on 10 hues obtained from the 5 "elementary hues" : B(blue), G(green), Y(yellow), R(red), P(purple). The 10 hues are given by 2 letters (out of the 5 elementary hues), given in this order , with a "5" before to specify that the color is pure, that is: 5B, 5BG, 5G, 5GY, 5Y, 5YR, 5R, 5RP, 5P, 5PB. These hues are divided in 40 evenly-spaced sub-steps (secondary hues). For instance one may go from yellow to red through the following shades: **5Y**, 2.5Y, 10YR, 7.5YR, **5YR**, 2.5YR, 10R, 7.5R, **5R**. The shade 7.5YR is yellower than 5YR and is located at 2.5 steps from the orange shade 5YR and at 7.5 steps from the yellow 5Y: it corresponds to a "yellowish orange" shade.

Warning

Value is plotted on a numerical **axial** scale consistent with the eye's subjective perception of value, going from 0 for black to 10 for white. **Chroma** is plotted along a **numerical radial** scale which is also divided in evenly-spaced steps representing the visual perception of "equal chroma steps" between colors. The *Munsell book of colors,* realized under the form of a collection of colored samples in 1915, has been corrected in 1943 [[6]] in order to be consistent with the international system CIE 1931. Since that time, the Munsell atlas is named the **Munsell renotation system**.

2. Color systems RGB and CIE XYZ

a) A vector space for colors

Because of the trichromatic nature of vision, a color can be perfectly defined by a set of only three numbers, which can be thought of as the coordinates of a vector in a 3-D vector space. More precisely, a color can be represented by a vector whose magnitude is proportional to the light level (value) and whose orientation in space is related to the color tone itself (see figure 10).

Figure 10

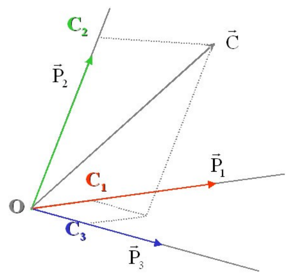

Figure 10 : Vector space used for the representation of colors

In the example given in figure 10, a color $\vec{C}$ is defined by a vector whose coordinates C_1, C_2 and C_3 (called trichromatic coordinates) are measured according to the basis vectors $\vec{P}_1$, $\vec{P}_2$ and $\vec{P}_3$, the “primary colors”. If the primary colors are non-metameric with respect to each other, they form a true basis for the vector space and one can write (1) :

$$\vec{C}=C_1\vec{P}_1+C_2\vec{P}_2+C_3\vec{P}_3 \quad (1)$$

Equation (1) means that one can match the color $\vec{C}$ (*i.e.* find a metameric color of $\vec{C}$) by a mixture of the three primary colors $\vec{P}_1$, $\vec{P}_2$ and $\vec{P}_3$ with the respective weights C_1, C_2 and C_3.

Besides, one can think of any light spectrum $L(\lambda)$ as being the result of an additive mixture of a large number of nearly-monochromatic light beams, adding up with a weighting function $f(\lambda)$ which represents the spectrum, sampled with the delta function δ_λ, so that :

$$L(\lambda)=\sum_\lambda f(\lambda)\delta_\lambda\Delta\lambda \quad (2)$$

If $\Delta\lambda\rightarrow 0$, we see that one can reproduce any polychromatic color $\vec{C}$ by the following linear combination:

$$\vec{C}=\sum_\lambda f(\lambda)\vec{P}_\lambda \quad (3)$$

where $\vec{P}_\lambda$ is a monochromatic primary color source of wavelength λ. Any monochromatic source $\vec{P}_\lambda$ can be expressed in turn as a combination of three non-monochromatic primaries $\vec{P}_1$, $\vec{P}_2$ and $\vec{P}_3$ according to (1) which leads to :

$$\vec{P}_\lambda=c_1(\lambda)\vec{P}_1+c_2(\lambda)\vec{P}_2+c_3(\lambda)\vec{P}_3 \quad (4)$$

where $c_1(\lambda)$, $c_2(\lambda)$ and $c_3(\lambda)$ are the **spectral tristimulus values** which are determined once for all by the CIE from a set of human observers (see below). From (3) and (4) one gets:

$$\vec{C}=\sum_\lambda f(\lambda)c_1(\lambda)\vec{P}_1+\sum_\lambda f(\lambda)c_2(\lambda)\vec{P}_2+\sum_\lambda f(\lambda)c_3(\lambda)\vec{P}_3$$

From (1), one gets the components C_1, C_2 and C_3 :

Once the spectral tristimulus values $c_1(\lambda)$, $c_2(\lambda)$ and $c_3(\lambda)$ are known, any colored stimulus with a spectrum $f(\lambda)$ can be represented by a dot in a 3D space with coordinates C_1, C_2 and C_3.

$$\begin{cases} C_1 = \sum_{\lambda} c_1(\lambda) f(\lambda) \\ C_2 = \sum_{\lambda} c_2(\lambda) f(\lambda) \\ C_3 = \sum_{\lambda} c_3(\lambda) f(\lambda) \end{cases} \quad (5)$$

b) Measurement of spectral tristimulus values

An estimate of $c_1(\lambda)$, $c_2(\lambda)$ and $c_3(\lambda)$ was performed in 1926 by J. Guild [[7]] with 7 people with a limited visual field of 2°, so that the light was only incident on the central part of the retina, the macula (refer to the section "Basic mechanisms of color vision"). Primary colors were chosen to be monochromatic, at 700nm for red ($\vec{R}$), 546,1nm for green ($\vec{V}$) and 435,8nm for blue ($\vec{B}$). The observation field was divided into two sections (see figure 11), one of them was illuminated by a monochromatic source, while the other received a combination of the three primary sources, which superimposed by additive mixing on the eye of the observer. The observer could tune the light level of the primary sources (by changing the position of the sources or by directly varying the intensity through calibrated apertures) until **matching of visual perceptions** was reached.

