# Velocity and displacement measurements by heterodyne interferometry

Denis MOUNIER



2024/03/19

# Table des matières

# I.Présentation

## Module :

Interference and Diffraction

## Auteur(s) :

Denis MOUNIER - ENSIM – Le Mans Université

## Résumé :

The course covers instruments for measuring displacement or speed using heterodyne interferometry. It follows the course presenting displacement or speed measurements by homodyne interferometry. The instruments concerned are: industrial laser vibrometers, tracking lasers, Laser Doppler Anemometry (LDA) instrumentation. Measuring devices based on "self-mixing" or time-of-flight or phase shift methods are not discussed. We show how three-dimensional point measurements and rotation speed measurements are carried out. The metrological aspects of heterodyne laser vibrometers are presented: bandwidth, sensitivity, limit of detection of a displacement or a speed.

## Mots-clés :

Doppler effect, Homodyne interferometry, heterodyne interferometry, Vibrometry, laser velocimetry, Laser tracking, laser tracker, Laser ultrasound, Fringe counting, Photon noise, Bragg cell

## Pré-requis :

Wave optics and interference. Geometric optics. Photometry. Laser: laser sources, Gaussian beam optics, speckle. Polarization of light. Signal processing elements. Electronic noises.

## Objectif(s) pédagogique(s) :

Understand the possibilities and limits of displacement or speed measurement instruments that use heterodyne interferometry.

## Plan du cours :

- Introduction
- Reminder: frequency modulation of a light wave by the Doppler effect
- Two-wave interferometry
- Conclusion

## Conception & production :

Le Mans Université

## Licence :

# II.Principles implemented in displacement and speed measurements

The homodyne interferometer potentially has a very wide bandwidth which is limited in principle only by the response time of the photodetectors. The sensitivity to a displacement measurement is independent of frequency. Ultrasonic vibration measurements up to frequencies of a few GHz are possible with homodyne interferometry.

Ultrasound measurement with a homodyne interferometer requires servo control to stabilize the operating point of the interferometer at its maximum sensitivity in order to avoid drifts in the optical phase that its environment can create (vibrations, temperature fluctuations). This restricts its use to a minimally disturbed environment such as that of an air-conditioned laboratory.

In the vast majority of cases, it is necessary to carry out measurements in often disturbed environments, in an industrial environment or outdoors. Additionally, it is not always useful to have extremely high bandwidth. For a particular application, the user can limit their bandwidth. Most interferometers fitted to industrial instruments are **heterodyne interferometers**.

In this course, industrial measurement systems that use heterodyne interferometry will be presented: three-dimensional vibrometry, *in-plane* velocimetry, Laser Doppler Anemometry (LAD) and rotational velocimetry. Three-dimensional vibration measurements and rotational measurements are possible with multiple laser beams (Figure 1).

