Filtering in optics

GEORGES BOUDEBS

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I.Présentation

Module :

Interférences and diffraction

Auteur(s) :

Georges BOUDEBS - Maître de Conférences HDR - Laboratoire de Photonique d'Angers (LPHIA | UPRES EA 4464)

Résumé :

First of all, we review the used systems to spatially create the optical waves as well as an example illustrating the techniques capable to modify the luminous transmission in real time, by an optical or electronic command. Then, and in the second part of this grain, we will address the general domain of the treatment of the information and more particularly that achieved by an optical means. Diverse applications are proposed: Zernicke filtration, convolution by an optical path, recognition of forms, matrix-vector multiplication...Such applications rest on the amplitude of the optical systems to submit the general linear transformations to the entry givens.

Mots-clés :

spatial filtration, frequency spectrum, Fourier plan, convolution by an optical path

Pré-requis

Fourier Analysis Theory of linear systems Formation of images

Objectif(s) pédagogique(s) :

Show the richness of the modulated optical signal spatially in amplitude as well as in phase. Give the bases concerning the optical treatment of the signal as wells as understanding the used techniques in this domain. Illustrate several examples among the numerous applications of the optical filtration.

Plan du cours :

- Introduction
- Spatial modulation of light
- Spatial filtration and optical treatment of the information
- Convolution without displacement by optical path in incoherence
- Synthesis of a band pass filter in incoherence
- Optical treatment of information in coherent illumination
- The filter of Vander Lugt (1963)
- The joint transform correlator (Weaver et Goodman 1966)
- Multiplication matrix-vecteur
- Blurred photo
- Conclusion

Conception & production :

Le Mans Université

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II.Course

It is clear that the tools of linear systems and the frequent analysis are useful in the comprehension of optical systems destined to the formation of images. However, the study becomes much more interesting if it can be applied to synthesis problems. The production of linear optical systems with specific properties makes the ability of spatially adjusting light necessary. In particular, such an ability is necessary only to present the information (the object) to the entry of an optical system. In addition, for the coherent optical treatment of the information, we need to modify and to manipulate the complex amplitude of the luminous transmitted field in the focal plane of an objective. With the help of such a manipulation we can filter the given at the entry of diverse materials.

1. Spatial modulation of light

The first part of this chapter concerns the traditional methods (still used today) to spatially modulate optical waves in order to know the **photographic film**. The second part is consecrated to **optoelectronic systems**, more powerful in the optical treatment of information. These systems are called “SLM”. They are capable of modifying the luminous transmission in real time by an optical or electronic command.

1.1. Photographic film

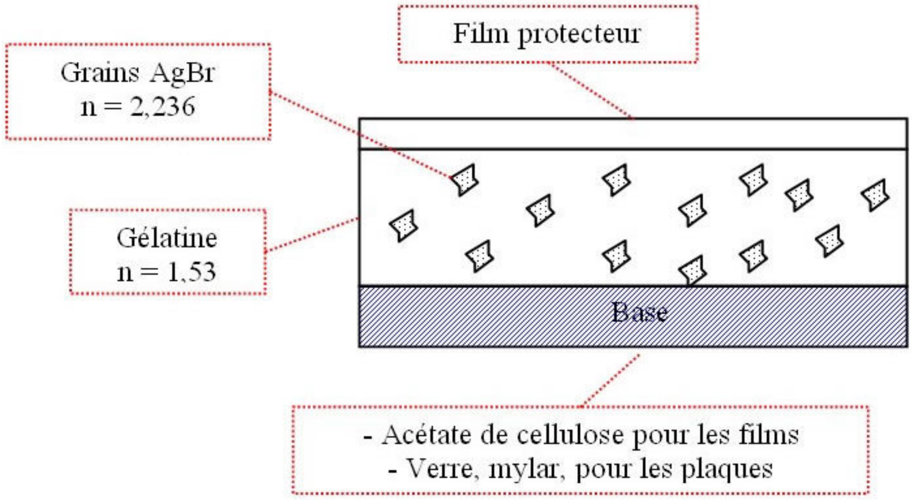
a) Introduction

It plays three essential roles in the treatment process of the information's optic. It can serve as:

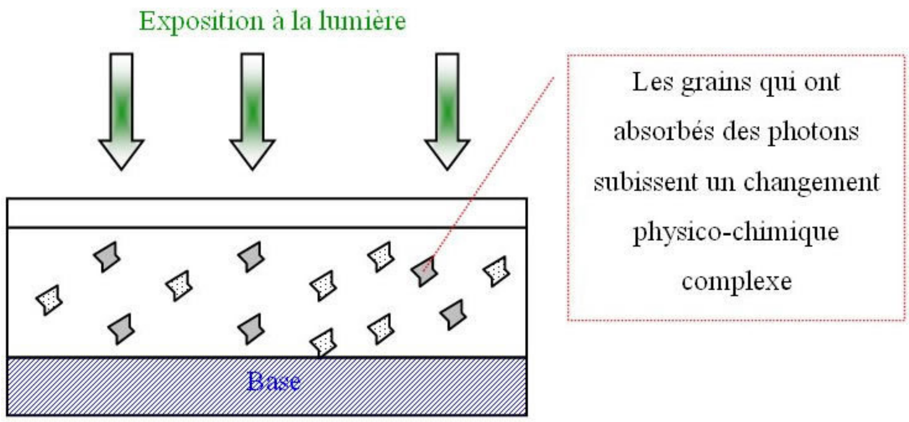
- A medium where the information is introduced in the optical system
- A filter in the plane of frequencies to achieve the necessary extenuations
- A medium to register the information at the exit of the system.

b) Process of registering and characterization

The structure of a black film is shown in the figure I-1.



An important quantity of halogen agent grains is in suspension in a gelatin support. The emulsion of a soft aspect is put between two supports to protect it. When the film is exposed to light, the grains that are absorbed by photons undergo a complex physio-chemical change (see figure I-2).



We are speaking of a registered latent image in the film while waiting for the development and the fixation.

The development is a chemical treatment that transforms the Ag halogen grain into metallic Ag. The grains that have not been irradiated do not transform.

- The fixation is a chemical treatment that eliminates the $AgBr$ remaining while preserving the metallic Ag. The Ag is strongly opaque in the optical frequencies. The opacity of the developed film depends therefore on the density of the silver grains in each region of the snapshot.
- The exposure is defined by $E(x,\ y) = I_e(x,\ y) \cdot T$. It is the incident energy by the unity of the surface : E is in J/m^2, I_e is in W/m^2, T is the duration of the exposure in s.
- The transmittance is defined by :

$$\tau\ (x, y) = \frac{I_{transmis}(x, y)}{I_{incident}(x, y)}$$

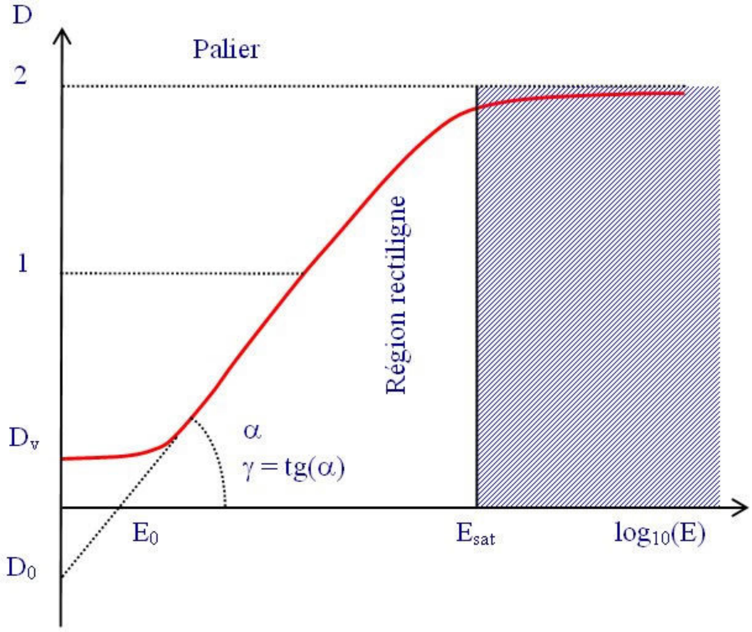
It is a local means, big before the size of the grain, but small before the fine structure of the details in the image.

- The photographic density is defined by :

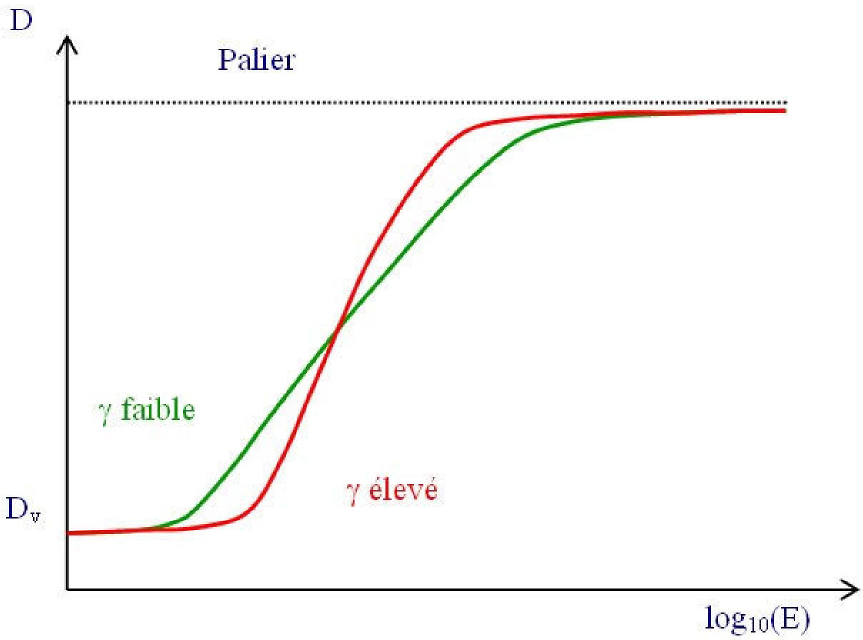
$$D = \log_{10}(1/\tau) \Leftrightarrow \tau = 10^{-D}$$

Hurter and Driffield (1890) show that D is directly proportional to the mass of Ag by the unity of the surface.

- Curve H-D : it is the graph of $D = f[log_{10}(E)]$(see figure I-3)



When $E < E_0 \Rightarrow D = D_v$ (density of the veil). The linear region is generally used where $D = \gamma log_{10}(E) - D_0$.



The γ of the film depends on several parameters: the type of the film, the developing bath used, the time of development...

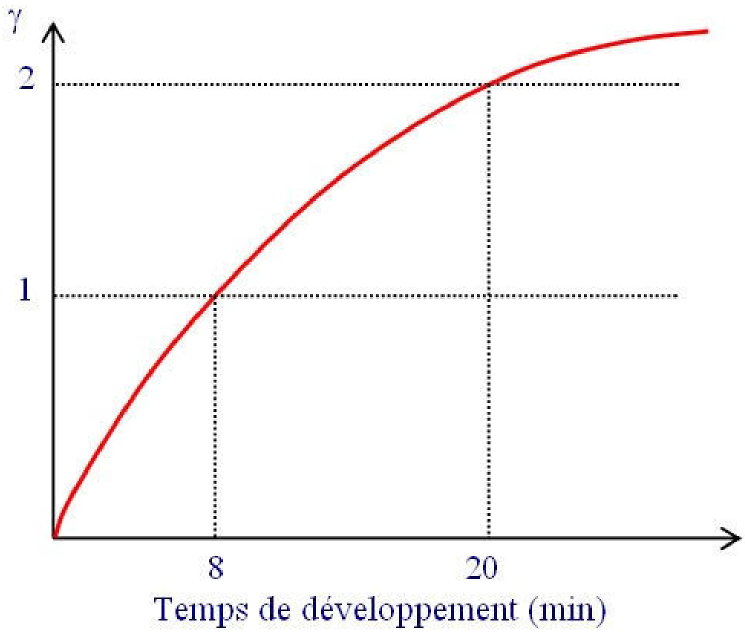


Figure I-5 shows the typical evolution of γ of a negative film in function of time of development.

c) The used film in incoherent optics

It transforms an intensity $I_e(x, y)$ received during the exposure into transmitted intensity after the development.

If the film is used in its linear part, then :

$D = \gamma_n \log_{10}(E) - D_0 = \gamma_n \log_{10}(I_e.T) - D_0$

$I_e$can be linked to the transmitted intensity by the intermediary of the function of the transmittance defined by the preceding paragraph :

$$D = \log_{10}(\frac{1}{\tau_n}) = \gamma_n \log_{10}(I_e.T) - D_0$$

$$\log_{10}(\tau_n) = D_0 - \gamma_n \log_{10}(I_e.T)$$

$$\tau_n = 10^{D_0}.10^{\log_{10}[(I_e.T)^{-\gamma_n}}]$$

$$\tau_n = 10^{D_0}.(I_e.T)^{-\gamma_n} = K_n I_e^{-\gamma_n}$$

$$\tau_n = \frac{I_{tra}}{I_{inc}} = K_n.(I_e)^{-\gamma_n}$$

γ_n and $K_n$being two positive constants, it is seen that the transmitted intensity by the film is not linear in function of I_e. For example, if $\gamma_n = 1 \Rightarrow I_t$.

To obtain a transmitted intensity that is proportional to the intensity of exposure, it is necessary to obtain a positive snapshot from the negative snapshot. For that, a second snapshot from the negative snapshot initially obtained is illuminated. If I_0 is the incident intensity :

$$\tau_n = K_n.(I_e)^{-\gamma_n} = \frac{I_t}{I_0}$$

$$I_t = I_0 \tau_n = I_{exposition\ du\ positif}$$

The transmittance in intensity of the second positive snapshot will be therefore :

$$\tau_p = K'_n.(I_{exposition\ du\ positif})^{-\gamma'_n} = K'_n.(I_0 \tau_n)^{-\gamma'_n}$$

By replacing the transmittance from the negative by its value in this last relationship :

$$\tau_p = K'_n.\{I_0[K_n.(I_e)^{-\gamma_n}]\}^{-\gamma'_n}$$

$\tau_p = K_n'(I_0)^{-\gamma_n}.(K_n)^{-\gamma_n}.(I_e)^{\gamma_n.\gamma_n'}$

$\tau_p = Kp.I_e^{\gamma_p}$

K_p and γ_p are two positive constants. Here γ_p is the resulting gamma. We see that the transmitted intensity by the film is linear in function of I_e. This is made possible by playing with the developing time. We can choose for example : $\gamma_n = 1/2$ et $\gamma_n = 2$.

d) The used film in coherent optics

It transforms the incident intensity during the exposure in complex amplitude transmitted after development. It can also transform the complex incident amplitude during the exposure into a complex amplitude transmitted after development (by using the interferometric methods). In the two cases, the film is characterized by its complex transmittance:

$t(x, y) = \sqrt{\tau(x, y)}e^{j\varphi(x,y)}$

where $\varphi(x, y)$ translates variations of phases that are introduced by the snapshot :

- either by the unpredictable variations of the thickness of the gelatin
- either by the variations of the thickness with the density of the Ag in the developed snapshot.

It is possible to eliminate the effects of these variation of thickness by using an immersion vat filled with oil at a convenient indication.