Figure 11

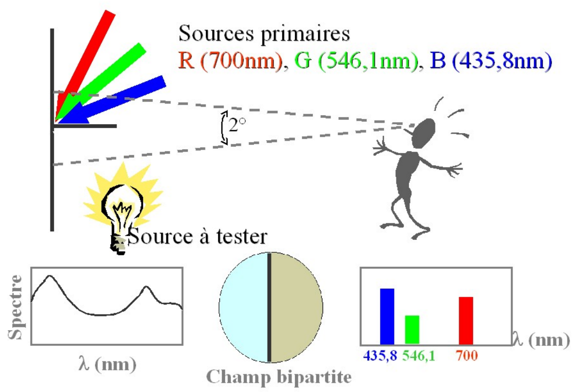

Figure 11: Principle of the measurement of trichromatic coordinates of a source.

When the "test source" to be matched for this experiment is a monochromatic source of wavelength λ, the trichromatic components obtained are called the **RGB tristimulus values** $\overline{r}(\lambda)$, $\overline{g}(\lambda)$ and $\overline{b}(\lambda)$. They were tabulated in 1931, so that this system is sometimes called RGB CIE 1931 system. They are plotted versus wavelength in figure 12.

Figure 12

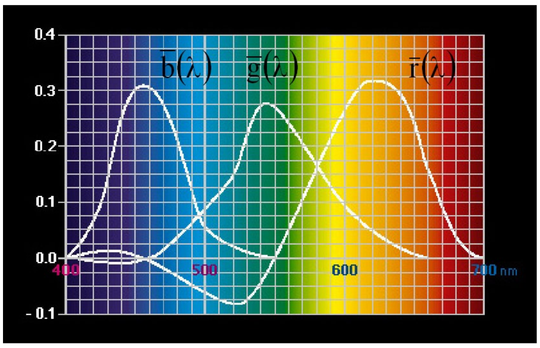

Figure 12: the RGB spectral tristimulus values

The Y scale is defined so that $\int \overline{r(\lambda)} d\lambda = \int \overline{g(\lambda)} d\lambda = \int \overline{b(\lambda)} d\lambda$.

This choice is justified by the fact that the visual perception of a neutral stimulus (white or gray color) corresponds to a balanced response of the three different cones. It has to be noticed that for wavelengths below 550nm, the color functions $\overline{g(\lambda)}$ and $\overline{r(\lambda)}$ take negative values. This is because it is impossible to match a monochromatic color below 550 nm by *adding* three monochromatic primaries. Therefore, in the setup shown in figure 11, matching of color perceptions in the two halves of the observation field would be obtained for some colors if one *adds* some light from one of the primaries to the "test light".

c) Graphical representation – Color triangle

From (5), any polychromatic radiation with a luminance $L_c(\lambda)$ is associated with a color $\vec{C}=R_c\vec{R}+G_c\vec{G}+B_c\vec{B}$ where the trichromatic coordinates in the RGB color system are given by:

$$\begin{cases} R_c=\int L_c(\lambda)\overline{r(\lambda)}d\lambda \\ G_c=\int L_c(\lambda)\overline{g(\lambda)}d\lambda \\ B_c=\int L_c(\lambda)\overline{b(\lambda)}d\lambda \end{cases} \quad (6)$$

Since only the relative values of these coordinates have a physical meaning in terms of chromaticity, it is usual to introduce the trichromatic coordinates r, g and b given by:

$$\begin{cases} r=\dfrac{R_c}{R_c+G_c+B_c} \\ g=\dfrac{G_c}{R_c+G_c+B_c} \\ b=\dfrac{B_c}{R_c+G_c+B_c} \end{cases} \quad (7)$$

In practice, it is easier to represent colors in a plane rather than in the 3-D space. We have, from (7):

$$r+g+b=1 \quad (8)$$

which is the equation of a plane defined by the 3 terminal points of the unit vectors $\vec{R}$, $\vec{G}$ and $\vec{B}$ (see figure 13).

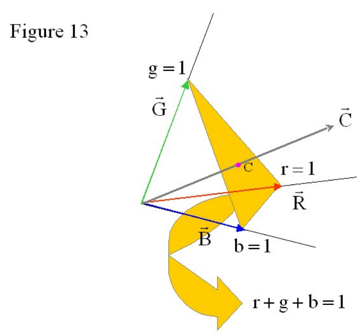

Figure 13: the color triangle

A color $\vec{C}$ is then represented by a point (C) in that plane. Since the component b can be straightforwardly derived from (8) provided that r and g are known, one generally prefers to display the plane $r+g+b=1$ under the form of a right isosceles triangle in which r and g lie along the right axes of the triangle.

The diagram obtained is called the **(r, g) chromaticity diagram**, displayed in figure 14. Monochromatic radiations (for $\lambda \in [380\text{nm}; 780\text{nm}]$) lie along the so-called **spectrum locus.** They correspond to the following coordinates (with $L_c(\lambda)=\delta_\lambda$) :

$$\begin{cases} r=\dfrac{\bar{r}(\lambda)}{\bar{r}(\lambda)+\bar{g}(\lambda)+\bar{b}(\lambda)} \\ g=\dfrac{\bar{g}(\lambda)}{\bar{r}(\lambda)+\bar{g}(\lambda)+\bar{b}(\lambda)} \\ b=\dfrac{\bar{b}(\lambda)}{\bar{r}(\lambda)+\bar{g}(\lambda)+\bar{b}(\lambda)} \end{cases} \quad (9)$$

At the three corners of the triangle appear the (monochromatic) primary colors red ($\vec{R}$; λ =700nm), green ($\vec{V}$; λ =546,1nm) and blue ($\vec{B}$; λ =435,8nm). The line connecting the red and blue primaries is called the **purple line.**

Figure 14

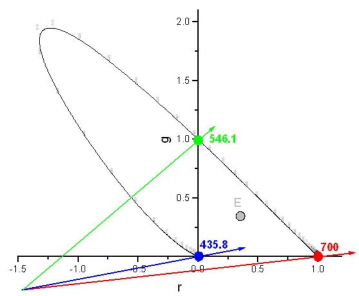

Figure 14: The (r, g) chromaticity diagram

All existing colors have their associated points inside the domain delimited by the spectrum locus and the purple line. Points outside of this area have no physical meaning and do not correspond to real stimuli.