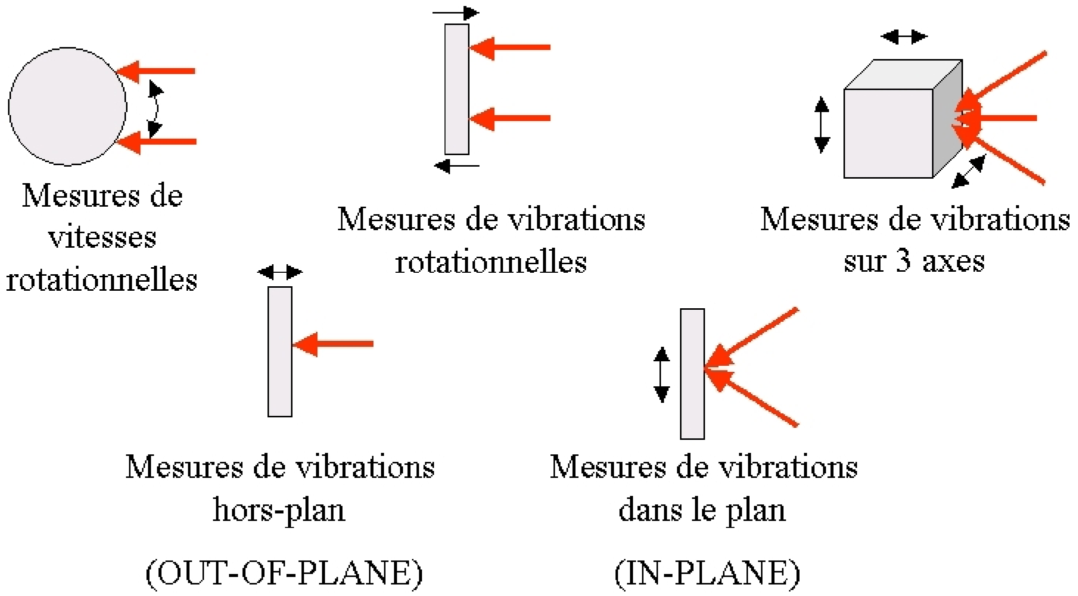


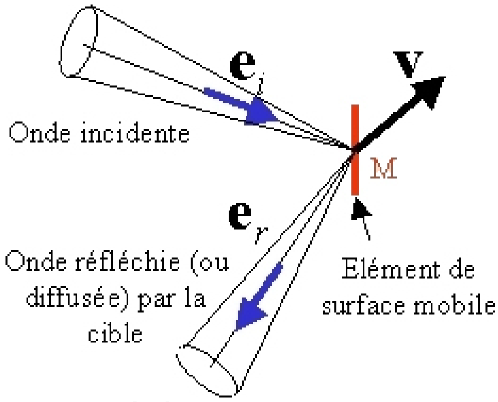

*Figure 1 - Types de déplacements mesurables au moyen de faisceaux optiques*

We will deal with frequency (or phase) demodulation which makes it possible to determine the modulating signal: speed (or movement) of the target.

## 1. Reminder: frequency modulation of a light wave by the Doppler effect

We consider a monochromatic light wave of frequency $\nu_0$ or quasi-monochromatic of spectral width $\delta\nu$ small compared to its central frequency $\nu_0$. Most continuous emission laser sources can be considered as quasi-monochromatic sources expressing the form

$s_i(\vec{r}, t) = A_i \cos(2\pi\nu_0 t + \varphi_0)$. It is assumed that the light wave is focused at a target point M of a diffusing surface. This point M has a speed $\vec{v}$ in the reference frame where the laser source is at rest. **The direction of incidence** of the wave on the target is given by the wave vector $\vec{\mathbf{k_i}}$ . We consider the light scattered in a particular direction: **the direction of observation**, characterized by the wave vector $\vec{\mathbf{k_r}}$.The unit vectors associated with the directions of incidence and observation are respectively: $\vec{\mathbf{e}_i} = \dfrac{\vec{k_i}}{\|\vec{\mathbf{k_i}}\|}$ *and* $\vec{\mathbf{e}_r} = \dfrac{\vec{\mathbf{k_r}}}{\|\vec{\mathbf{k_r}}\|}$ (see figure).




*Directions des faisceaux incident et réfléchi par une cible mobile intervenant dans la formule du décalage Doppler*

## *Fondamental*

**The light wave scattered or reflected by the moving target undergoes an instantaneous frequency shift $\delta\nu_D(t)$ called Doppler shift equal to:**

$$\frac{\vec{v}(t) \cdot (\vec{e_r} - \vec{e_i})}{\lambda}$$

**The instantaneous frequency of the reflected wave $\nu(t) = \nu_0 + \delta\nu_D(t)$ is frequency modulated by the motion of the target.**

The Doppler shift is maximal if the direction of the target velocity is collinear with the vector $\vec{\mathbf{e_r}} - \vec{\mathbf{e_i}}$ called **the sensitivity vector**.

The techniques of Laser Doppler Anemometry (LDA) and Laser Doppler Velocimetry (LDV) take advantage of this Doppler shift to measure the speeds of particles in suspension, carried by a moving gas or liquid (see §« *Laser Doppler Anemometry (LDA) and Laser Doppler Velocimetry (LDV)* »).

In the course ***Displacement and velocity measurements by homodyne interferometry***, we show how to measure such a Doppler shift with a Fabry-Pérot interferometer.