The transmittance of the vat and of the film can be written :

$t_n(x, y) = \sqrt{\tau_n(x, y)} = \sqrt{K_n}.(\sqrt{I_e})^{-\gamma_n} = K_n|U_e|^{-\gamma_n}$

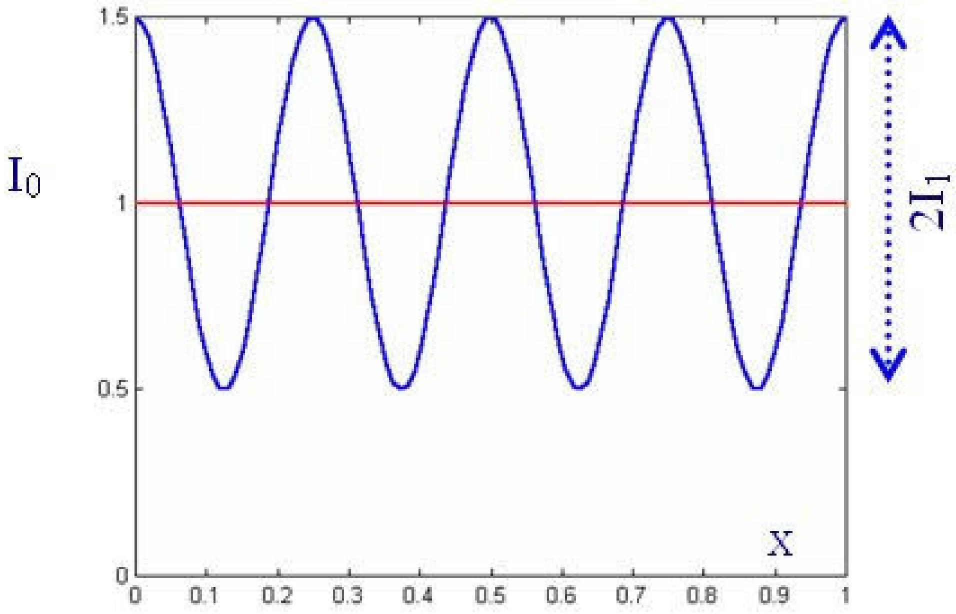
By realizing a positive:

$t_p(x, y) = \sqrt{\tau_p(x, y)} = \sqrt{K_p}.(\sqrt{I_e})^{\gamma_p} = K_p|U_e|^{\gamma_p}$

It is desired in numerous cases that the film transform the amplitude into the square of its module, it suffices to take $\gamma_p = 2 = \gamma_n\gamma_n' = 2.1$

e) The function of transfer of the modulation of the film

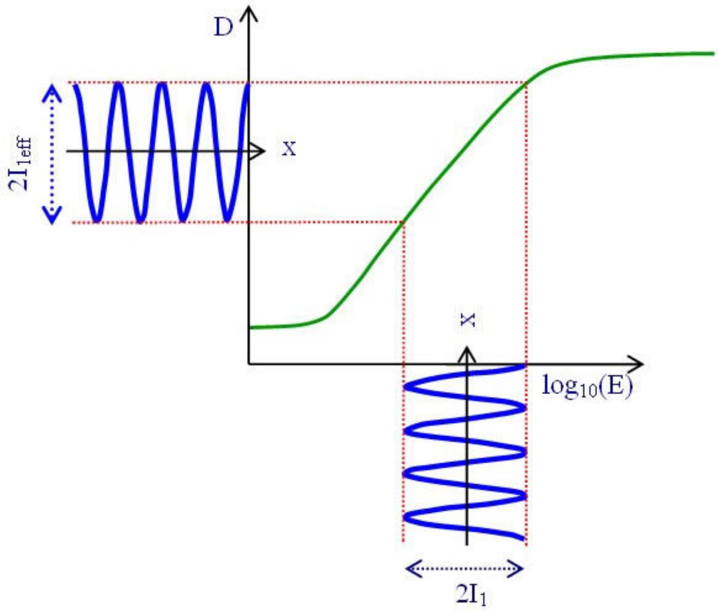
When the spatial period of variations of the luminous intensity becomes too small, it is possible that no other corresponding variation of density appears in the final snapshot. That is a sinusoidal variation of the incident intensity (see figure I-6) :



$I(x) = I_0 + I_1\cos(2\pi f x)$

The rate of modulation of this intensity is defined by : $M_i = \frac{I_1}{I_0}$

We think back to the curve H-D of the film that is known for deducting the sinusoidal distribution of the effective intensity seen in the film (see figure I-7) :



$I_{eff}(x) = I_{0eff} + I_{1eff}\cos(2\pi fx)$

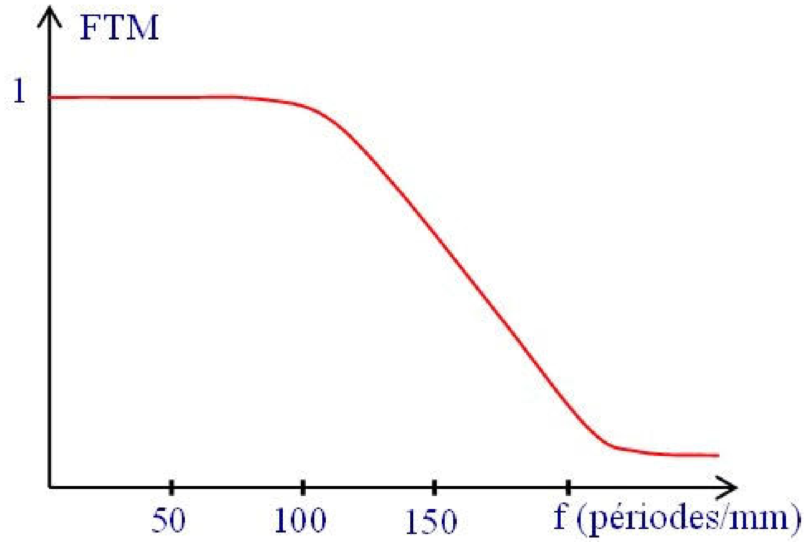
The registered rate of modulation is effectively deduced :

$$M_{eff}(f) = \frac{I_{1eff}}{I_{0eff}}$$

The function of transfer of modulation is defined by :

$$M(f) = \frac{M_{eff}(f)}{M_i(f)}$$

By varying f (the spatial frequency of the incident intensity) this operation can be repeated to determine the variation of the FTM of the film in function of the frequency. Typically we have an allure of a type passing low like the curve shows it in figure I-8.



The frequency of disconnection is variable (between 50 and $2500 mm^{-1}$for example for the photo plaque Kodak 649 F)

f) Whitening of photo snapshots

After the developing and the fixation of the film, a weak variation of thickness in the gelatin is established in the places where the Ag metallic remains. **The process of whitening** consists of removing the metallic Ag by a chemical treatment to keep this thickness variation in place.

Another whitening process is established to replace the metallic Ag of the gelatin by another salt of transparent Ag to the light, but presenting a bigger clue than the environmental gelatin.

The result is a structure of spatial variation of the constituent indication "**a pure phase image**." Differences of the phase can be reached with this technique until 2π and a resolution going to $2500\ mm^{-1}$.

1.2. Spatial light modulators (SLM)

The inconvenient principal of the photo snapshots is the time relatively long necessary for the chemical treatment.

It is preferable to use the electro-optical properties of certain crystals to create or register in real time the given optics that intervene in the system destined to the optical treatment of the signal.

Définition

Two categories of SLM are distinguished :

- SLM at electric addressing, used if the information is collected by the optoelectronic components (photo diodes, camera CCD, numeric simulation)
- SLM at optical addressing, used if the information is under the optical form (exit from a video monitor, exit from no matter what imaging system).

In all the cases, by definition, the exit (of the SLM component) is always optical. For ample information concerning this subject, the reader can consult the reference [[Spatial light modulator technology]]

a) Usages of the SLM

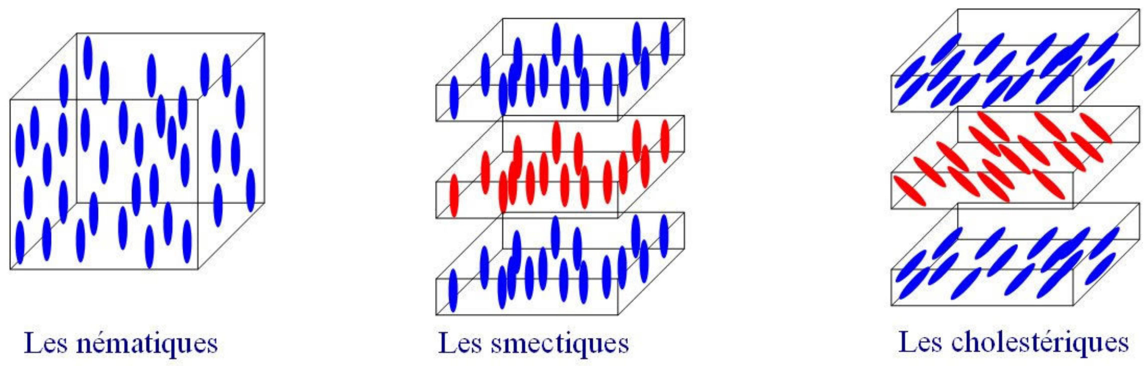
At the origin, the spatial modulators of light were developed for a usage in optical processors such as :

1. Converting an incoherent image into a coherent image
2. Amplifying a weak image
3. Converting the wavelengths (pass from the I.R. into the visible)
4. Modifying the spatial filter used in the Fourier plane (spectral)
5. ...

But, it is the development of the modulators for big public applications like video projectors that permitted the research work on the optical processors of recognition of forms that we know today.

b) Properties of liquid crystals

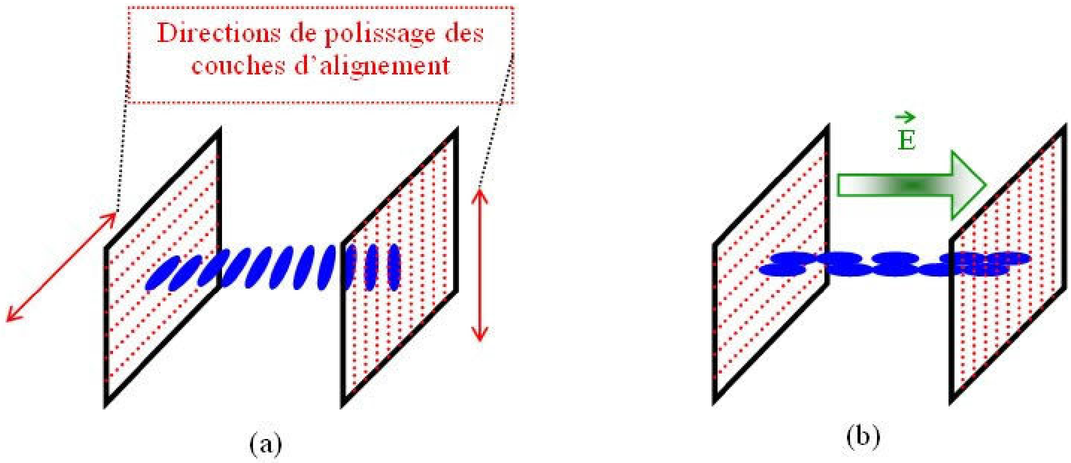
The use of liquid crystals is widespread (digital display, screens...). The applied tension to the electrodes provokes a variation in the intensity of the transmitted or applied light by the display unit. The liquid crystals can be seen as composed of ellipsoidal molecules. These molecules regroup between those of different means forming two classes (or phases) of liquid crystals: nematic, smectic, and the cholesteric (chiral).



The SLM principally use the nematic crystals (NLC) and a special class of smectic (C*) called the "Ferroelectric Liquid Cristal" (FLC).

Properties of the NLC :

It is possible to impose limiting conditions to orient nematic crystal liquids by polishing the surfaces of the layers of alignment in a given direction. The axis of the molecules in contact with the inner wall have a tendency to align with the little scratches of polishing at the level of the surface. To keep a continuity at the level of alignment of the axis, we can apply a torsion to the cell like is shown in figure I-10(a). By applying an electric field, a electric dipole is introduced in each molecule. The big axis of the molecule (or the dipole appears) aligns itself with the electric field (figure I-10(b)).



Les propriétés optiques des NLC

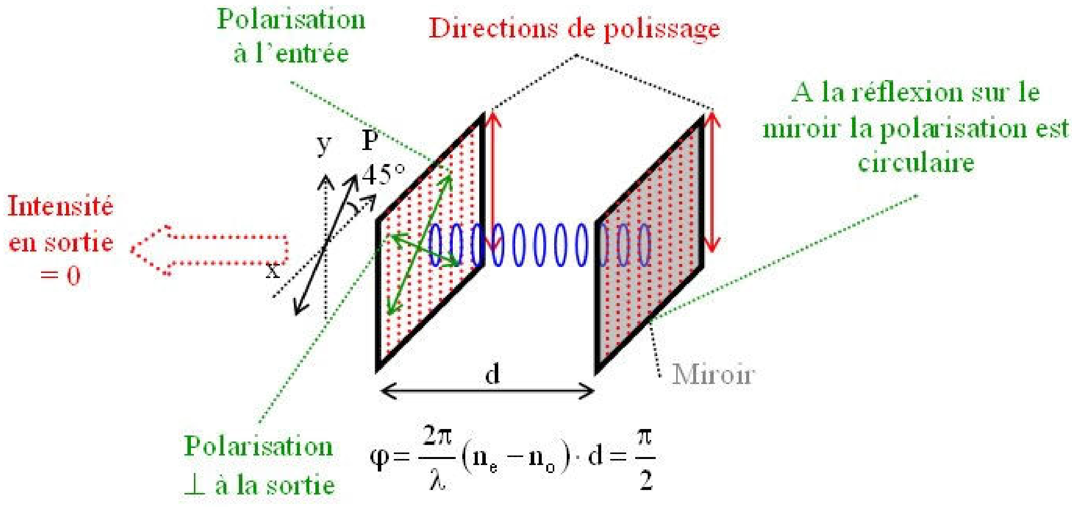
- The elongated molecules provoke an anisotropy inducing an important birefringence. The variation of indication is elevated which permits us to have thicknesses at the level of the relatively weak cells $\Delta n = n_e - n_0 = 0.2$ ($n_e$following the axis of the molecule and n_0 perpendicular to the axis)
- If the molecules are disposed in a helical way (figure I-10 (a)) we have an important rotary power.

By combining these two properties, one can see modulations of light intensity.