d) XYZ color space

As it can be seen on figure 14, many colors have negative trichromatic coordinates in the RGB system, as a result of the choice of three monochromatic primary colors. In order to eliminate this drawback, a linear transformation over the RBG space is made, leading to the tristimulus values $\overline{x(\lambda)}$, $\overline{y(\lambda)}$ and $\overline{z(\lambda)}$ which take only positive values. Moreover, the primaries $\vec{X}$, $\vec{Y}$ and $\vec{Z}$ (no longer monochromatic) have been chosen in such a way that the plane $O\vec{X}\vec{Y}$ corresponds to a zero-luminance plane, and that the axis $O\vec{Y}$ becomes the axis of visual luminances. Indeed, for a stimulus having a luminance $L_e(\lambda)$ in energetic units, we know that the luminance in visual units $L_v(\lambda)$ is:

$$L_v(\lambda)=K_m\int L_e(\lambda).V(\lambda).d\lambda=Y$$

with $K_m=683\,lm/W$ (in photopic vision), where $V(\lambda)$ is the luminous efficiency function (refer to a basic photometry course to learn more about $V(\lambda)$). Using our usual trichromatic notations, we see that if we define the Y coordinate as $Y=K_m\int L_e(\lambda).\overline{y(\lambda)}.d\lambda$ then $\overline{y(\lambda)}$ is identical to $V(\lambda)$, so that one of the trichromatic coordinates will give directly the visual luminance.

The linear transformation $[\overline{r(\lambda)};\overline{g(\lambda)};\overline{b(\lambda)}]\rightarrow[\overline{x(\lambda)};\overline{y(\lambda)};\overline{z(\lambda)}]$ is defined by [[8]] :

$$\begin{cases}\overline{x(\lambda)}=2{,}7688\,\overline{r(\lambda)}+1{,}7517\,\overline{g(\lambda)}+1{,}1301\,\overline{b(\lambda)}\\ \overline{y(\lambda)}=\overline{r(\lambda)}+4{,}5907\,\overline{g(\lambda)}+0{,}0601\,\overline{b(\lambda)}\\ \overline{z(\lambda)}=0{,}00001\,\overline{r(\lambda)}+0{,}0565\,\overline{g(\lambda)}+5{,}5942\,\overline{b(\lambda)}\end{cases} \quad (10)$$

which gives the tristimulus values vs. wavelength shown in figure 15.

Figure 15

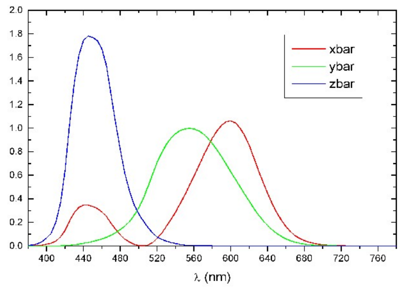

Figure 15: tristimulus values xyz CIE 1931

A color is represented by a vector whose trichromatic coordinates X, Y, Z are:

$$\begin{cases}X=K_m\int L_e(\lambda).\overline{x(\lambda)}.d\lambda\\ Y=K_m\int L_e(\lambda).\overline{y(\lambda)}.d\lambda\\ Z=K_m\int L_e(\lambda).\overline{z(\lambda)}.d\lambda\end{cases} \quad (11)$$

from which one can define the trichromatic coordinates x , y , z such as:

$$\begin{cases}x=\dfrac{X}{X+Y+Z}\\ y=\dfrac{Y}{X+Y+Z}\\ z=\dfrac{Z}{X+Y+Z}\end{cases} \quad (12)$$

Since $x+y+z=1$, colors are represented in a plane (xy) whose axes x and y form right angles. Monochromatic colors ($\lambda\in[380\text{nm};780\text{nm}]$) which have coordinates given by

$$\begin{cases}x=\dfrac{\bar{x}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)}\\ y=\dfrac{\bar{y}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)}\\ z=\dfrac{\bar{z}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)}\end{cases} \quad (13)$$

lie along the spectrum locus. As for the RGB system, the surface inside which one may find all real physical colors is delimited by the spectrum locus and by the purple line. This surface is embedded within a circumscribing triangle whose corners X, Y, Z represent the three primaries:

- the primary $X: x=1 ; y=0$

- the primary $Y : x=0 ; y=1$
- the primary $Z : x=0 ; y=0 \Rightarrow z=1$

These primaries are non real since they are outside of the chromatic gamut (cf. figure 16).

Figure 16

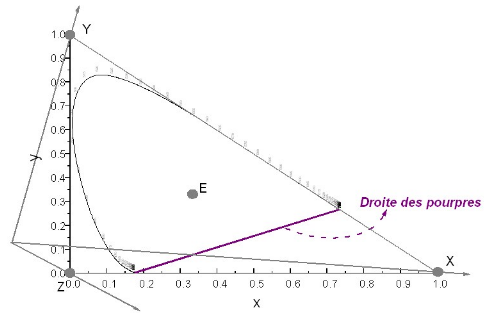

Figure 16: Chromaticity diagram xy CIE 1931

Figure 17 shows the chromaticity diagram xy CIE 1931 with the corresponding color names for an observer in daylight conditions [[9]]. The central zone around the position of white (equienergetic stimulus) corresponds to the CIE normalized illuminants.