The amplitude of the light wave reflected by the target can be written as $s_r(\vec{\mathrm{r}}, t) = A_r \cos\left[2\pi\nu_0 t + \Delta\varphi(t) + \varphi_0\right]$ with :

$$\Delta\,\varphi(t) = \frac{2\pi}{\lambda}(\vec{e_r} - \vec{e_i}) \cdot \Delta\vec{r}$$

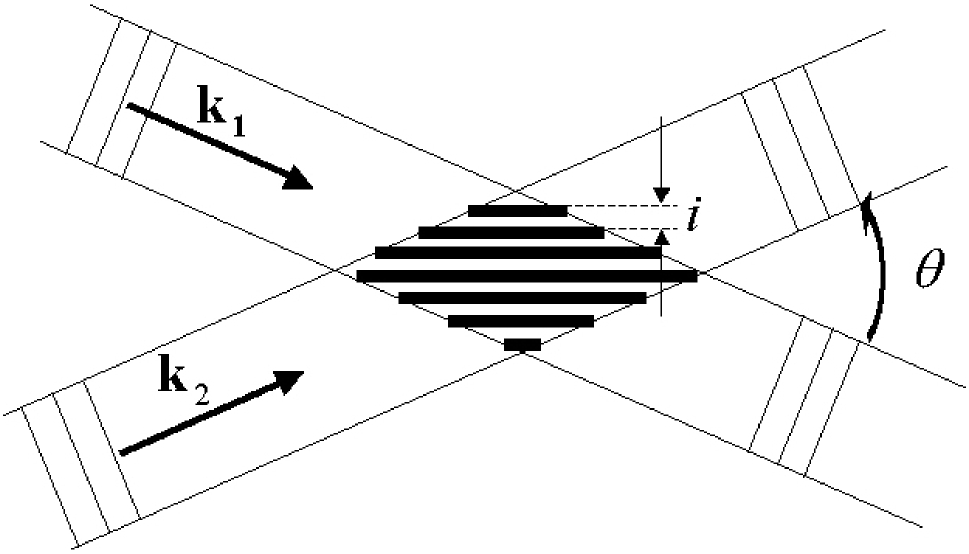
where $\Delta\varphi(t)$ is the wave **phase modulation** and $\Delta\vec{\mathrm{r}}$ is the target displacement.

This phase modulation is measurable using a two-wave interferometer where one of the two waves, **the reference wave**, has a constant frequency (or phase) while the other wave, **the probe wave**, undergoes frequency (or phase) modulation after reflection from the moving target.

# 2. Two-wave interferometry

## 2.1. Interference of two plane waves

We consider two collimated monochromatic light beams whose propagation directions are coplanar and form an angle $\theta$ between them. Since the two beams are collimated, we consider them as two monochromatic plane waves (see figure).




*Champ d'interférences de deux ondes planes*

Each wave is defined by the constant frequencies $\nu_1$ and $\nu_2$ and the wave vectors $\overrightarrow{k_1}$ and $\overrightarrow{k_2}$ . We further assume that the frequencies are neighbors $|\nu_1 - \nu_2| \ll \nu_1, \nu_2$.

The associated wavelengths are: $\lambda_1 = \frac{2\pi}{\|\overrightarrow{k_1}\|}$ and $\lambda_2 = \frac{2\pi}{k_2}$ . The real representations of the wave fields are: $s_1 = A_1 \cos(2\pi\nu_1 t - \overrightarrow{k}_1 \cdot \overrightarrow{r} + \varphi_{01}) and s_2 = A_2 \cos(2\pi\nu_2 t - \overrightarrow{k}_2 \cdot \overrightarrow{r} + \varphi_{02})$ .

These vibrations can be represented by a complex expression:

$s_i = \Re e\{A_i \exp[j(2\pi\nu_i t - \overrightarrow{k}_{\mathbf{i}} \cdot \overrightarrow{r} + \varphi_{0i}]\}$, with $i = 1, 2$, where $\Re e$ means: **real part of.**

The complex representation is translated by a Fresnel diagram (see figure). The superposition of the two wave fields results in a vector sum of the Fresnel vectors $\overrightarrow{OM}_1$ and $\overrightarrow{OM}_2$. The argument of the complex amplitude $s_i is \varphi_i = 2\pi\nu_i t - \overrightarrow{k}_{\mathrm{i}} \cdot \overrightarrow{r} + \varphi_{0i}$ . The square of the modulus of the vector $\overrightarrow{OM}$ gives the resulting luminous intensity (a quantity proportional to the luminous power density)