Exemple

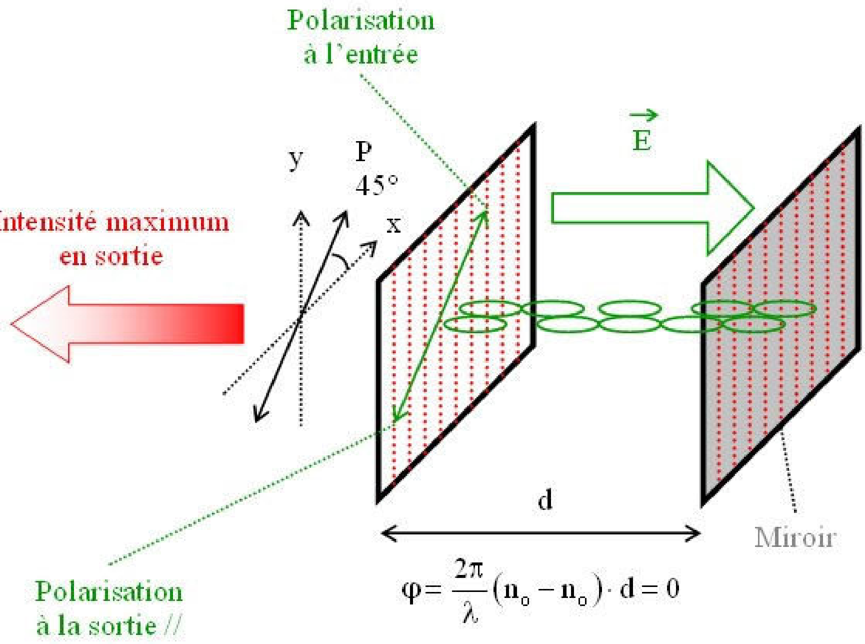
Example of function :

In the absence of an electric field (figure I-11), the polarized light at 45 on the axis xOy meets the axis of the molecule oriented in the vertical.



It sees therefore the big as well as the small axis of the molecule. The components of the luminous field following x and y undergo a different phase difference (following $n_e$and n_o). At the level of the mirror at the back of the cell, the light will be polarized circularly if one arranges it so that the phase difference φ is equal to $\pi/2$. This is achieved by choosing *d* a thickness of suitable cell. After the round trip (reflection on the mirror) the light will be polarized rectangularly and oriented at 90 of the incident light. The light meets the polarizer P placed at the entrance in a crossed position. The intensity will be minimal at the exit of the cell.

On the other hand and in the presence of a sufficient electric field, the molecule will be aligned with the applied electric field. This time, the components on the x and y of the light meet the small axis of the molecule (see figure I-12). There is no phase difference between the components of the luminous field. The light is polarized rectangularly by reflecting itself on the mirror and stays parallel to itself arriving at the polarizer P after the round trip. The exiting luminous intensity is maximal.



If the electric field is not sufficient to align all the molecules in the cell, a partial reflection results.

Typical technical characteristics :

- Tension to apply : $5\text{–}10\ V$
- Thickness of the cells : $1 - 10\ \mu m$
- Response time of alignment of the molecules : $50 - 100\ \mu s$
- Response time of relaxation of the molecules : $20\ ms$

The number of pixels or of cells in a SLM (typically 600×800, commonly called and at high resolution) can vary depending on the expected application.

Remarque

It is necessary to note that even though spatial modulators of light have been developed for use in optical processors, it is the development of modulators for application like video projectors that permitted the numerous research on the optical processors of recognition of forms over the last 15 years.

2. Spatial filtration and optical treatment of the information

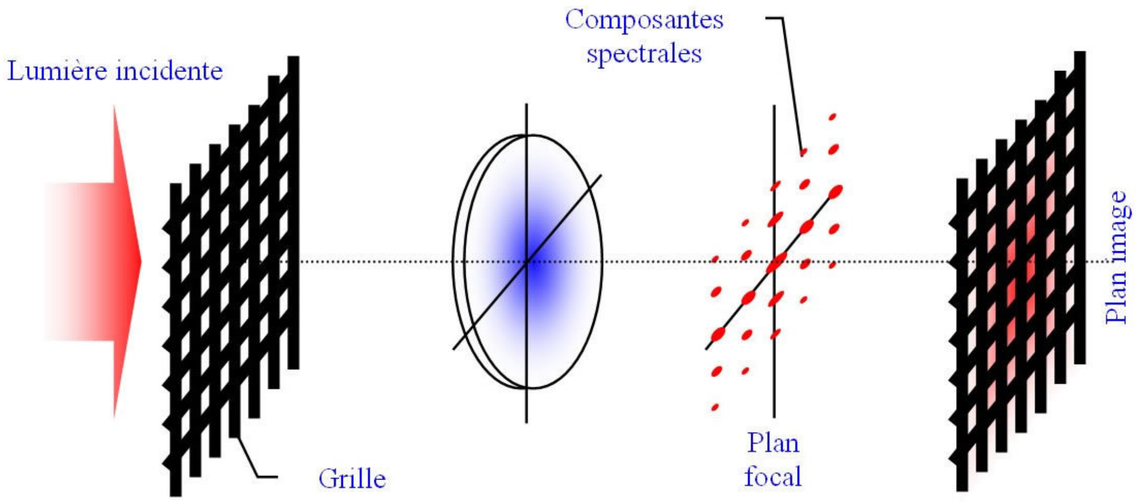
Important applications exist that are not in the domain of the formation of images strictly stated, but these applications reveal more of the general domain of the **treatment of information**. Such applications lie on the aptitude of the optical systems to subject the general linear transformations to those given at entry. In certain cases, the simultaneous treatment of a grand number of givens can, because of this number, multiply the efficiency of the human observer. A linear transformation can play a crucial role in the reduction of a large quantity of givens giving information on certain particular points of information that interest the observer. One finds an example of this type of application in the study of the recognition of characters. In other cases, a group of givens can appear under a form that a human observer cannot directly use, even though a linear transformation of these givens can make them useful.

We will only expose several general principles by choosing examples to illustrate them. We will omit inevitably a certain number of interesting and very useful techniques; the interested reader can consult the references [[OpticalProcessing of Information]],[[Optical and Electro-optical Information Processing]] to have a more complete view of the subject. We limit ourselves here to several general principles, largely useful.

For more details, the books [[Introduction to Fourier Optics]],[[Optical signal processing]], [[Optical informations processing-Fundamentals]],[[Optical signal Processing]]

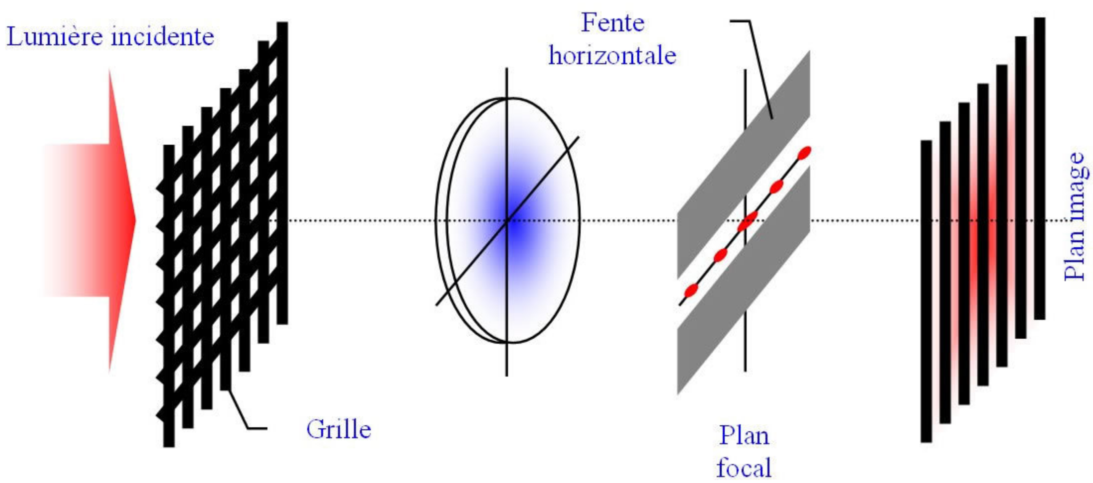
2.1. The experiences of Abbe and Porter

In the figure II-1, one can see the montage used by Abbe (1893) and Porter (1906) to achieve their experiments.

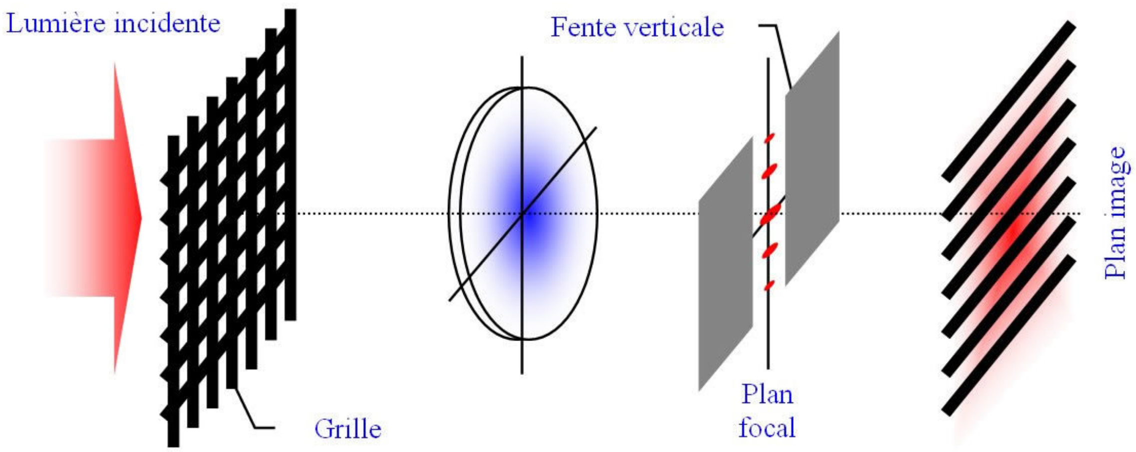


A grid is placed in the object plane of a convergent lens. In the focal plane, one re finds the spectrum of the grid (see the grain “formation of images”, part: “properties of the relative lenses at the Fourier transformation”). The spectrally made formations spread from the focal plane toward the image plane, interfering between them, to form an image that is an attenuated replica of the object. We are going to neglect the function of the optical transfer. By placing several obstacles (diaphragms, slits, screens) in the focal plane, it is possible to modify the spectrum (the image) in different ways.

By putting a slit horizontally (figure II-2), the image only contains the vertical structure of the grid.



By returning the slit to the vertical direction (figure II-3), the image only contains the horizontal structure of the grid.



2.2. The microscope at a phase contrast (F. Zernike 1935)

For example, a transparent object (bacteria). When the light crosses the object, there is an effect of phase variation $(\varphi(x,\ y))$. This effect is not visible at the exit of a classic microscope.

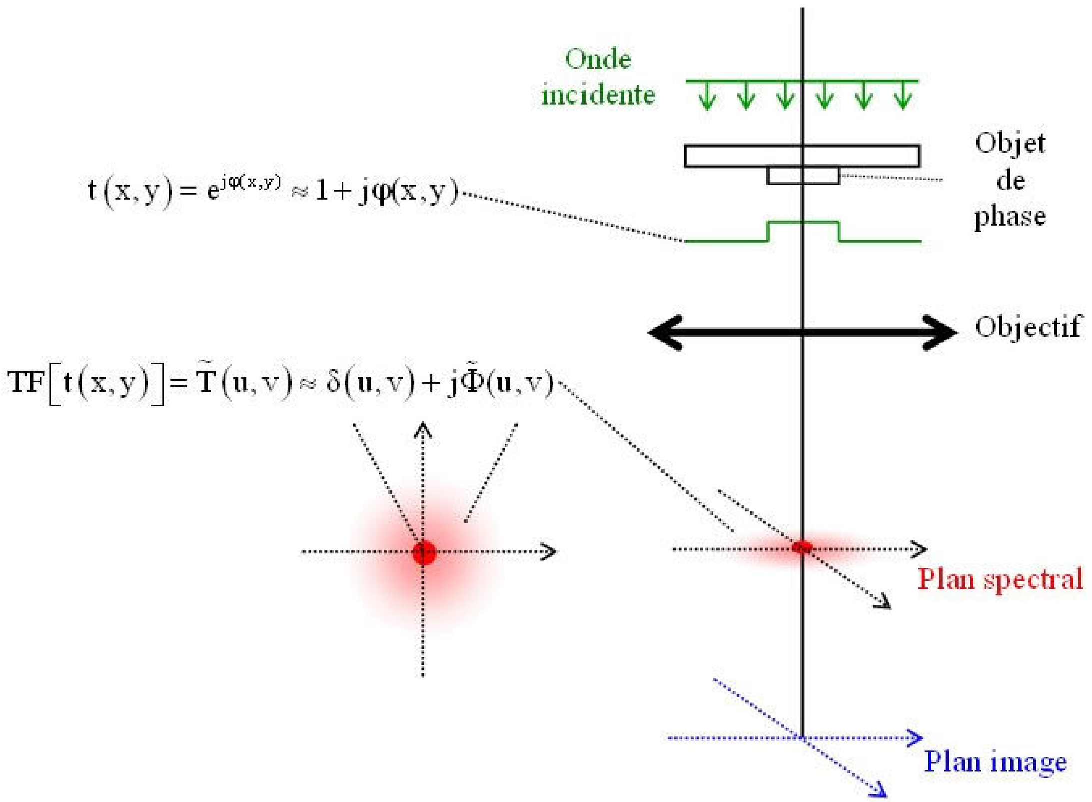
Let's suppose an entry pupil of a infinite image system. That is an incident wave, plane and monochromatic $U_i = 1$, so the transmitted wave is $U_t = exp[j\varphi(x,\ y)]$ whose intensity is $I_t = |exp[j\varphi(x,\ y)]|^2$. The transmitted intensity is therefore monotonous and always constant. There are no spatial variations of the intensity at the exit therefore the object is invisible.

The phase contrast rests on the spatial filtration.

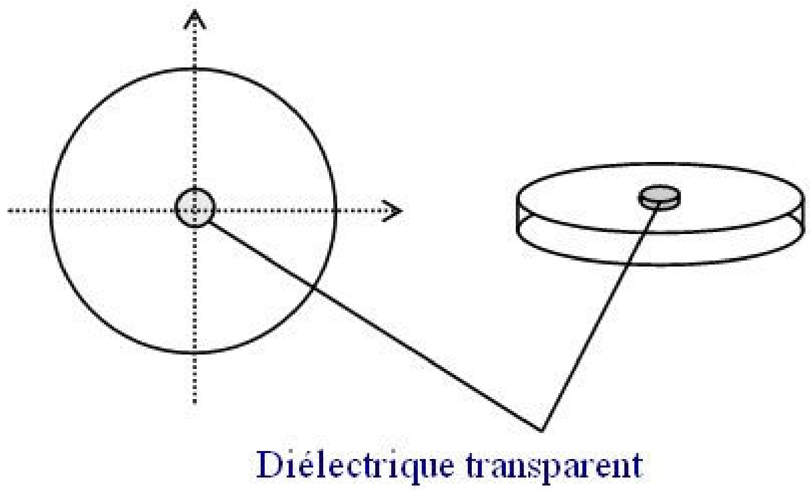
This is a technique that has the advantage of linearly relinking the variation of the introduced phase by the object at the observed intensity. The necessary hypothesis at this linearity is : $\varphi(x,\ y) \ll 1$.

The transmission of the object is : $t(x,\ y) = e^{j\varphi(x,\ y)} \sim 1 + j\varphi(x,\ y)$

The wave after crossing the object possesses a continual background (the 1) that will be focused at the center in the focal plane of the objective. It is the term in dark red representing $\delta(u,\ v)$ on the figure II-4. Even though $\varphi(x,\ y)$ whose spectrum is $\widetilde{\phi}$, its spectral components are much less intense (in light red on the figure). $\widetilde{\phi}$ is diffracted far from the center because of the high spatial frequencies contained in the object (the bacteria possess a different indication in the environment in which they bathe, the bacteria is also of a relatively smaller dimension than the illuminated field).