Figure 17

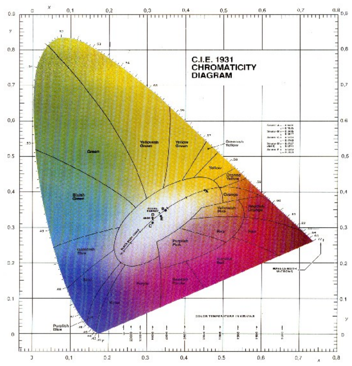

Figure 17: Colored zones of the xy CIE1931 chromaticity diagram

e) Equienergetic stimulus

By definition, the spectrum of an "equienergetic stimulus" E (perfect white) is constant over the whole visible spectrum, that is $L_E(\lambda)=L_E=cste$, or

$$\begin{cases} R_E=L_E\int \overline{r(\lambda)}d\lambda \\ G_E=L_E\int \overline{g(\lambda)}d\lambda \\ B_E=L_E\int \overline{b(\lambda)}d\lambda \end{cases}$$

and

$$r_E=g_E=b_E=1/3$$

since

$$\int \overline{r(\lambda)}d\lambda=\int \overline{g(\lambda)}d\lambda=\int \overline{b(\lambda)}d\lambda$$

This is valid in the RGB system, but it remains true in the X, Y, Z color system, in which we may write $x_E = y_E = z_E = 1/3$. The representative point of stimulus E is located at the center of the RGB or XYZ triangle (see figures 14 and 16).

f) Dominant color – complementary color

Within the (xy) diagram, a color $\vec{C}$ resulting from the additive mixing of two colors $\vec{C}_1$ and $\vec{C}_2$, such as $\vec{C} = \vec{C}_1 + \vec{C}_2$ will result in three representative points C, C_1 and C_2 aligned along the same line (see figure 18).

Figure 18

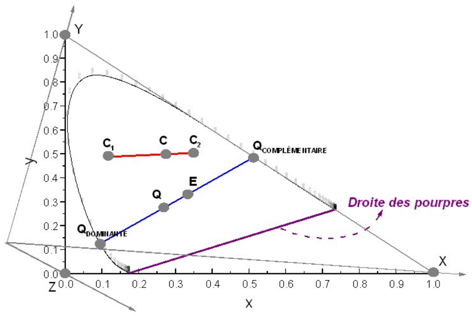

Figure 18: Positions of dominant and complementary colors

In a similar fashion, any color $\vec{Q}$ can be thought of as an additive mixing of the white equienergetic stimulus $\vec{E}$ with a pure color lying along the spectrum locus. The latter is called the **dominant color** $\vec{Q}_{DOMINANT}$ with $\vec{Q} = \vec{Q}_{DOMINANT} + \vec{E}$.

Conversely, the white equienergetic stimulus $\vec{E}$ could theoretically be obtained upon additive mixing of any color $\vec{Q}$ with a pure color called the **complementary color** $\vec{Q}_{COMPLEMENTARY}$ such as $\vec{E} = \vec{Q} + \vec{Q}_{COMPLEMENTARY}$.

The ratio $\frac{\overline{QE}}{\overline{Q_{DOMINANT}E}}$ is a measure of the purity of the color $\vec{Q}$. This ratio is equal to 0 for the equienergetic white and is equal to 1 for a pure monochromatic color.

3. Systems CIELAB and CIELUV

The eye's ability to separate two distinct colors depends of the hue, saturation and brightness of the observed color. As a consequence, the sensitivity to color differences will vary from one region to another within the CIExy diagram. Many studies have tried to study these variations: Mac Adam [[10]] defined 25 points, spread out inside the (x,y) diagram, around which he drew ellipses with different sizes and orientations: all the colors inside an ellipse appear to be matched with the central color of the ellipse for a standard observer (see figure 19).

In the (xy) diagram, colors which appear to be different “by the same amount” are represented in the diagram as dots separated by very different distances depending on the color hue, which means that the (xy) space is not uniform. Transformations of the color space CIE XYZ 1931 have been proposed in order to obtain a better correlation between the “color difference” (as perceived by an observer) and the length separating two representative points.

Figure 19

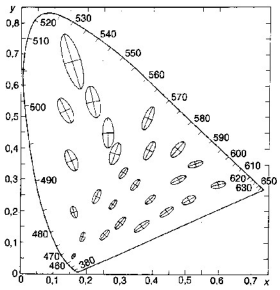

Figure 19 : Minimum discriminable color differences (Mac Adam's discrimination ellipses, magnified 10x)

a) Chromatic space L*u*v* 1976 (CIELUV)

In 1976 the *Commission Internationale de l'Eclairage* (CIE) has defined a nearly uniform chromatic space by defining two coordinates $u*$ and $v*$ such as:

$$\begin{cases} u* = 13\, L*(u' - u_n') \\ v* = 13\, L*(v' - v_n') \end{cases} \quad (14)$$

with

$$\begin{cases} u' = \dfrac{4x}{(-2x + 12y + 3)} \\ v' = \dfrac{9y}{(-2x + 12y + 3)} \end{cases}$$

$L*$ is the CIE 1976 lightness [[11]] defined by $L* = 116(Y/Y_n)^{1/3} - 16$ if $L* \geqslant 8$ or $L* = 903{,}3\ Y/Y_n$ if $L* \leqslant 8$.

u_n', v_n' and Y_n stand for the values of u', v' and Y for a given illuminant (see examples in table1):

Illuminant	u_n'	v_n'	Y_n
A	255,97	524,29	100
C	200,89	460,89	100
D_{65}	197,83	468,34	100

Table 1 : Values of u', v' and Y for the most widely used normalized illuminants.

The CIE recommends using the CIELUV color space for the characterization of color displays.

b) Chromatic space L*a*b* 1976 (CIELAB)

The CIELAB space is obtained by representing in a Cartesian coordinates system the parameters $a*$ and $b*$ defined by:

$$\begin{cases} a* = 500[(X/X_n)^{1/3} - (Y/Y_n)^{1/3}] \\ b* = 200[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}] \end{cases} \quad (15)$$

with

$$\begin{cases} X/X_n \\ Y/Y_n > 0{,}008856 \\ Z/Z_n \end{cases}$$

and L^* has the same definition as in the CIELUV color space.

X_n, Y_n and Z_n are the values of X, Y and Z corresponding to the illuminant, as tabulated in table 2:

Illuminant	X_n	Y_n	Z_n
A	109,85	100	35,58
C	98,07	100	118,23
D_{65}	95,04	100	108,88

Table 2 : Values of X, v' and Y for the most widely used normalized illuminants.