$I = s \cdot s^* = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos[\Phi(\overrightarrow{r}, t)], where I_1 = A_1^2, I_2 = A_2^2 and \Phi(\overrightarrow{r}, t) = 2\pi(\nu_2 - \nu_1)t + (\overrightarrow{k}_2 - \overrightarrow{k}_1) \cdot \overrightarrow{r} + \Phi_0$ . The light intensity in the interference field undergoes a double sinusoidal modulation: temporal at the frequency $\nu_2 - \nu_1$ because of the term $2\pi(\nu_2 - \nu_1)t$.) and spatial according to the spatial frequency vector: $\frac{\overrightarrow{k}_2 - \overrightarrow{k}_2}{2\pi}$, because of the term $(\overrightarrow{k}_2 - \overrightarrow{k}_2) \cdot \overrightarrow{r}$ . The spatial modulation period is by definition the interfringe: $i = \frac{2\pi}{\|\overrightarrow{k}_2 - \overrightarrow{k}_1\|}$.

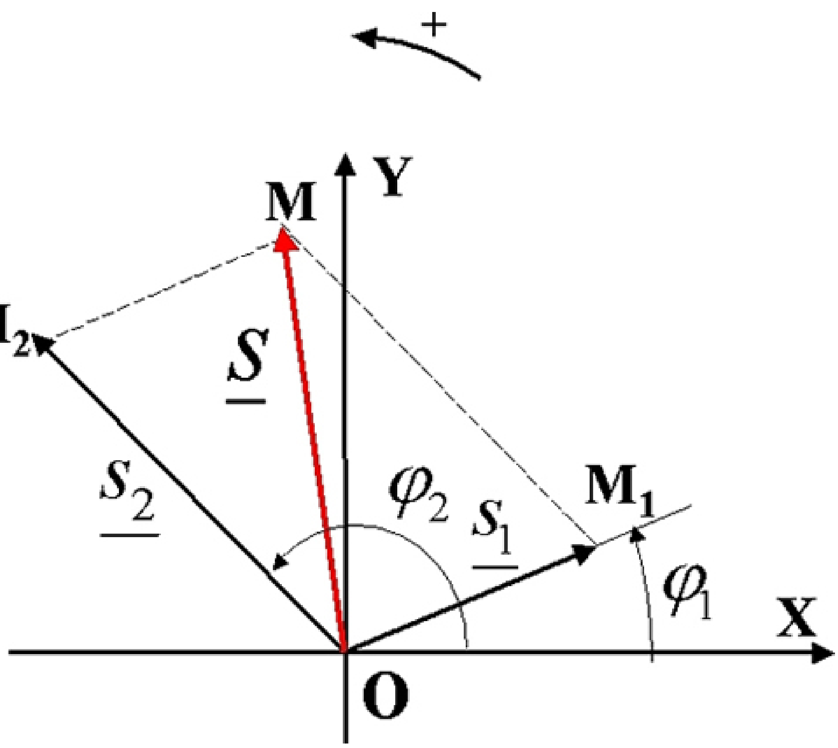



*Diagramme de Fresnel*

It is expressed as a function of the average wavelength $\lambda$ of the two waves and the angle \ theta $\theta$ between the two beams:

$$i = \frac{\lambda}{2 \cdot \sin(\frac{\theta}{2})}$$

## *Fondamental*

**The light intensity of the interference field can be written as $I = I_0(1 + m \cos \Phi)$ where $m = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$ is the interference contrast and $I_0 = I_1 + I_2$ is the average intensity.**

* *

*

To learn more, you can consult the following references: (1 [Laser Ultrasonics],2 [Optical Remote measurement of Ultrasound],3 [Génération et détection optiques d'ondes élastiques],4,5)

# III.Heterodyne interferometry

## 1. Heterodyne interferometer

### Définition

In a **heterodyne interferometer**, at least one of the two waves (reference or probe) undergoes a frequency change $\pm\nu_b$ , positive or negative, typically of the order of $40\ \mathrm{MHz}$ to $100\ \mathrm{MHz}$ .

### Exemple

Let us consider as an example the Royer-Dieulesaint interferometer described in Figure 2 [3 [Génération et détection optiques d'ondes élastiques]].