The spatial filtration consists to make the continuous background change phases (the δ in the spectrum) of $\pi/2$ or $-\pi/2$ in relation to the diffracted wave. For that, a transparent dielectric is placed at the center of a thin blade (figure II-5).



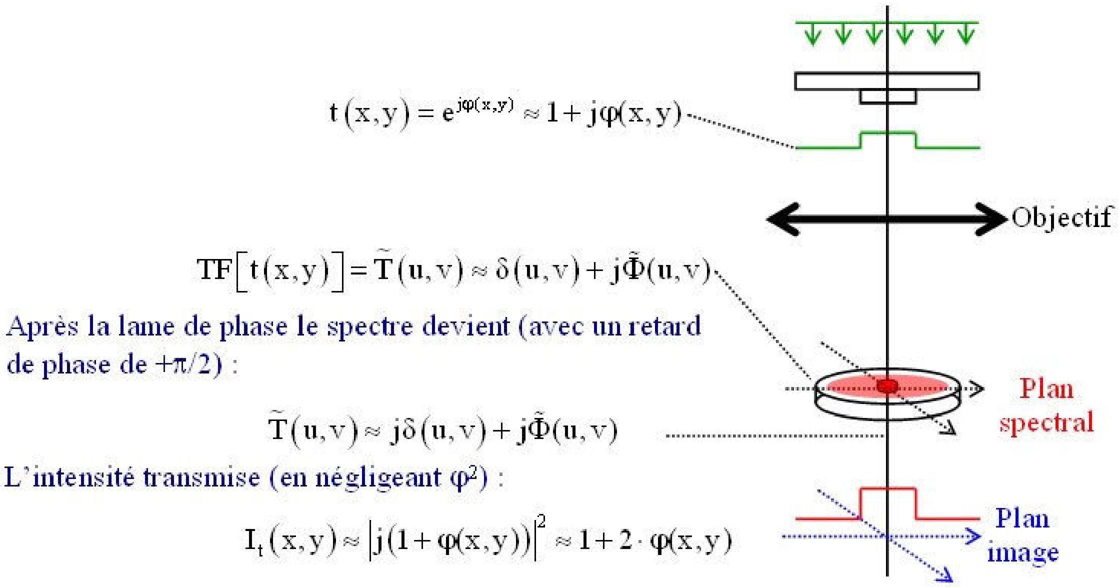
Its thickness and index are calculated in order to produce a delay of phase of $+\pi/2$. The spectrum after crossing this device :

$\tilde{T}\ (u,v) \approx j\ \delta\ (u,v) + j\ \tilde{\Phi}\ (u,v)$

(that comes back to multiply the $\delta(u,\ v)$ by $j = exp(j\pi/2)$.

The transmitted intensity (neglecting φ^2) then becomes :

$I_t\ (x,y) \approx |j\ (1 + \varphi\ (x,y))|^2 \approx 1 + 2.\varphi\ (x,y)$



If the phase delay is of $+3\pi/2$ (one multiplies by $-j = exp(j3\pi/2)$) :

$I_t\,(x, y) \approx |j\,(-1 + \varphi\,(x, y))|^2 \approx 1 - 2.\varphi\,(x, y)$

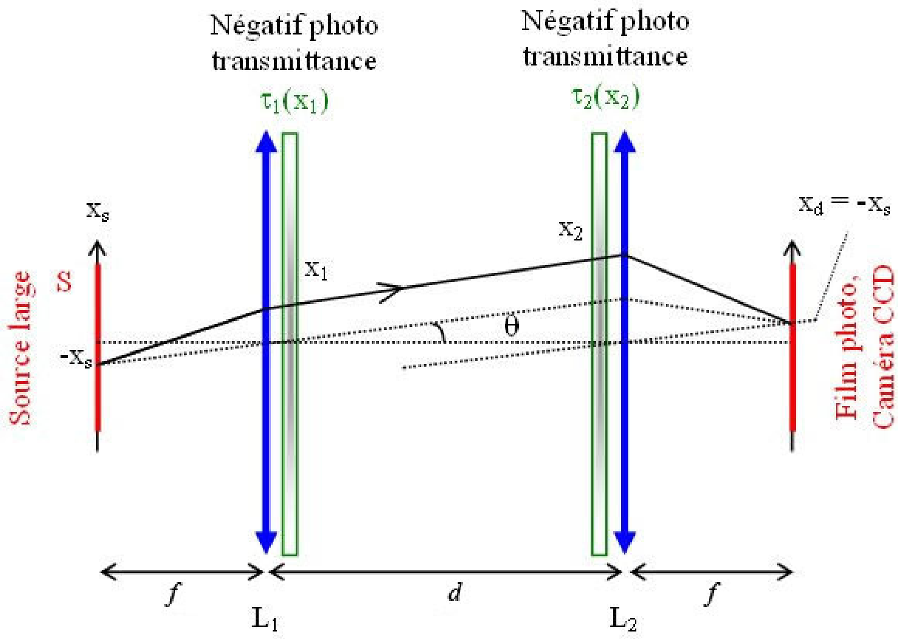
Here we speak of a negative phase contrast even though for the preceding case the phase contrast is positive.

Remarque

It is possible to improve the contrast of the image by making the phase blade partially absorbent to reduce the intensity of the continuous background.

3. Convolution without displacement by optical path in incoherence

Let us place two transparencies τ_1 and τ_2 respectively against the lenses L_1 and L_2as indicated in figure II-7.



That is, a ray coming from S a large source situated at $-x_s$, it emerges from L_1 in x_1. The corresponding transmission is $\tau_1(x_1)$. It arrives on L_2 in x_2 to converge into x_d in the plane of the detector. $x_d = -x_s$ because the two lenses have the same focus. The intensity in this point is :

$$I\,(x_s) = \tau_1\,(x_1)\,\tau_2\,(x_2)$$

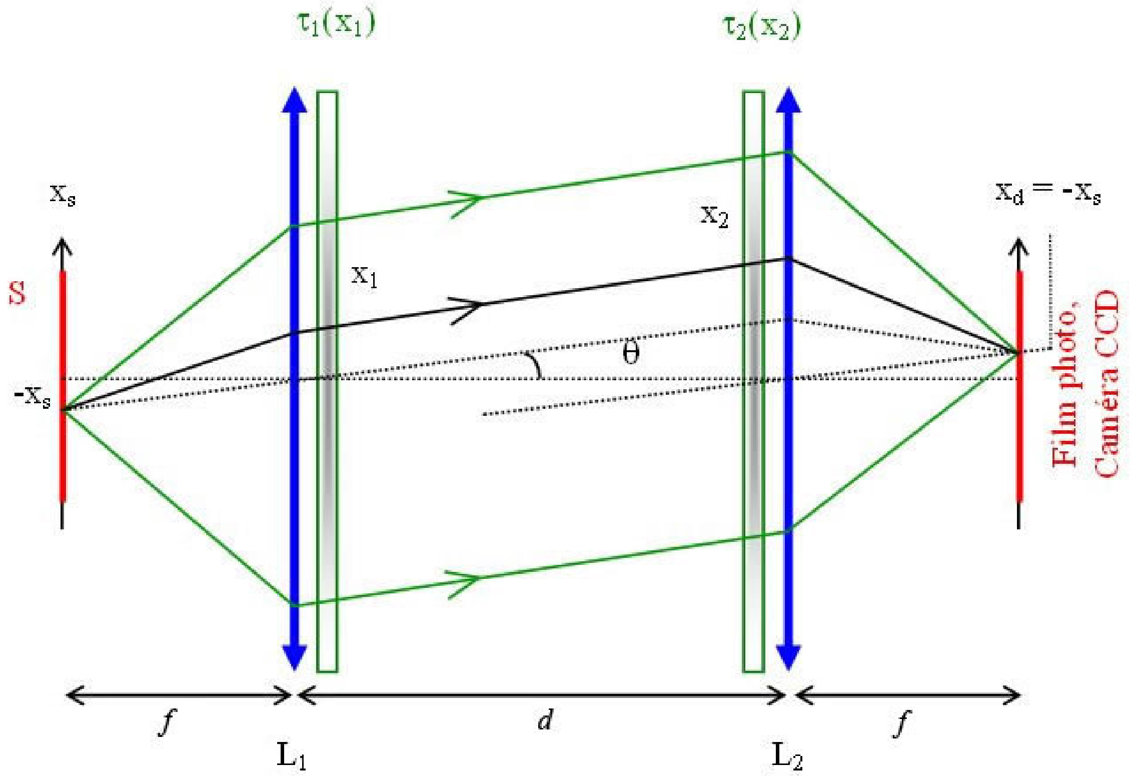
or $tg(\theta) = \dfrac{x_s}{f} \rightarrow x_s = x_1 + \dfrac{d}{f}x_s$ therefore

$$I\,(x_s) = \tau_1\,(x_1)\,\tau_2\,(x_1 + \frac{d}{f}x_s)$$

So $x = x_1 + \dfrac{d}{f}x_s \rightarrow x_1 = x - \dfrac{d}{f}x_s \rightarrow$

$$I\,(x_s) = \int \tau_1\,(x - \frac{d}{f}\,x_s)\,\tau_2\,(x)\,dx$$

If the spherical wave issued is considered from the same source point in $-x_s$, the corresponding waves all come to focus themselves at the same point $-x_d$ of the detector that will make the sum of all the intensities corresponding to different rays (different x) (see figure II-8).



$$I\,(x_s) = \int \tau_1\,(x - \frac{d}{f}\,x_s)\,\tau_2\,(x)\,dx$$

The generalization on two dimensions is immediate :

$$I\,(x_s, y_s) = K \int\int \tau_1\,(x - \frac{d}{f}x_s, y - \frac{d}{f}y_s)\,\tau_2\,(x, y)\,dx\,dy$$

Remarque

We have achieved the bi dimensional convolution operation by optical path. This operation is relatively long to do even today by numerical path. The rapid execution of this operation is above all linked to the inherent parallelism in the optical methods that carry out the treatment on all the points of the plane (x, y) at the same time. It suffices from one top clock of a matrix detector to see the image of the convolution $(\tau_1 * \tau_2)$.

4. Synthesis of a band pass filter in incoherence

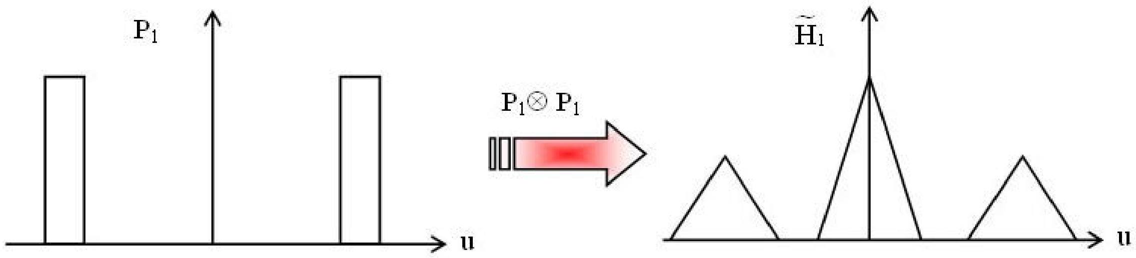
The frequent spectrum of the image intensity is written in incoherence (see grain "Formation of images", part "Frequential analysis of optical systems forming images):

$$TF\left(I_i\right)=\left(\widetilde{H}\otimes\widetilde{H}\right).\left(G_g\otimes G_g\right)$$

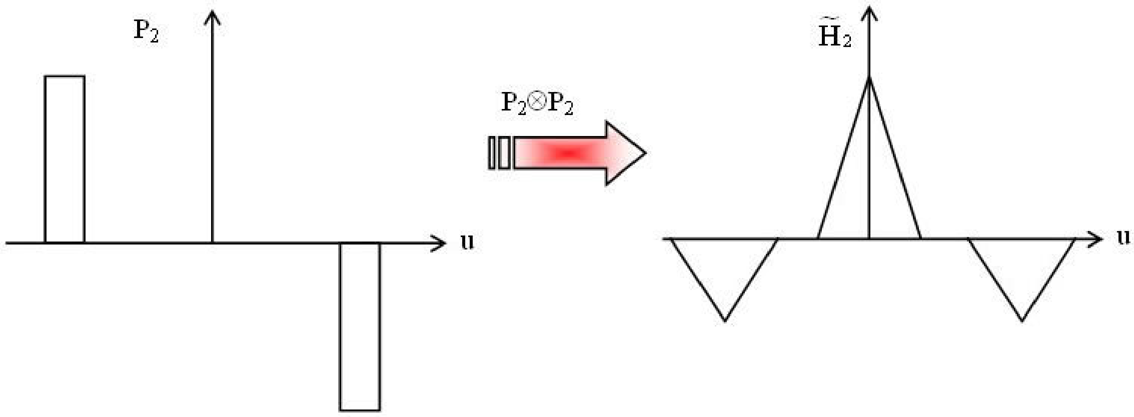
$\widetilde{G}_g$ is the spectrum of the object, $\widetilde{H}$ is the function of coherent transfer with :

$$\widetilde{H}\left(u,v\right)=P\left(\lambda d_i u, \lambda d_i v\right)$$

So a pupil P_1 having the form of Young slits and $\widetilde{H}_1$ the FTM of the system as indicated in figure II-9.



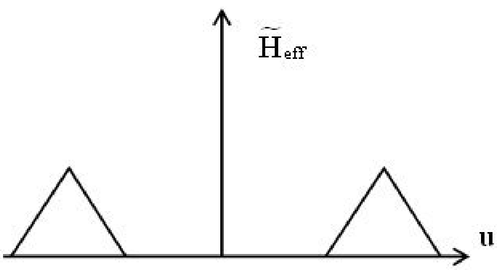
Let us suppose that we place a lamp at a phase of 180 ($\lambda/2$) on one of the slits ($e^{j\pi} = -1$), the FTM of the system comes after the auto correlation composed of a central positive triangle and of two negative lateral triangles as indicated in the figure II-10.



The subtraction of the obtained image by $\widetilde{H}_1$ from the obtained image by $\widetilde{H}_2$ gives :

$$\begin{aligned} TF(I_{i1}) - TF(I_{i2}) &= (P_1 \otimes P_1).(\widetilde{G}_g \otimes \widetilde{G}_g) - (P_2 \otimes P_2).(\widetilde{G}_g \otimes \widetilde{G}_g) \\ &= (\widetilde{H}_1 - \widetilde{H}_2).(\widetilde{G}_g \otimes \widetilde{G}_g) \\ &= \widetilde{H}_{eff}.(\widetilde{G}_g \otimes \widetilde{G}_g) \end{aligned}$$

It is shown that for this effective FTM, (see figure II-11) the continuous background and the low spatial frequencies disappeared.



This FTM is a function of band passing transfer. The obtained image only contains the highest spatial frequencies, the possible applications concerning the detection of the outlines.