The CIE recommends using the CIELAB color space for the characterization of colored surfaces and dyes.

D. Normalized Illuminants

The color of a non-self-emitting object is meaningful only if some light coming from a light source is sent onto the object, which will scatter, reflect or transmit the light towards the eye of the observer. This means that defining the color of an object means specifying the illuminant as well: it is then of utmost importance to be able to specify precisely which illuminant is used when attempting to quantify the color of an object.

The simplest illuminant would certainly be the equienergetic white source (see § C.2.e. "Equienergetic stimulus") since its spectral distribution is constant throughout the whole visible spectrum. However, such an ideal source does not exist in practice. In cases where an object is illuminated by artificial incandescent light, the CIE recommends using the **illuminant A** which corresponds to a blackbody with a **color temperature** of 2856 K. This illuminant is realized with an incandescent tungsten light bulb. The **illuminant C**, close to daylight, can be realized by combining the illuminant A with a liquid filter, in order to achieve a **correlated color temperature** of 6774K. The **normalized illuminants D50, D55, D65, D75** represent different phases of daylight, corresponding to correlated color temperatures of 5004, 5504, 6504 and 7504K, respectively. The spectral composition of the main normalized illuminants is shown in figure 20.

Figure 20

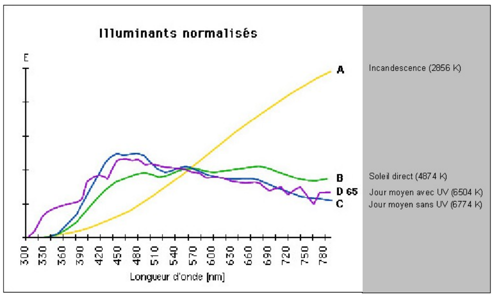

Figure 20: Emission spectra of normalized illuminants.

For illuminants, or more generally for any self-emitting object, it is useful to introduce the relative energetic spectral distribution $S(\lambda)$ in such a way that the trichromatic coordinates X_n, Y_n and Z_n of the illuminant are:

$$\begin{cases} X_n = k \int S(\lambda)\bar{x}(\lambda)d\lambda \\ Y_n = k \int S(\lambda)\bar{y}(\lambda)d\lambda \\ Z_n = k \int S(\lambda)\bar{z}(\lambda)d\lambda \end{cases} \quad (16)$$

or, if $S(\lambda)$ is tabulated with a sampling pitch $\Delta\lambda$:

$$\begin{cases} X_n = k \sum_{\lambda} S(\lambda)\bar{x}(\lambda)\Delta\lambda \\ Y_n = k \sum_{\lambda} S(\lambda)\bar{y}(\lambda)\Delta\lambda \\ Z_n = k \sum_{\lambda} S(\lambda)\bar{z}(\lambda)\Delta\lambda \end{cases} \quad (17)$$

The scaling factor k is then defined by:

$$k = \frac{100}{\sum_{\lambda} S(\lambda)\bar{y}(\lambda)\Delta\lambda} \quad (18)$$

so that the Y_n component of the reference illuminant is equal to 100. Under these conditions, the X_n and Z_n components are the ones that are reported in table 2. If Y_n is to be equal to the visual luminance $L_v(\lambda)$ and if $S(\lambda)=L_e(\lambda)$ then k must be equal to $K_m=683\, lm/W$.

1. Color differences

The parameters L*u*v* (or L*a*b*) of the CIELUV (CIELAB) color spaces may be represented on 3 orthogonal axes. The case of the CIELAB space is shown in figure 21.

Figure 21

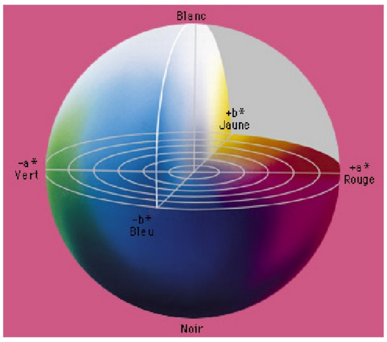

Figure 21: CIELAB color space

In these color spaces, a colored stimulus may be specified either by its cartesian coordinates L*u*v* (or L*a*b*) or by its cylindrical coordinates (see figure 22) :

- **The hue angle** h (h=0° for the red hue and +a* (or +u*) direction ; h=90° pour the yellow hue and +b* (or +v*) direction; h=180° for the green hue and -a* (or -u*) direction; h=270° for the blue hue and -b* (or -v*) direction.)
- **The chroma** C^* or **saturation** s (iso-chroma lines C^*_{ab} (or C^*_{uv}) are concentric circles centered at the origin a*=b*=0 (or u*=v*=0) corresponding to the white reference stimulus).

- **The lightness** L* (défined in § I.C.3.a).

Figure 22

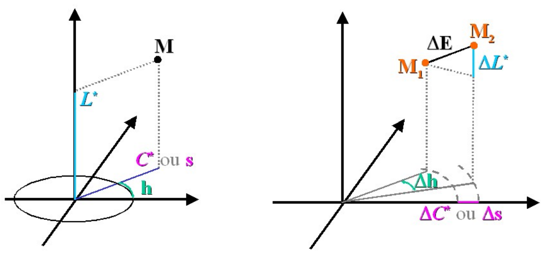

Figure 22: Polar coordinates in CIE 1976 systems

From figure 22, and given two stimuli M_1 and M_2, having coordinates in the CIELUV (CIELAB) system L*u*v* (L*a*b*, respectively), the differences ΔC^*, Δs, Δh, ΔH^* and ΔE^* are defined by the following relations (see table 3):