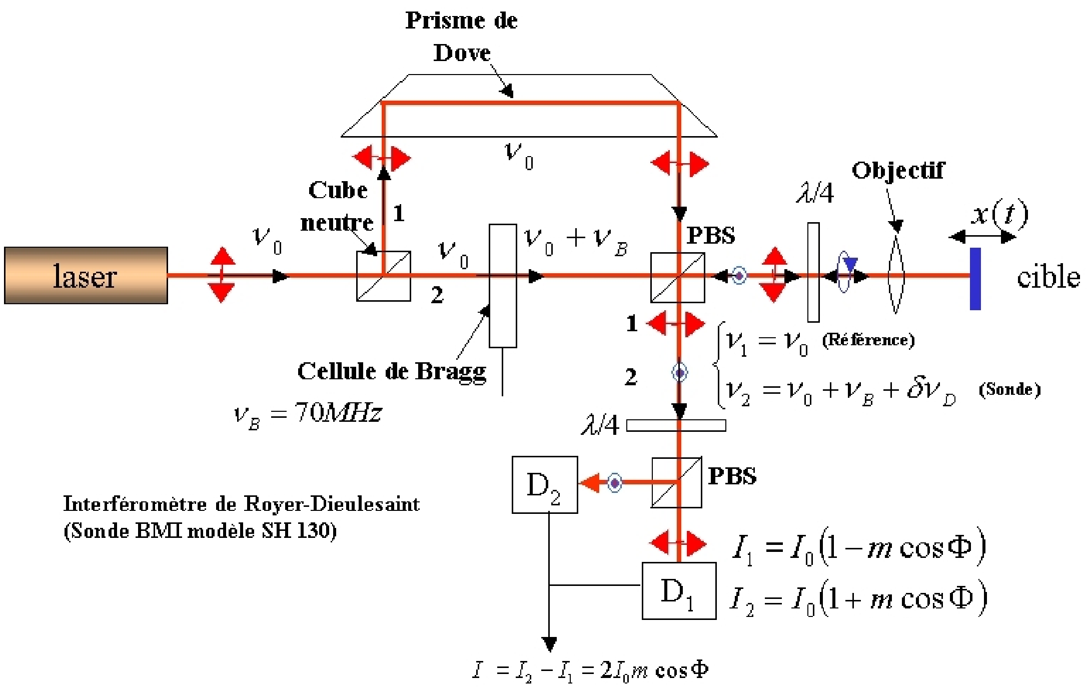



*Figure 2 - Interféromètre hétérodyne de Royer-Dieulesaint*

This heterodyne interferometer forms the basis of a laser vibrometer. It was designed for measuring ultrasonic vibrations up to frequencies of $35\ \mathrm{MHz}$ . The original optical setup of this interferometer is a hybrid Mach-Zehnder and Michelson interferometer. The optical assembly uses a laser wave polarized in the horizontal plane. It is divided by a neutral cube, without change of polarization state, into a reference wave which is reflected and a probe wave which is transmitted. The reference wave undergoes two total reflections in a Dove prism and is then returned to the photodiodes. The frequency of the reference wave is $\nu_1 = \nu_0$ .

The probe wave passes through a Bragg cell which shifts its frequency by $\nu_B = +70\ \mathrm{MHz}$. After the Bragg cell, the frequency is $\nu_0 + \nu_B$. The probe wave is completely transmitted towards the target by a polarization splitter cube with horizontal polarization. It passes through a quarter-wave plate before being focused on the target where it undergoes the Doppler shift $\delta\nu_D$ . The backscattered light is reflected by the polarization splitter cube because its polarization has become vertical after the double crossing of the quarter-wave plate. Finally, the probe wave, which has frequency $\nu_2 = \nu_0 + \nu_B + \delta\nu_D$, interferes with the reference wave of frequency $\nu_1 = \nu_0$ .

The difference in currents is $I = 2I_0 m \cos\Phi$ with a phase shift $\Phi = 2\pi\nu_B t + 2\pi \int_0^t \delta\, nu_D(t)dt + \Phi_0$ with $\delta\nu_D(t) = \dfrac{\overrightarrow{v}(t).(\overrightarrow{e_r} - \overrightarrow{e_i})}{\lambda}$.

D'où $\Phi = 2\pi\nu_B t + \dfrac{4\pi}{\lambda}(\overrightarrow{e_r} - \overrightarrow{e_i}) \cdot \int_0^t \overrightarrow{v}(t')dt' + \Phi_0 = 2\pi\nu_B t + \dfrac{4\pi}{\lambda}(\overrightarrow{e_{er}} - \overrightarrow{e_i}) \cdot \Delta\overrightarrow{r} + \Phi_0$.