The subtraction in incoherent optics by optical path is impossible to do (the intensities are positive and always add themselves together). On the other hand, it is possible to transform the incoherent images into coherent images with the help of a SLM and to make interference of these two images by adding a phase difference of π to I_{i2}. The result is an intensity representing the amplitude to the square of the difference. The synthesis of other FTM is possible, for more details consult the bibliography [[Optical informations processing-Fundamentals]] .

5. Optical treatment of information in coherent illumination

5.1. Introduction

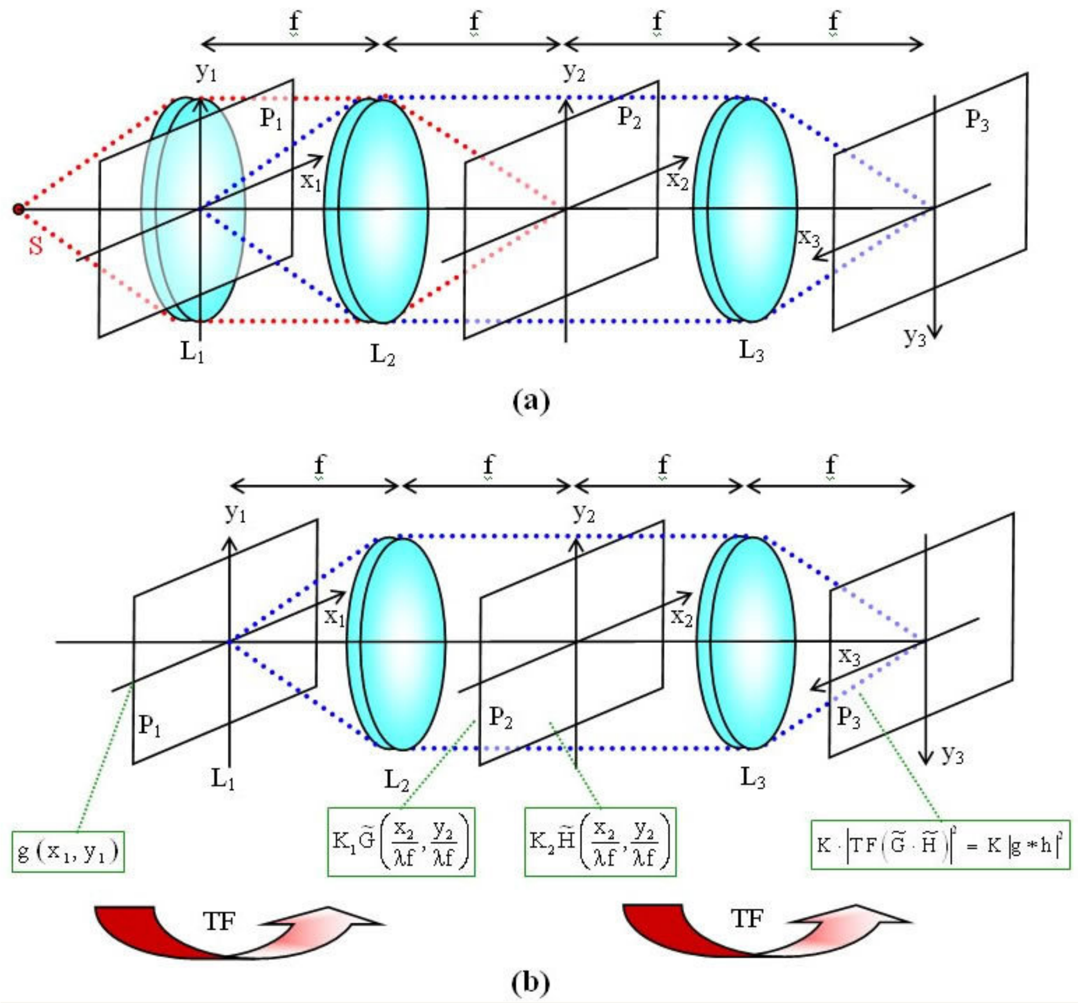
In coherent illumination the filtration operations are achieved by directly manipulating the complex amplitude of the spectrum in the Fourier plane of a lens.

5.2. Used Architectures

The coherent systems, linear in complex amplitude, are capable of achieving the operation of the form :

$$I(x,y)=K\left|\iint g(u,v)h(x-u,y-v)dudv\right|^2$$

Several configurations can be used to achieve this kind of operation.

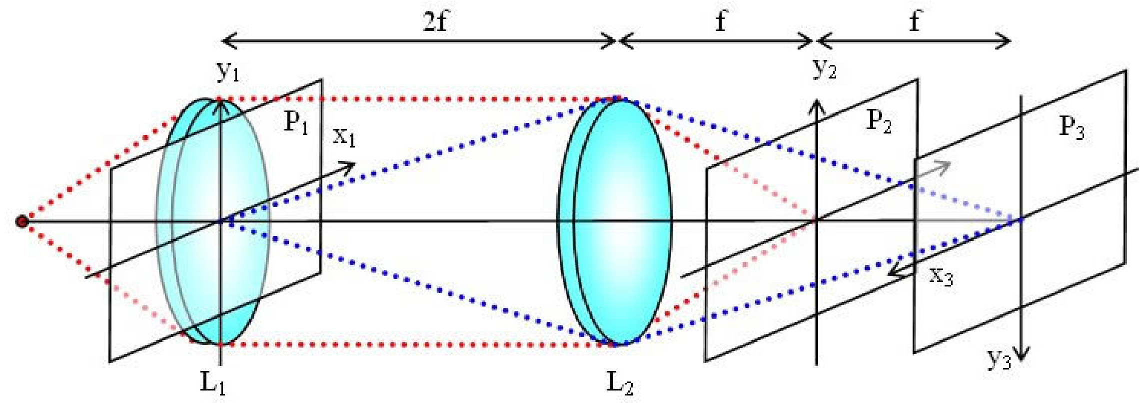


In the figure II-12(a) the most direct filtration architecture can be seen, called «system 4-f », because there are 4 lengths f separating the object of the image. The lens L_1serves to collimate the beam to obtain a wave plane at the entrance of the system (red beam). The object to be treated is placed against L_1 in P_1. Its image finds itself in P_3. Notice that the spectral plane (the Fourier plane), is the plane which contains the image from the source (red beam).

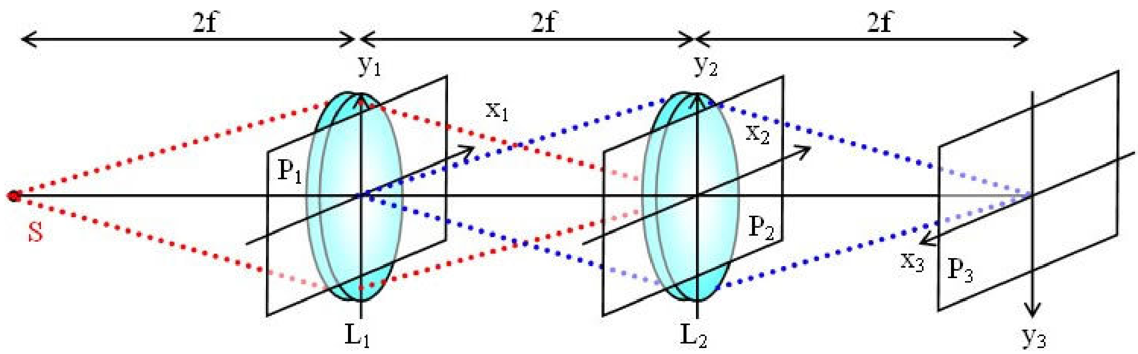
Let us consider a transparency at the entrance as illustrated in figure II-12(b) (having a transmittance in the amplitude $g(x_1, \ y_1)$). This plane is the object focal plane of L_2. In the focal plane image of this lens we have $\widetilde{G}$ the spectrum of g. So $\widetilde{H}$ the function of transfer that is desired to apply is placed in P_2 a transmittance filter :

$$t_f\left(x_2, y_2\right) = K_2 \cdot \widetilde{H}\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right)$$

The field after P_2 is therefore $\widetilde{G} \cdot \widetilde{H}$. The lens L_3 produces the TF of the modified spectrum in the plane P_3. It is in this plane that the filtered image is obtained (treated). Note that the coordinates are inverted in the plane P_3 to remember that the image is inverted : $TF(TF(g(x_1))) = g(-x_1)$.



In this second architecture (see figure II-13) we have the same length but one less lens. L_2 produces the TF in the P_2 and the image in P_3. at the same time. Note that here the spectrum is associated with the quadratic phase factor $exp(-jk(x^2+y^2)/2f)$ since the entry is not in the object focal plane (see the grain « formation of images, » part: « properties of the relative lenses in the Fourier transformation »).



In this third and last architecture (see figure II-14), two lenses are always used but it requires a supplementary length f ($6f$ in total). L_1collects the light and produces the TF in the plane P_2(source image). In this plane, the filter is placed against L_2 that gives a image ofP_1in P_3. In this configuration the term of quadratic phase does not appear in P_2. In fact, the exact calculation shows that this term disappears because the wave that illuminates$P_1$is spherical and contains the same term of quadratic phase but of the opposite sign.

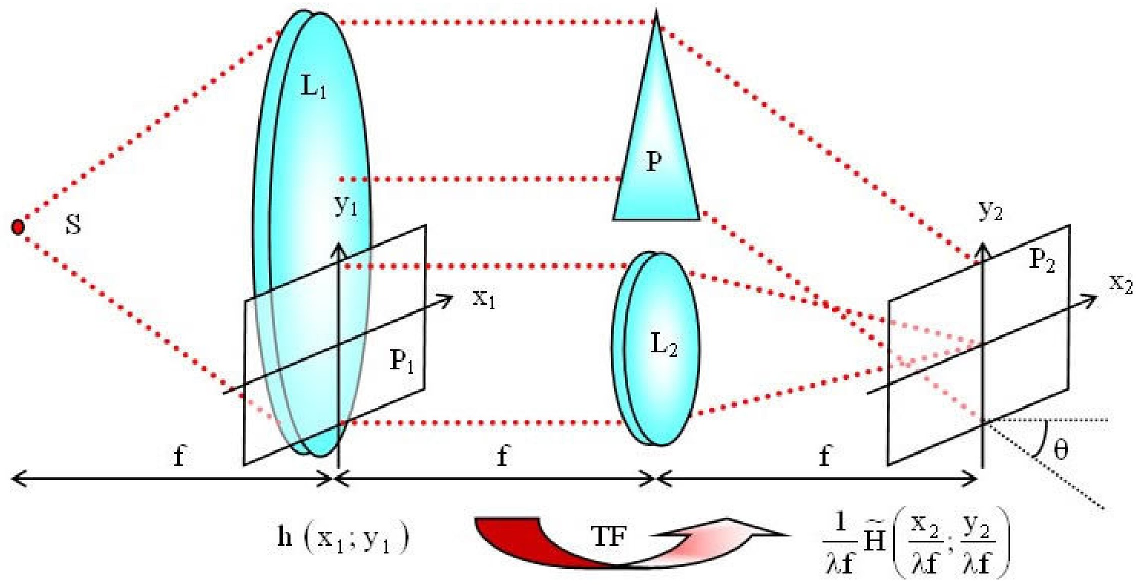
6. The filter of Vander Lugt (1963)

6.1. Introduction

The filters produced have the remarkable property to permit the efficient control of the amplitude and of the phase of a transfer function at the same time even if they are only formed in purely absorbent figures.

6.2. Achievement of the function of transfer

The **frequent mask** (of the **filter of Vander Lugt**) is produced with the help of a interferometric system that can be seen in the figure II-15.



The information coding the phase finds itself in the term of interference. L_1 serves as a collimator. A part of this light falls on the plane P_1in which a transmittance in amplitude equal to the impulsion desired response h is found. The lens L_2 produces the TF of h on the film placed in P_2. The other part of the light crosses the prism P and arrives at P_2 under the incidence θ.

The total incident intensity at the level of the film is determined by the interference of two distributions of amplitude (coming from L_2 and from P). The inclined wave plane is represented by its complex amplitude in the plane P_2.

$$U_r(x_2, y_2) = r_0 e^{j\vec{k}\cdot\vec{r}_2} = r_0 e^{j\frac{2\pi}{\lambda}\left(-\sin\theta\cdot\vec{j}_2 + \cos\theta\cdot\vec{k}_2\right)\cdot\left(x_2\cdot\vec{i}_2 + y_2\cdot\vec{j}_2\right)}$$

$$U_r(x_2, y_2) = r_0 e^{-j\frac{2\pi}{\lambda}\sin\theta\cdot y_2}, \quad \text{en posant} \quad \frac{\sin\theta}{\lambda} = \alpha \Rightarrow$$

$$U_r(x_2, y_2) = r_0 e^{-j2\pi\alpha\cdot y_2}$$

The distribution of the intensity is therefore written as :

$$I(x_2, y_2) = \left| r_0 e^{-j2\pi\alpha\cdot y_2} + \frac{1}{\lambda f}\tilde{H}\left(\frac{x_2}{\lambda f}; \frac{y_2}{\lambda f}\right)\right|^2$$

$$I(x_2, y_2) = r_0^2 + \frac{1}{(\lambda f)^2}\left|\tilde{H}\left(\frac{x_2}{\lambda f}; \frac{y_2}{\lambda f}\right)\right|^2 + \frac{r_0}{\lambda f}\tilde{H}\left(\frac{x_2}{\lambda f}; \frac{y_2}{\lambda f}\right)\exp(j2\pi\alpha y_2) + \frac{r_0}{\lambda f}\tilde{H}^*\left(\frac{x_2}{\lambda f}; \frac{y_2}{\lambda f}\right)\exp(-j2\pi\alpha y_2)$$

$\widetilde{H}$ being complex, in the general case, one can pose that :

$$\widetilde{H}\left(\frac{x_2}{\lambda f};\frac{y_2}{\lambda f}\right)=A\left(\frac{x_2}{\lambda f};\frac{y_2}{\lambda f}\right)\exp\left[j\varphi\left(\frac{x_2}{\lambda f};\frac{y_2}{\lambda f}\right)\right]$$

Which gives the intensity :

$$I\left(x_2,y_2\right)=r_0^2+\frac{A^2}{(\lambda f)^2}+\frac{2r_0}{\lambda f}A\left(\frac{x_2}{\lambda f},\frac{y_2}{\lambda f}\right)\cos\left(2\pi\alpha\, y_2-\varphi\left(\frac{x_2}{\lambda f},\frac{y_2}{\lambda f}\right)\right)$$

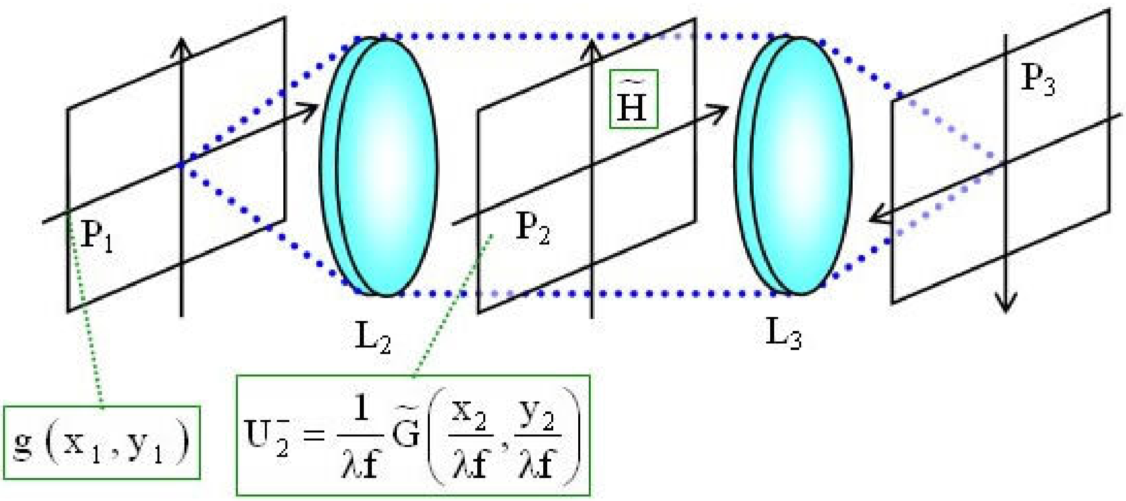
This form shows how the interferometric process permits the registration of a complex function on a detector sensitive to the intensity. The information relative to the amplitude and to the phase is registered under the form of a amplitude modulation and of a wave phase carrying a high frequency that is introduced by the wave of inclined “reference” coming from the prism. To produce this filter, the film of such a type is developed so that its transmittance in the amplitude is proportional to the intensity of exposition : $t(x_2,\ y_2)\propto I(x_2,\ y_2)$

In the term of interference (c. to d. in t) we have the information required to make the impulsion response filter equal to h

The remaining problem is to show how and under what conditions this information can be extracted from the other terms that are present in I.