	Symbol	CIELUV	CIELAB
Hue angle	h	$\arctan(v^*/u^*)$	$\arctan(b^*/a^*)$
Chroma	$C*$	$\sqrt{u^{*2}+v^{*2}}$	$\sqrt{a^{*2}+b^{*2}}$
Saturation	s	C^*_{uv}/L^*	-
Hue angle difference	Δh	h_1-h_2	h_1-h_2
Chroma difference	ΔC^*	$C_1^*-C_2^*$	$C_1^*-C_2^*$
Saturation difference	Δs	s_1-s_2	-
Lightness difference	ΔL^*	$L_1^*-L_2^*$	$L_1^*-L_2^*$
Hue difference	ΔH^*	$\sqrt{(\Delta E*)^2-(\Delta L*)^2-(\Delta C*)^2}$	$\sqrt{(\Delta E*)^2-(\Delta L*)^2-(\Delta C*)^2}$
COLOR difference	ΔE^*	$\sqrt{(\Delta L*)^2+(\Delta u*)^2+(\Delta v*)^2}$	$\sqrt{(\Delta L*)^2+(\Delta a*)^2+(\Delta b*)^2}$

Table 3: Formulas for computing color and hue differences

The color difference ΔE^* between the two stimuli can be thought of as being composed of a difference of lightness ΔL^*, a difference in chroma ΔC^* and a difference in hue ΔH^*.

In the graphic arts and printing industry, the maximum acceptable color difference in the CIELAB space is:

- ΔE^* =1,5 for most demanding works (reproduction of works of art...),
- ΔE^* =2,0 for magazines,
- ΔE^* =2,5 for newspapers.

E. Colorimetry and instrumentation

A color is defined by the threefold combination “source-object-observer.”

- Source: reference illuminant with a relative spectral energetic distribution $S(\lambda)$.
- Object: in general opaque, with a diffuse reflectance $R(\lambda)$.
- Observer: normalized by the CIE in 1931 (vision field of 2°) and later in 1964 (vision field of 10°) with the colorimetric functions $\overline{x(\lambda)}$, $\overline{y(\lambda)}$ and $\overline{z(\lambda)}$.

This leads to the following trichromatic components X, Y and Z:

$$\begin{cases} X = k \sum_{\lambda} S(\lambda) \overline{x}(\lambda) R(\lambda) \Delta\lambda \\ Y = k \sum_{\lambda} S(\lambda) \overline{y}(\lambda) R(\lambda) \Delta\lambda \\ Z = k \sum_{\lambda} S(\lambda) \overline{z}(\lambda) R(\lambda) \Delta\lambda \end{cases} \quad (19)$$

By convention the scaling factor k is defined by:

$$k = \frac{100}{\sum_{\lambda} S(\lambda) \overline{y}(\lambda) \Delta\lambda} \quad (18)$$

Notwithstanding the "color atlas" that one may use in order to visually compare colors, all colorimetry systems based on photoelectric measurements are based on the calculation of the trichromatic components X, Y and Z, derived from (19).

The main manufacturers of colorimetric measurement systems are Hunterlab and Gretag Macbeth. Other companies are now commercializing such systems, like Minolta, Zeiss or X-rit. Datacolor International is specialized in color control tools for textiles.

1. Visual color matches

In many companies, an extremely simple way to control color is still being used, which consists in seeking the best visual match among a set of printed colored samples. The use of such color atlases has two important drawbacks: 1) it is not easy to guarantee that the different editions of the same atlas will be perfectly identical; 2) the colored samples have to be compared with a perfectly well-defined illuminant, since metamerism depends on the spectral composition of the light source.

2. Colorimeters with tri-stimuli filters

Two kinds of such colorimeters can be found:

- Colorimeters made of one single detector and a filter wheel. The latter comprises three filters, whose transmission spectra are equal to the colorimetric functions $\overline{x(\lambda)}$, $\overline{y(\lambda)}$ and $\overline{z(\lambda)}$. By spinning the filter wheel, the three filters are then periodically set in front of the detector.
- Colorimeters made of three different photodetectors, working simultaneously, having spectral sensitivities equal to the colorimetric tristimulus functions.

Figure 23

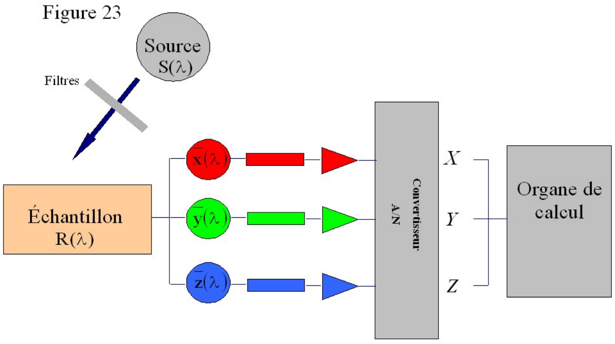

Figure 23: Principle of a colorimeter based on tristimuli filters

In both cases, the trichromatic components of a sample are measured after calibration with a reference source whose spectrum is close to the spectrum of a normalized illuminant. These colorimeters are in general less accurate than those based on a monochromator (see below).

3. Spectro-photocolorimeters

They combine a monochromator with a normalized light source. For each wavelength, the light reflected by the sample is compared to the light reflected by a reference white surface under the same conditions of lighting and observation (see figure 24). One can then determine the (diffuse) reflectance spectrum $R(\lambda)$ of the sample.

Figure 24

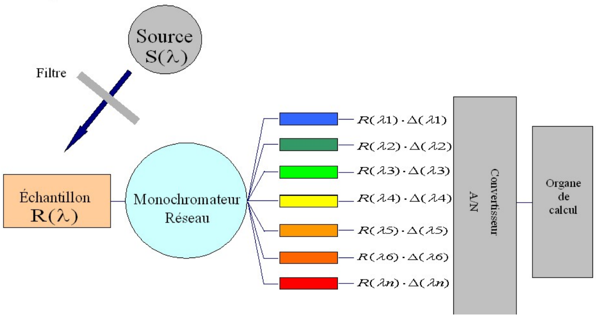

Figure 24: Principle of a spectrocolorimeter.

Two types of spectrocolorimeters can be found, according to the type of photodetector used:

- A single detector is capturing the light dispersed by the monochromator: in this case the the diffraction grating of the monochromator is rotated thanks to a precision stepper motor.
- A photodiode array is set at the exit of the monochromator in order to acquire the spectrum in one single step (Optical Multichannel Analyzer).

Furthermore, different measurement heads can be coupled to the monochromator:

- For diffusive materials (either in reflection or in transmission), the samples (the sample to be measured and the reference sample) is directly fixed onto the measurement head, which is connected to the monochromator either directly or through an optical fiber.
- For self-emitting sources (LCD or CRT displays for instance), the measurement head is independent of the monochromator, and connected to the latter with an optical fiber.

Etude de Cas

II

A. Color mixing from the R,G,B channels of a TV screen

If the relative spectral distributions, or emission spectra, $S_R(\lambda)$, $S_G(\lambda)$ and $S_B(\lambda)$ of the three R,G,B sub-pixels of a TV screen are known, one can write the trichromatic components (X, Y, Z) of the color aR+bG+cB obtained upon additive mixing of the three channels R,G,B (see equation (17)):

$$\begin{cases} X = k \sum_{\lambda} [a S_R(\lambda) + b S_G(\lambda) + c S_B(\lambda)] \bar{x}(\lambda) \Delta\lambda \\ Y = k \sum_{\lambda} [a S_R(\lambda) + b S_G(\lambda) + c S_B(\lambda)] \bar{y}(\lambda) \Delta\lambda \\ Z = k \sum_{\lambda} [a S_R(\lambda) + b S_G(\lambda) + c S_B(\lambda)] \bar{z}(\lambda) \Delta\lambda \end{cases}$$

with (see equation (18))

$$k = \frac{100}{\sum_{\lambda} S(\lambda) \bar{y}(\lambda) \Delta\lambda}$$

where $S(\lambda)$ is the relative spectral distribution of the reference white light, which is in practice one of the normalized illuminants, *e.g.* D_{65}. a, b, c are constants ranging from 0 to 1, proportional to the radiant flux emitted (or transmitted) by each sub-pixel R,G,B of the screen. Let's tune the color of the display in order to obtain a white tone as close as possible of the reference illuminant D_{65}, that is:

$$S(\lambda) = a S_R(\lambda) + b S_G(\lambda) + c S_B(\lambda)$$

If the color mixture consists of equal amounts of each subpixel, then $a = b = c = p_0$ and $S(\lambda) = p_0 [S_R(\lambda) + S_G(\lambda) + S_B(\lambda)]$. We have then reproduced a white light with maximum luminance ($Y = 100$) for $p_0 = 1$.

Exercices

III

A. Questions

Answer the following questions:

Question 1

[Solution n°1 p 29]

Consider 2 stimulus $\vec{Q}_0$ and $\vec{Q}_1$ defined by their trichromatic coordinates (x_0, y_0) and (x_1, y_1) as well as by their luminance Y_0 and Y_1 , respectively. Give the vectorial expression of the stimulus $\vec{Q}$ obtained through additive mixing of the 2 stimulus $\vec{Q}_0$ and $\vec{Q}_1$.

Question 2

[Solution n°2 p 29]

For a stimulus $\vec{Q}$ with trichromatic coordinates (x, y) and luminance Y, we define Σ as $\Sigma = X + Y + Z$. Show that Σ can also be written $\Sigma = Y / y$

Question 3

[Solution n°3 p 29]

We choose as the stimulus $\vec{Q}_0$ the reference illuminant D_{65}. From table 1, give the trichromatic coordinates (x_0, y_0) of this stimulus.

Illuminant	X_n	Y_n	Z_n
A	109,85	100	35,58
C	98,07	100	118,23
D_{65}	95,04	100	108,88

Table 1 : Trichromatic coordinates of normalized illuminants.

Question 4

[Solution n°4 p 30]

Which polychromatic source is close to the stimulus $\vec{Q}_0$?

Question 5

[Solution n°5 p 30]

The stimulus $\vec{Q}_1$ is monochromatic at 585nm. From the portion of the CIE 1964 table displayed below, give the trichromatic coordinates (x_1, y_1) of this stimulus.

λ (nm)	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$
580	1,01	0,87	0
585	1,07	0,83	0
590	1,12	0,78	0

Tableau 2 : Tristimulus values.

Question 6

[Solution n°6 p 30]

Show that the stimulus $\vec{Q}=\vec{Q}_0+\vec{Q}_1$ has the following coordinates :

$$\begin{cases} x=\dfrac{\Sigma_0 x_0+\Sigma_1 x_1}{\Sigma_0+\Sigma_1} \\ y=\dfrac{Y_0+Y_1}{\Sigma_0+\Sigma_1} \\ Y=Y_0+Y_1 \end{cases}$$

Question 7

[Solution n°7 p 31]

Calculate the numerical values of x , y and Y for $Y_0=14{,}5$ and $Y_1=30$.

Question 8

[Solution n°8 p 32]

From table 1, which illuminant is closer to the stimulus $\vec{Q}$?

Question 9

[Solution n°9 p 32]

Calculate the purity p of the color $\vec{Q}$.

Answers to exercises

> Solution n°1 (exercice p. 27)

$$\vec{Q}=\vec{Q}_0+\vec{Q}_1$$

> Solution n°2 (exercice p. 27)

The (x, y, z) trichromatic coordinates are defined by:

$$\begin{cases} x=\dfrac{X}{X+Y+Z}=\dfrac{X}{\Sigma} \\ y=\dfrac{Y}{X+Y+Z}=\dfrac{Y}{\Sigma} \\ z=\dfrac{Z}{X+Y+Z}=\dfrac{Z}{\Sigma} \end{cases}$$

or

$$\Sigma=Y/y$$

> Solution n°3 (exercice p. 27)

From table I, we have:

$$\begin{cases} X_0=95{,}04 \\ Y_0=100 \\ Z_0=108{,}89 \end{cases}$$

or

$$\Sigma_0=303{,}93$$

which leads to:

$$\begin{cases} x_0=0{,}3127 \\ y_0=0{,}3290 \\ z_0=0{,}3583 \end{cases}$$

> Solution n°4 (exercice p. 27)

The trichromatic coordinates (x_0, y_0, z_0) indicate that the stimulus is close to the equienergetic white, whose coordinates are $x_0 = y_0 = z_0 = 0{,}33$.