6.3. Treatment of the given

Let us introduce the mask $\widetilde{H}$ that we just produced in the montage 4-f (in the plane P2) (see the figure II-16).



Where g is the function of entry that we wish to filter.

The amplitude $U_2$of the transmitted field by the mask $\widetilde{H}$ is written $(U_2 = t_2\cdot\widetilde{G})$:

$$U_2\left(x_2,y_2\right)\propto\frac{r_0^2}{\lambda f}\widetilde{G}+\frac{1}{(\lambda f)^3}\widetilde{G}\cdot|\widetilde{H}|^2+\frac{r_0}{(\lambda f)^2}\widetilde{G}\cdot\widetilde{H}\exp\left(j2\pi\alpha\, y_2\right)+\frac{r_0}{(\lambda f)^2}\widetilde{G}\cdot\widetilde{H}^*\exp\left(-j2\pi\alpha\, y_2\right)$$

L_3produces the TF of U_2in P_3 and knowing that $TF[exp(j2\pi au)] = \delta(x+a,v)$, the amplitude of the field is written :

$$U_3(x_3,y_3) \propto r_0^2 g(x_3,y_3) + \frac{1}{(\lambda f)^2}\left\{h(x_3,y_3) * h^*(-x_3,-y_3) * g(x_3,y_3)\right\}$$

$$+\frac{r_0}{\lambda f}\left\{h(x_3,y_3) * g(x_3,y_3) * \delta(x_3,y_3+\alpha\lambda f)\right\} + \frac{r_0}{\lambda f}\left\{h^*(-x_3,-y_3) * g(x_3,y_3) * \delta(x_3,y_3-\alpha\lambda f)\right\}$$

Remarque

L_2divides the amplitude of the spectrum by λf and reduces the coordinates by λf following each axis.L_3 always divides the amplitude of the spectrum by λf it puts back this time the coordinates at the same level. The theorem of similarity tells us that it is necessary to multiple by $(\lambda f)^2$. At the total, the result comes back to multiply by λf.

Knowing that δ is the natural neutral element of the product of convolution with which the following translation is done following y_3 the third and fourth term being :

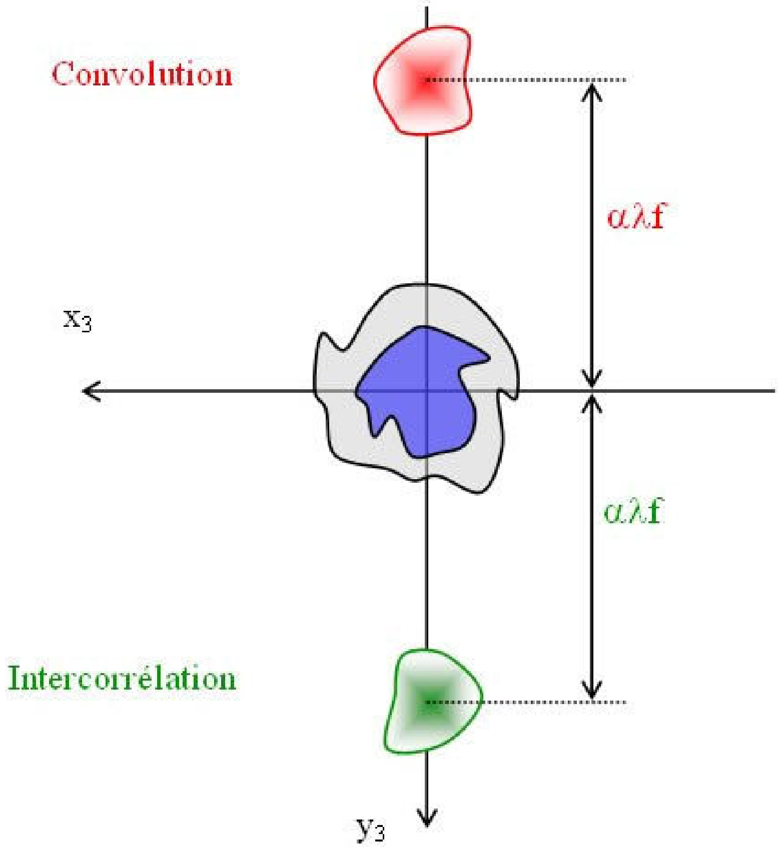
$$U_3(x_3,y_3) \propto r_0^2 g(x_3,y_3) + \frac{1}{(\lambda f)^2}\left\{h(x_3,y_3) * h^*(-x_3,-y_3) * g(x_3,y_3)\right\}$$

$$+\frac{r_0}{\lambda f}\underbrace{\left\{h(x_3,y_3) * g(x_3,y_3) * \delta(x_3,y_3+\alpha\lambda f)\right\}}_{\iint h(x_3-u,\, y_3+\alpha\lambda f - v)\, g(u,v)\,\mathrm{dudv}} + \frac{r_0}{\lambda f}\underbrace{\left\{h^*(-x_3,-y_3) * g(x_3,y_3) * \delta(x_3,y_3-\alpha\lambda f)\right\}}_{\iint h^*(u-x_3,\, v-y_3+\alpha\lambda f)\, g(u,v)\,\mathrm{dudv}}$$

Ce terme donne le produit de convolution de h *par* g *centré au point de coordonnées* $(0,-\lambda\alpha f)$

Ce terme donne le *produit d'intercorrélation de* h *par* g *centré au point de coordonnées* $(0,\lambda\alpha f)$

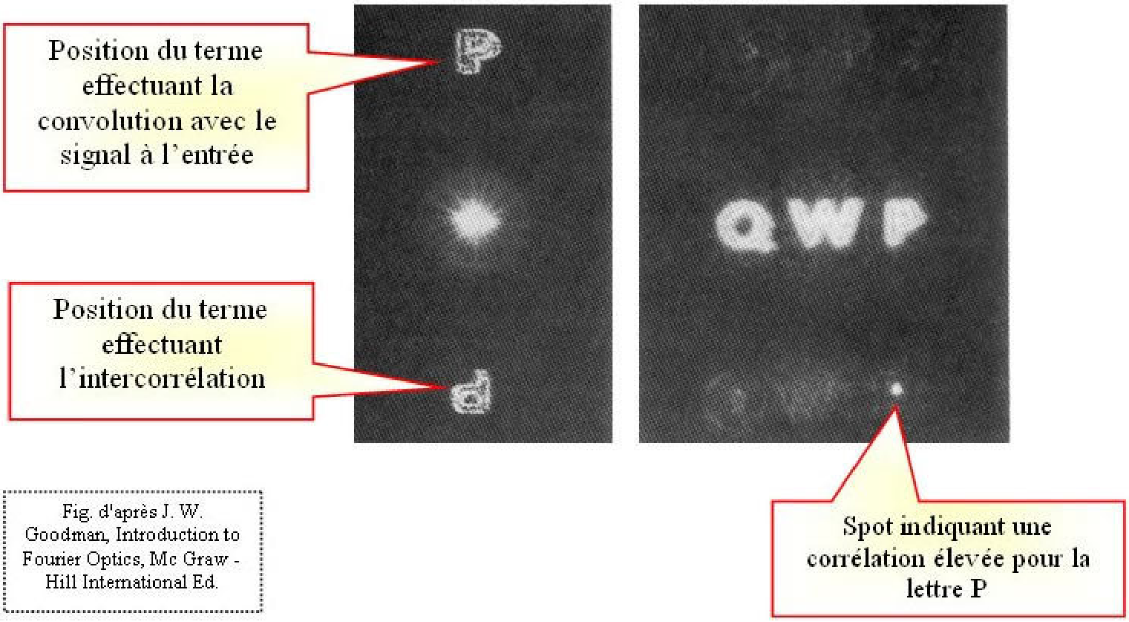
The first two terms centered at the origin do not have any particular interest for filtration. In P_3three regions where the amplitude is different from 0 are observed. These regions do not cover themselves if $\lambda f\alpha$ is sufficiently large before the spatial extension of h and of g (voir figure II-17).



Here the principal advantage of this filter is to produce a function of transfer at complex values with an absorbent filter. The transmittance of phase in the Fourier plane is more simple to produce technically (no control of dimension, of thickness, or of indication).

6.4. Application to the recognition of forms

It is about knowing the placement of a letter (the P) presented at the entry of a system 4-f among different letters (three in this example).

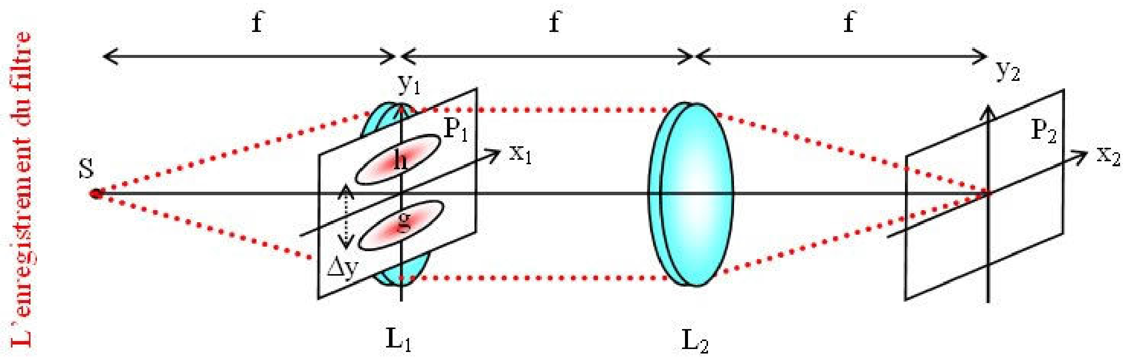


In the figure II-18, the image of the impulsion response of a filter of Vander Lugt (at the left) can be seen that will be introduced in the filtration plane. The response of this filter to the letters Q, W and P (image at the right) at the exit of the system. Note the presence of a intense spot under P (at the level of the term of inter correlation) indicating a degree of resemblance elevated between this letter and the chosen filter.

It is necessary to note that the inconvenient principal of such a method of recognition using the transformation of Fourier is its big sensitivity to the rotation and to the changing of levels. Others that are transformed (such as that of Mellin) offer less sensitivity to the enlargement [[Practical holography]].

7. The joint transform correlator (Weaver and Goodman 1966)

As for the filter of Vander Lugt, correlations and convolutions are produced with the help of this method. The difference comes from the fact that the impulsion response desired and the givens to filter are simultaneously present during the registration. (see figure II-19). figure II-19



L_1sends a wave plane that illuminates two transparencies : h for the impulsion desired response and g for the entry givens to filter. One notes Δy as the distance between the centers of h and g. The composite signal present at the entrance is written :

$$U_1(x_1,y_1)=h\left(x_1,y_1-\frac{\Delta y}{2}\right)+g\left(x_1,y_1+\frac{\Delta y}{2}\right)$$

In $P_2$the TF of the proceeding signal is produced. The amplitude of the field is written :

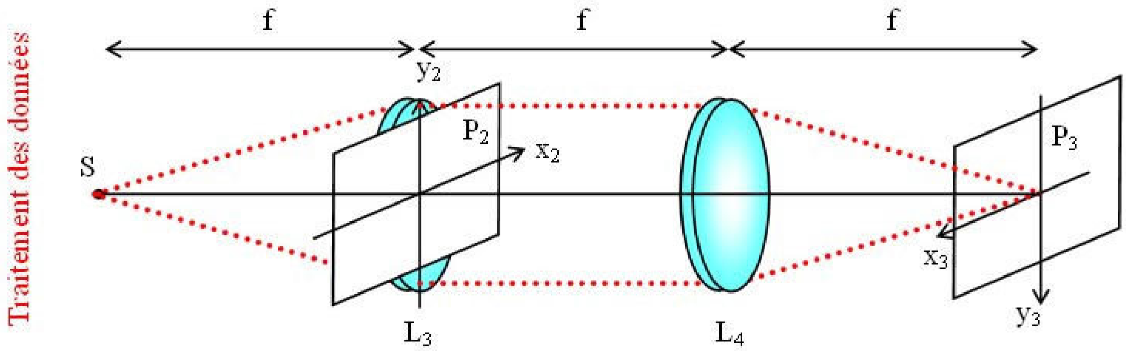
$$U_2(x_2,y_2)=\frac{1}{\lambda f}\widetilde{H}\left(\frac{x_2}{\lambda f},\frac{y_2}{\lambda f}\right)e^{-j\frac{\pi y_2\Delta y}{\lambda f}}+\frac{1}{\lambda f}\widetilde{G}\left(\frac{x_2}{\lambda f},\frac{y_2}{\lambda f}\right)e^{+j\frac{\pi y_2\Delta y}{\lambda f}}$$

The SLM or the film serving to the registration is sensitive to :

$$I(x_2,y_2)=\frac{1}{(\lambda\,\mathrm{f})^2}\left\{|\widetilde{H}|^2+|\widetilde{G}|^2+\widetilde{H}\,\widetilde{G}^*\cdot e^{-j\frac{2\pi y_2\Delta y}{\lambda f}}+\widetilde{G}\,\widetilde{H}^*\cdot e^{j\frac{2\pi y_2\Delta y}{\lambda f}}\right\}$$

In the case where it is a film, the negative result is supposed to have a transmittance in an amplitude proportional to the I the intensity of exposition.

The film is then placed in the plane $P_2$of the montage of the figure II-20 where L_4produces the TF in P_3.