> Solution n°5 (exercice p. 27)

For a pure (monochromatic) color, the trichromatic coordinates (x, y, z) are given by:

$$\begin{cases} x = \dfrac{\bar{x}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)} \\ y = \dfrac{\bar{y}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)} \\ z = \dfrac{\bar{z}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)} \end{cases}$$

hence: $x_1 = 0{,}5654$, $y_1 = 0{,}4346$, $z_1 = 0{,}0000$

> Solution n°6 (exercice p. 28)

For an additive mixture of two colors $\vec{Q}_0$ et $\vec{Q}_1$ whose trichromatic coordinates are (X_0, Y_0, Z_0) and (X_1, Y_1, Z_1) respectively, the components of the resulting stimulus $\vec{Q}$ are:

$$\begin{cases} X = X_0 + X_1 \\ Y = Y_0 + Y_1 \\ Z = Z_0 + Z_1 \end{cases}$$

Since the trichromatic coordinates are:

$$\begin{cases} x_0 = \dfrac{X_0}{\Sigma_0} \\ y_0 = \dfrac{Y_0}{\Sigma_0} \\ z_0 = \dfrac{Z_0}{\Sigma_0} \end{cases}$$

for stimulus $\vec{Q}_0$ and

for stimulis $\vec{Q}_1$

Then for stimulus $\vec{Q}$ we have

$$\begin{cases} x_1 = \dfrac{X_1}{\Sigma_1} \\ y_1 = \dfrac{Y_1}{\Sigma_1} \\ z_1 = \dfrac{Z_1}{\Sigma_1} \end{cases}$$

With

$$\Sigma = X + Y + Z = (X_0 + Y_0 + Z_0) + (X_1 + Y_1 + Z_1) = \Sigma_0 + \Sigma_1$$

which leads to:

$$\begin{cases} x = \dfrac{X_0 + X_1}{\Sigma_0 + \Sigma_1} \\ y = \dfrac{Y_0 + Y_1}{\Sigma_0 + \Sigma_1} \\ z = \dfrac{Z_0 + Z_1}{\Sigma_0 + \Sigma_1} \end{cases}$$

or

$$\begin{cases} x = \dfrac{\Sigma_0 x_0 + \Sigma_1 x_1}{\Sigma_0 + \Sigma_1} \\ y = \dfrac{Y_0 + Y_1}{\Sigma_0 + \Sigma_1} \end{cases}$$

And, by definition:

$$Y = Y_0 + Y_1$$

> Solution n°7 *(exercice p. 28)*

From the expressions derived in question 6, we may write:

$$\begin{cases} x = \dfrac{x_0 Y_0 / y_0 + x_1 Y_1 / y_1}{Y_0 / y_0 + Y_1 / y_1} \\ y = \dfrac{Y_0 + Y_1}{Y_0 / y_0 + Y_1 / y_1} \end{cases}$$

Besides we know that:

$$\begin{cases} x_0 = 0{,}3127 \\ y_0 = 0{,}3290 \end{cases}$$

(from question 3) and

$$\begin{cases} x_1 = 0{,}5654 \\ y_1 = 0{,}4346 \end{cases}$$

(from question 5)
then we can finally write:

$$\begin{cases} x = 0{,}4669 \\ y = 0{,}3935 \end{cases}$$

> Solution n°8 *(exercice p. 28)*

From table 1, the trichromatic coordinates of the A, C et D_{65} illuminants are:

Illuminant	x_n	y_n	z_n
A	0,45	0,41	0,15
C	0,31	0,32	0,37
D_{65}	0,31	0,33	0,36

Tableau 3 : Trichromatic coordinates of normalized illuminants.

In the plane (x, y) the distances $\overline{QA}$, $\overline{QC}$ et $\overline{QD}_{65}$ are:

$$\begin{cases} \overline{QA} = \sqrt{(x_A - x)^2 + (y_A - y)^2} \\ \overline{QC} = \sqrt{(x_C - x)^2 + (y_C - y)^2} \\ \overline{QD}_{65} = \sqrt{(x_{D_{65}} - x)^2 + (y_{D_{65}} - y)^2} \end{cases}$$

Then $\overline{QA} = 0{,}024$; $\overline{QC} = 0{,}175$; $\overline{QC}_{65} = 0{,}167$
Therefore, the stimulus $\vec{Q}$ whose trichromatic coordinates are:

$$\begin{cases} x = 0{,}4669 \\ y = 0{,}3935 \end{cases}$$

is closer to the A illuminant.

> Solution n°9 *(exercice p. 28)*

The purity is defined by:

$$p = \frac{\overline{QE}}{\overline{Q_{DOMINANT}E}}$$

With the equienergetic stimulus $\vec{E}$ defined by its coordinates equal to:

$$\begin{cases} x_E = 0{,}3333 \\ y_E = 0{,}3333 \end{cases}$$

and

$$\overline{QE}=\sqrt{(x_E-x)^2+(y_E-y)^2}$$

In this case,

$$\vec{Q}_{DOMINANT}=\vec{Q}_1$$

so:

$$\overline{Q_1 E}=\sqrt{(x_E-x_1)^2+(y_E-y_1)^2}$$

Here $\overline{Q_1}E=0{,}2532$ and $\overline{Q_E}=0{,}1465$ so that $p=0{,}58$

We may also calculate the purity p by considering that $\vec{Q}=\vec{Q}_0+\vec{Q}_1$ with $\vec{Q}_0\simeq\vec{E}$ and then

$$p=\frac{\overline{QQ_0}}{\overline{Q_1 Q_0}}$$

with $\overline{Q_1 Q_0}=0{,}2739$ and $\overline{Q\,Q_0}=0{,}1671$.

this leads to $p=0{,}61$.

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