The amplitude of the field in this plane is written :

$$U(x_3;y_3)=\frac{1}{(\lambda f)}[h(x_3;y_3)*h^*(-x_3;-y_3)+g(x_3;y_3)*g^*(-x_3;-y_3)$$

$$+\underbrace{h(x_3;y_3)*g^*(-x_3;-y_3)*\delta(x_3;y_3-\Delta y)}_{\iint h(u,v)g^*(u-x_3,v-y_3+\Delta y)\,\mathrm{dudv}}+\underbrace{h^*(-x_3;-y_3)*g(x_3;y_3)*\delta(x_3;y_3+\Delta y)}_{\iint h^*(u-x_3,v-y_3-\Delta y)g(u,v)\mathrm{dudv}}]$$

These last two terms represent the product of inter correlation of h by g centered to the coordinate points $(0,\pm\Delta y)$. One of these terms can be seen as the image of the other across a mirror placed on the optical axis.

The advantage of this system is that it is not necessary to align the filter in the Fourier plane compared to the method of Vander Lugt.

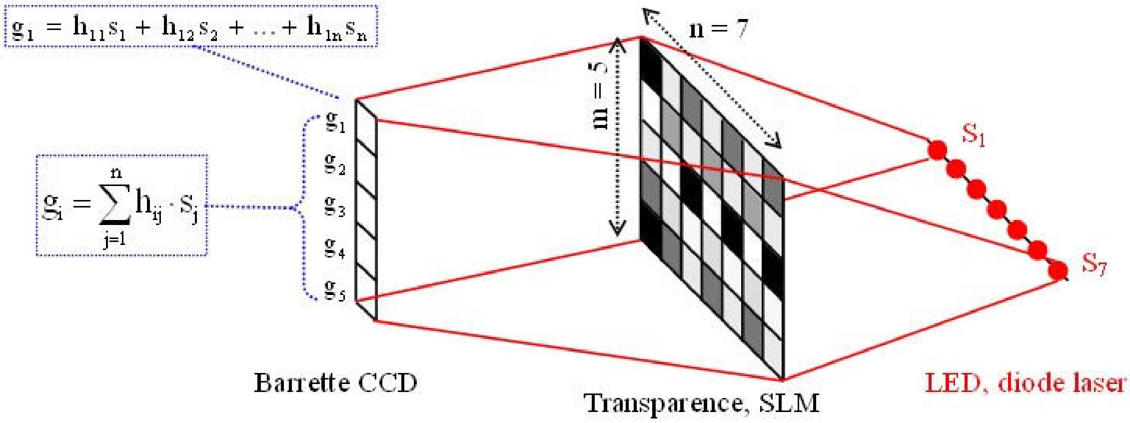
Another advantage worth noting is the ability to use this last approach in real time by placing a SLM in the plane P_1, a photo refractive material in P_2 and a camera CCD in P_3.

8. Multiplication matrix-vector

Where the following operation to produce :

$$\begin{bmatrix} g_1 \\ g_2 \\ . \\ . \\ g_m \end{bmatrix} = \begin{pmatrix} h_{11} & . & . & . & h_{1n} \\ h_{21} & . & . & . & h_{2n} \\ . & & & & \\ . & & & & \\ h_{m1} & . & . & . & h_{mn} \end{pmatrix} . \begin{bmatrix} s_1 \\ s_2 \\ . \\ . \\ s_n \end{bmatrix}$$

An incoherent processor entirely parallel to multiplication matrix-vector is illustrated in figure II-21.

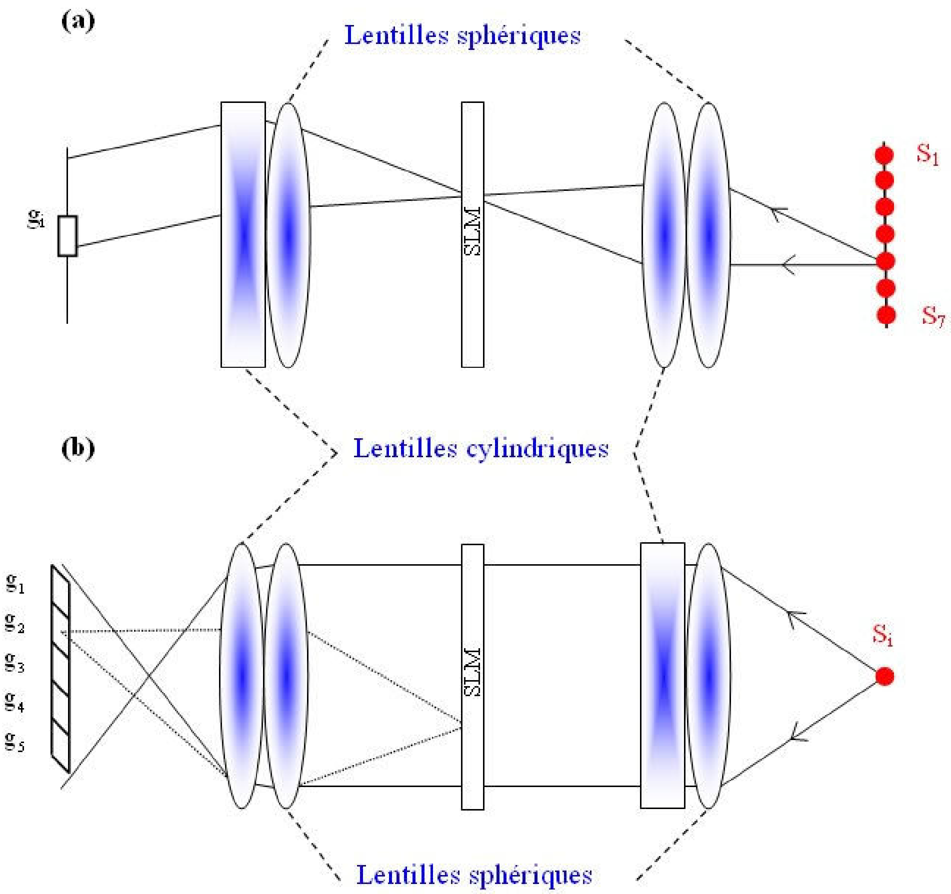


In this figure, the details of the used optical elements are temporarily omitted. The intensity of the source S_i is proportional to the elements of the vectors s_i. So that the transmittance of the cell hij in the SLM is proportional to the element h_{ij} of the matrix. The h_{ji} element of the barrette CCD in exit receives a luminous intensity proportional to the signal :

$$g_i = \sum_{j=1}^{n} h_{ij} . s_j$$

This represents the result of the multiplication.

So that the beams can expand vertically and focus themselves horizontally following the coats, one uses a combination of spherical and cylindrical lenses placed side by side using the same focus (see figure II-22).



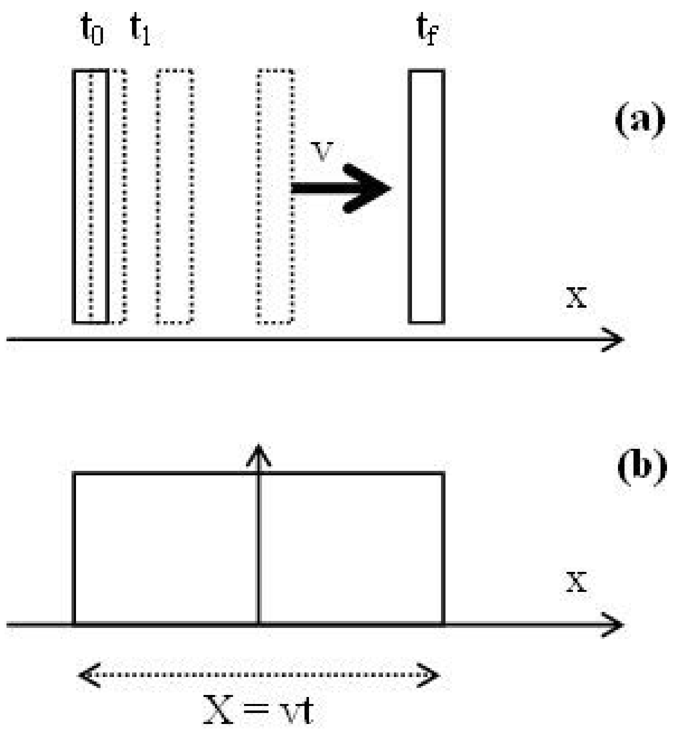
This combination possesses a focus of $f/2$ in the direction where the cylindrical lens converges the rays, and a focus of f in the direction where the cylindrical lens has no action. Since we can see in figure II-22 (a) that for a horizontal coat the sources are imaged in the plane containing the SLM. Then the beam is collimated since the second cylindrical lens, turned at 90 in relation to the first, has no convergent power in this direction to the equivalent focus of the second pair of lenses is of f. The side view shown in the figure II-22(b) shows how first the beam is collimated following a vertical coat and then with the aid of the second pair of lenses one makes the image at each column of the SLM in the barrette.

Remarque

- Here the calculation is made in parallel. In a clock cycle, one sends the givens and one recollects the product $1\ cycle \simeq 10\ ns$ which explains the rapidity of the calculation.

9. Blurred photo

If a photo is hazy because of a "movement" or because it is taken from a plane (or a satellite), it is possible to produce an inverse filtration to make it more clean.



By neglecting the function of the optical transfer, the image is given by :

$$I = O * rect\left(\frac{x}{X}\right)$$

$$TF(I) \propto TF(O).\,\mathrm{sinc}(X.u)$$

$$TF(O) \propto \frac{TF(I)}{\mathrm{sinc}(X \cdot u)}$$

$$O \propto TF^{-1}\left\{\frac{TF(I)}{\mathrm{sinc}(X \cdot u)}\right\}$$

This operation is called **inverse filtration** or **deconvolution**. The same process can be used when there is a default of adjustment . Nevertheless, a difficulty exists at the level of zeros present in the sinus cardinal function. The object that is deconvoluted with the help of a central lobe does not contain the high spatial frequencies. The spatial resolution finds itself weakened.

* *
*

In conclusion, even if the optical processors for the treatment of images acquired a certain maturity and a culminating point in terms of activity of research, they have not seen the rapid industrial development that was hoped for, since the competition with the numerical processors is very tough. Yet, this work on optical treatment permitted multiple positive spillovers for the development of algorithms and for new methods of imagery [[Tome1_Chapitre05.pdf]].

In the future, it is necessary to determine the applications that are susceptible to benefit from a contribution of the optic and to propose new architectures, surely hybrid, associating an optical treatment with an electronic treatment. The potential applications concern the biomedical sector, defense, access to given multimedia, and security to only cite a few.

III.Case study

1. Application for the recognition of the characters

The recognition of the characters constitutes a particular application of optical treatment of which there are several years of interest. As we see it, this application constitutes an excellent example of a case where the operations of treatment have simple impulsive responses, but not necessarily the functions of simple transfer. The synthesis technique of Vander Lugt is therefore particularly well adapted to this application.

1.1. The adapted filter

The concept of the adapted filter plays an important role in the problems of recognition of characters.

Définition

By definition, a linear filter spatially invariant is called adapted to the particular signal $s(x, y)$ if its impulsive response $h(x, y)$ is given by :

$$h(x,y)=s^*(-x,-y)$$

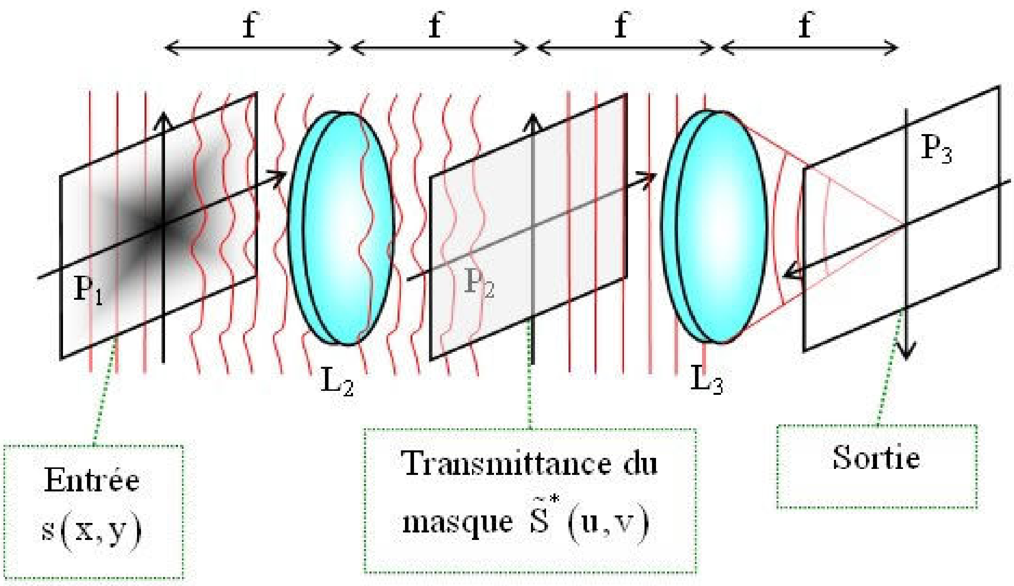
If a signal of entry $g(x, y)$ is applied to an adapted filter at $s(x, y)$, one finds an exit signal $v(x, y)$ of the form :

$$v(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(x-\alpha, y-\beta)\, g(\alpha,\beta)\, d\alpha\, d\beta$$
$$=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} s^*(\alpha-x,\beta-y)\, g(\alpha,\beta)\, d\alpha\, d\beta$$

where the product of inter correlation of functions g and s is recognized.

Historically, the notion of the adapted filter first appears in the domain of detection of signals; if one has to detect a signal of a known form, drowned in a “white” noise, the usage of an adapted filter permits the achievement of a linear operation that gives the maximal relationship of the instantaneous power of the signal (to a given instant) to the medium power of noise. However, in the present application, the characters will be assumed to not contain noise, and the usage of a particular mode of filtration justifies itself differently.

The optical interpretation given by the figure EC1 permits to better understand the mechanism of adapted filtration.



Let us suppose that one has to achieve an adapted filter at the entry signal $s(x, y)$ by means of a frequent mask disposed in the spectral plane of a classic system of coherent treatment. The Fourier transformation of the impulsive response (Eq.(1)) shows that the function of necessary transfer is :

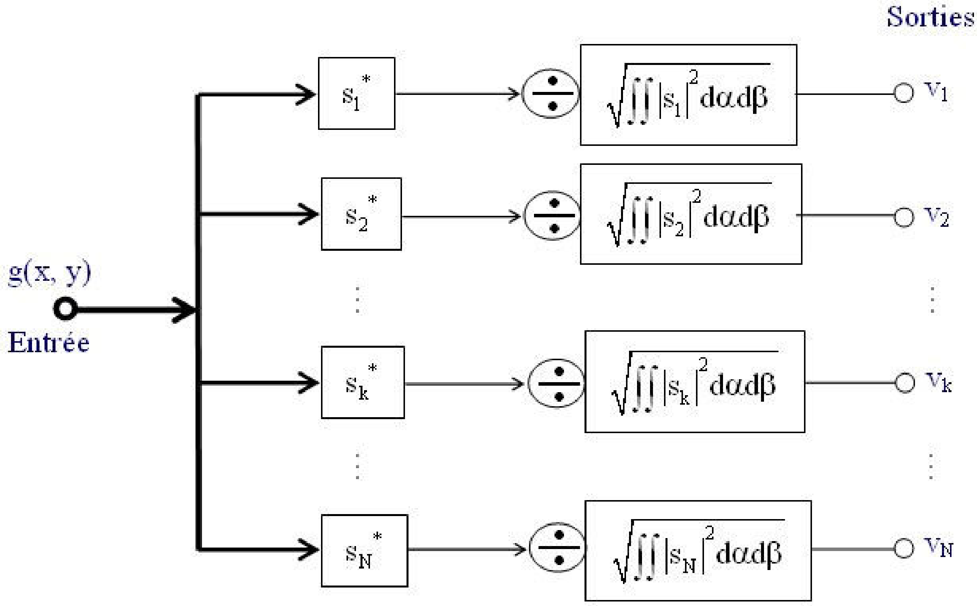
$$\tilde{H}(u,v)=\tilde{S}^*(u,v)$$

Where $\tilde{H}(u, v) = TF[h(x, y)]$ and $\tilde{S}(u, v) = TF[s(x, y)]$. The frequent mask must therefore have a transmission in proportional amplitude to $\tilde{S}^*$.

Let us consider at present the particular nature of the amplitude of the transmitted wave by the mask when we apply the signal s (to which the filter is adapted) to the entry of the system. This mask receives a complex amplitude proportional to $\tilde{S}$ and transmits a proportional distribution to $\tilde{S}\tilde{S}^*$. This last quantity is real, which implies that the frequent mask compensates exactly for curve of the incident wave $\tilde{S}$. he complex amplitude transmitted represents therefore a wave plane that the last lens of the system transforms into a brilliant spot in the exit plane of the system. When we apply an entry signal other than $s(x, y)$, the curve of the wave is not, in general, compensated by the frequent mask and the transmitted light is not concentrated into a brilliant spot by the last lens. We therefore conceive that we can detect the presence of the signal s by measuring the luminous intensity in the focal plane of the last lens of the system. (if the signal of entry s is not centered at the origin, the brilliant spot in the exit plane displaces itself from a equal distance of the center of the signal s at the axis).

1.2. At the axis

Let us consider the following particular problem: the entry signal g of the system of treatment is constituted by one no matter the N characters possible s_1, $s_2 ... s_N$ and we propose to determine which of these characters constitute effectively the signal g. As we are going to show, the process of identification can be achieved by applying the entry signal to a series of N filters, each of them being adapted to one of these characters being able to constitute the entry signal (see figure EC2).



The figure EC2 represents a block diagram of a device of recognition. The entry signal is applied simultaneously (or successively) to N filters adapted to functions of transfer $\widetilde{S}_1^*,\ \widetilde{S}_2^* \ldots \widetilde{S}_N^*$. The response to each of these filters is normalized by the square root of the total energy corresponding to the character to which it is adapted. This normalization, who can be achieved by electronic path after detection of signals exiting from the filters, takes into consideration the fact that the diverse characters serving from the signal of entry do not generally let the same energy pass. Finally, we compare the squares of the modules of the exit signals $|v_1|^2,\ |v_2|^2 \ldots |v_N|^2$ in each of the points where we think to find the maximum of the exit signals (we suppose therefore that the character to which they are adapted is present in each case). We are going to show that, if the particular character s_k. is effectively present in the entry signal, that is to say if, $g(x,\ y) = s_k(x,\ y)$ the corresponding value $|v_k|^2$ is the most elevated of the N responses.

Indeed, lets us first note that after Eq. (2), the maximum of the exit signal $v_k$of the adapted filter that is suitable is given by :

$$|v_k|^2 = \frac{\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |s_k|^2 d\alpha\, d\beta\right]^2}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |s_k|^2 d\alpha\, d\beta} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |s_k|^2 d\alpha\, d\beta$$

From another part, the response $|v_n|^2$ with $n \neq k$, from one adapted filter to a different character of s_k is given by :

$$|v_n|^2 = \frac{\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} s_k s_n^* d\alpha\, d\beta\right]^2}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |s_n|^2 d\alpha\, d\beta}$$

By using the inequality of Schwarz, we can write :

$$\left|\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} s_k s_n^* d\,\alpha\, d\,\beta\right|^2 \leq \left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |s_k|^2 d\,\alpha\, d\,\beta\right]\cdot\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |s_n|^2 d\,\alpha\, d\,\beta\right]$$

We easily deduct that :

$$|v_n|^2 \leq \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |s_k|^2 d\,\alpha\, d\,\beta = |v_k|^2$$

the equality achieved if and only if :

$$s_n(x, y) = \kappa \cdot s_k(x, y)$$

This result shows therefore that the adapted filter furnishes a means of recognition, among a lot of possible characters, which is really presented to the system.

IV.Exercice

1. Knowledge test

Filtration and optical treatment of the signal

Answer the following questions

Question 1

[Solution n°1 p 40]

The method of observation of the objects of the phase called "somber field" consists to place a small opaque screen in the focal image plane of the lens forming the image of the object, to stop the non diffracted light. By supposing that the variations of the phase of the object are always very inferior to $1\ radian$, determine the intensity of the observed image in function of the variations of the phase of the object.

Question 2

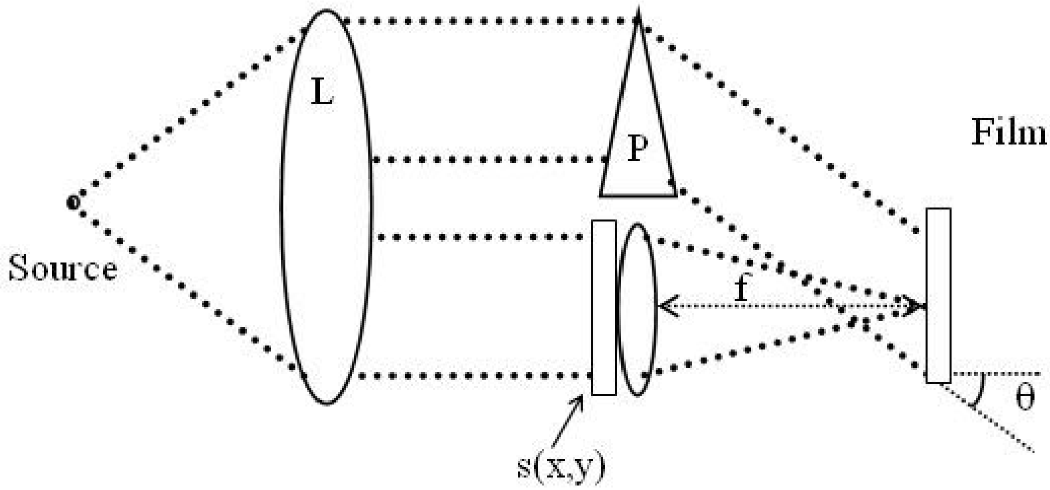
[Solution n°2 p 40]

Find an expression giving the intensity of the image in a microscope of phase contrast when the blade of the phase corresponding to the filter of Zernicke is also partially absorbent with a transmission in equal intensity to α (to simplify the problem, one will take a pupil of an infinite imaging system).

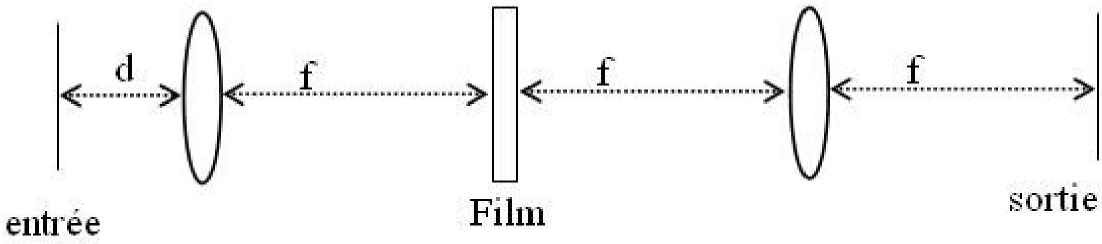
Question 3

[Solution n°3 p 41]

One uses the Vander Lugt method to achieve a frequent filter. As indicated in the figure TC1, one places the snapshot "signal" of transmission in amplitude $s(x,\ y)$ against a convergent lens (instead of placing it in its object focal plane) and the photographic plate registering the luminous intensity in the focal image plane.



One makes sure that the transmission in amplitude of the developed snapshot is proportional to the exposition intensity and one places this snapshot in the system of the figure TC2.



By examining in each case the part of the plane of exit that is suitable, what must be the distance d between the input plane and the first lens to achieve :

- a) an impulsive response filter $s(x,\ y)$?
- b) an impulsive response filter $s*(-x,\ -y)$?

Solution des exercices

>Solution n°1 (exercice p. 38)

The transmission of the object of phase is written :

$$t(x,y)=e^{j\varphi(x,y)}\approx 1+j\varphi(x,y)$$

in the focal plan of the lens, the amplitude of the field is proportional to :

$$TF[t(x,y)]=\tilde{T}(u,v)\approx\delta(u,v)+j\tilde{\Phi}(u,v)$$

by stopping the non diffracted light with the help of an opaque screen, the field becomes :

$$T(u,v)\approx j\tilde{\Phi}(u,v)$$

The intensity in the image plane is therefore written :

$$I_t(x,y)\approx\left|j\varphi(x,y)\right|^2\approx\varphi(x,y)^2$$

>Solution n°2 (exercice p. 38)

The contrast of the phase rests on the spatial filter.
The necessary hypothesis is that the object of the phase presents a weak phase difference : so $\varphi(x,\ y)\ll 1\ rad$. In these conditions

$$t(x,y)=e^{j\varphi(x,y)}\approx 1+j\varphi(x,y)$$

In the spectral plane :

$$TF[t(x,y)]=\tilde{T}(u,v)\approx\delta(u,v)+j\tilde{\Phi}(u,v)$$

After the phase blade, the spectrum becomes (with a delay of phase of $+\pi/2$ and a transmission in equal intensity toα)

$$\tilde{T}(f_x,f_y)\approx j\sqrt{\alpha}\,\delta(u,v)+j\tilde{\Phi}(u,v)$$

By making the TF to pass in the image plane, the transmitted intensity (by neglecting φ^2) :

$$I_t(x,y)\approx\left|j\left(\sqrt{\alpha}+\varphi(x,y)\right)\right|^2\approx\alpha+2\sqrt{\alpha}\cdot\varphi(x,y)$$

>Solution n°3 *(exercice p. 38)*

The wave focused by the lens is written (object placed against the lens: presence of the curve of quadratic phase before the spectrum of s) :

$$U_f(u,v)=\frac{e^{j\frac{k}{2f}[u^2+v^2]}}{j\lambda f}\int_{-\infty}^{+\infty}\int s(x,y)e^{-j\frac{2\pi}{\lambda f}[xu,yv]}dxdy$$

$$U_f(u,v)=\frac{e^{j\frac{k}{2f}[u^2+v^2]}}{j\lambda f}\tilde{S}\left(\frac{u}{\lambda f};\frac{v}{\lambda f}\right)$$

The wave that comes from the prism (wave plane inclined to θ) is written :

$$U_r(u,v)=r_0 e^{-j2\pi\alpha v}\text{ ; avec }\frac{\sin\theta}{\lambda}=\alpha$$

The registered intensity by the film will be therefore (interference of the proceeding waves) :

$$I(u,v)=\left|r_0 e^{-j2\pi\alpha\cdot v}+\frac{e^{j\frac{k}{2f}[u^2+v^2]}}{j\lambda f}\tilde{S}\left(\frac{u}{\lambda f};\frac{v}{\lambda f}\right)\right|^2$$

After development the transmission of the negative is written :

$$t(u,v)=t_1(u,v)+t_2(u,v)+t_3(u,v)+t_4(u,v)$$

with

$$t_1(u,v)+t_2(u,v)=r_0^2+\frac{1}{(\lambda f)^2}\left|\tilde{S}\left(\frac{u}{\lambda f};\frac{v}{\lambda f}\right)\right|^2$$

$$t_3(u,v)=+\frac{r_0}{j\lambda f}e^{j\frac{k}{2f}[u^2+v^2]}\tilde{S}\left(\frac{u}{\lambda f};\frac{v}{\lambda f}\right)\exp(j2\pi\alpha v)$$

$$t_4(u,v)=-\frac{r_0}{j\lambda f}e^{-j\frac{k}{2f}[u^2+v^2]}\tilde{S}^*\left(\frac{u}{\lambda f};\frac{v}{\lambda f}\right)\exp(-j2\pi\alpha v)$$

To see the impulsive response at the exit, it is necessary to place a source point at the entry $\delta(x,y)$.

In the plane just before the film (developed snapshot) the amplitude of the field is written (object placed at a distance d before the lens) :

$$\begin{aligned} U_f(u,v) &= \frac{1}{j\lambda f}\cdot TF(\delta)\cdot e^{j\frac{k}{2f}(u^2+v^2)(1-\frac{d}{f})} \\ &= \frac{1}{j\lambda f}\cdot e^{j\frac{k}{2f}(u^2+v^2)(1-\frac{d}{f})} \end{aligned}$$

a) After the passage in the film of transmission in an amplitude proportional to $I(u,v)$, the amplitude of the field just after the film due at $t_3$is written :

$$U_{f3}=\left[\frac{1}{j\lambda f}\cdot e^{j\frac{k}{2f}(u^2+v^2)(1-\frac{d}{f})}\right]\left[\frac{r_0}{j\lambda f}e^{j\frac{k}{2f}[u^2+v^2]}\tilde{S}\left(\frac{u}{\lambda f};\frac{v}{\lambda f}\right)\exp(j2\pi\alpha v)\right]$$

by regrouping the arguments of the exponentials representing the curve of the quadratic phase :

$$U_{f3}=-\left[\frac{r_0}{(\lambda f)^2}\cdot e^{j\frac{k}{2f}(u^2+v^2)(1-\frac{d}{f}+1)}\right]\left[\tilde{S}\left(\frac{u}{\lambda f};\frac{v}{\lambda f}\right)\exp(j2\pi\alpha v)\right]$$

$$U_{f3}=-\left[\frac{r_0}{(\lambda f)^2}\cdot e^{j\frac{k}{2f}(u^2+v^2)(2-\frac{d}{f})}\right]\left[\tilde{S}\left(\frac{u}{\lambda f};\frac{v}{\lambda f}\right)\exp(j2\pi\alpha v)\right]$$

In the exit plane, one is going to find the TF of this term. By putting $d=2f$ one cancels the term of the quadratic phase and one re finds $s(x,y)$ translated on $y(TF[exp(j2\pi\alpha v)]=\delta(x,y+\alpha))$

b) By following the same step, the amplitude due to t_4 is written :

$$U_{f4}=\left[\frac{1}{j\lambda f}\cdot e^{j\frac{k}{2f}(u^2+v^2)(1-\frac{d}{f})}\right]\left[-\frac{r_0}{j\lambda f}\cdot e^{-j\frac{k}{2f}[u^2+v^2]}\tilde{S}^*\left(\frac{u}{\lambda f};\frac{v}{\lambda f}\right)\exp(-j2\pi\alpha v)\right]$$

$$U_{f4}=\left[\frac{r_0}{(\lambda f)^2}\cdot e^{-j\frac{k}{2f}(u^2+v^2)\frac{d}{f}}\right]\tilde{S}^*\left(\frac{u}{\lambda f};\frac{v}{\lambda f}\right)\exp(-j2\pi\alpha v)$$

By putting this time $d=0$, one re finds $s*(-x,-y)$ translated in reversed sense compared to the last question.

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