The interference signal is therefore a sinusoidal signal of frequency $\nu_B$ modulated in phase by the term $\dfrac{4\pi}{\lambda}(\overrightarrow{e}_r - \ overrightarrowe_i) \cdot \Delta\overrightarrow{r}(t)$.

The **phase modulation** is provided by the movement of the target.

From another point of view, we can also consider the interference signal as a frequency modulated sinusoidal signal, with a carrier frequency $\nu_B$and a frequency modulation $\delta\nu_D(t) = \dfrac{\overrightarrow{v}(t) \cdot (\overrightarrow{e}_r - \overrightarrow{e}_{i)}}{\lambda}$ depends on the instantaneous speed $\mathbf{v}(t)$.

We therefore have the following two equivalent associations:

1. Phase modulation <==> shift
2. frequency modulation <==> speed

Ultrasonic vibration speeds are generally less than $1\ \mathrm{m/s}$ which corresponds to Doppler shifts less than $4\ \mathrm{MHz}$(for $\lambda = 500\ \mathrm{nm}$). This frequency is generally lower than the Bragg frequency $\nu_B$ . To extract the speed, it is necessary to demodulate the interference signal. This is a frequency demodulation similar to that carried out in FM radio receivers. Figure 3 shows the simulation of a heterodyne interference signal obtained with a target in harmonic vibration (continuous red curve) with a frequency equal to 1/20 of the Bragg frequency $\nu_b$ and an amplitude equal to a optical wavelength.

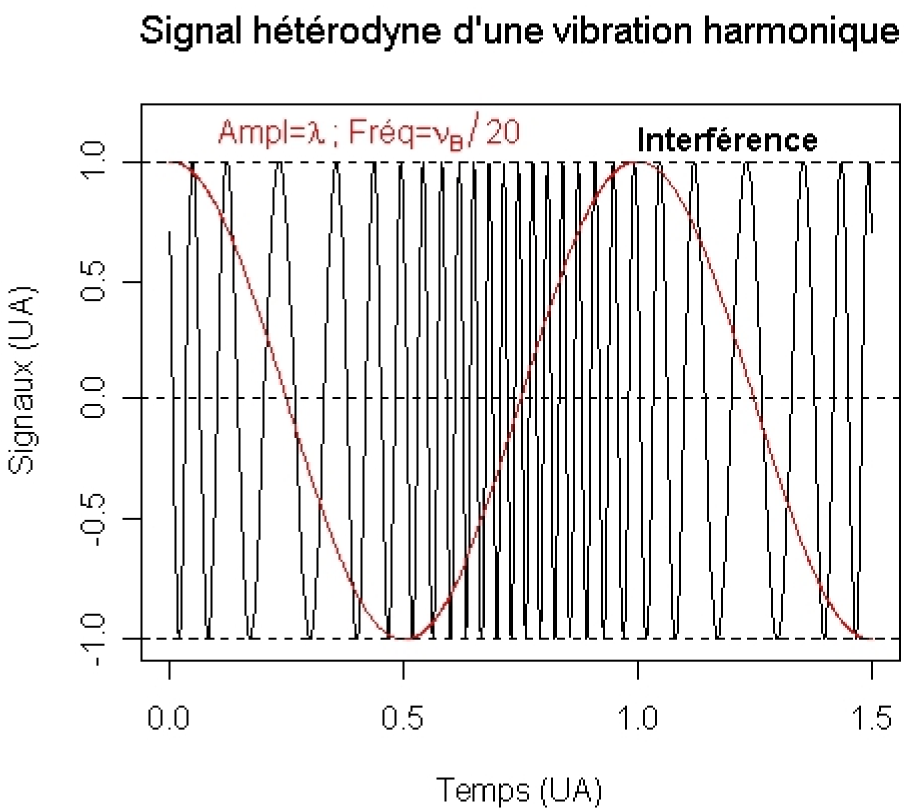



*Figure 3 - Signal d'un interféromètre hétérodyne pour une cible en oscillation sinusoïdale*

The maximum Doppler shift is equal to $\pm\frac{\pi}{5}\nu_B$. The corresponding spectrum is shown in Figure 4.

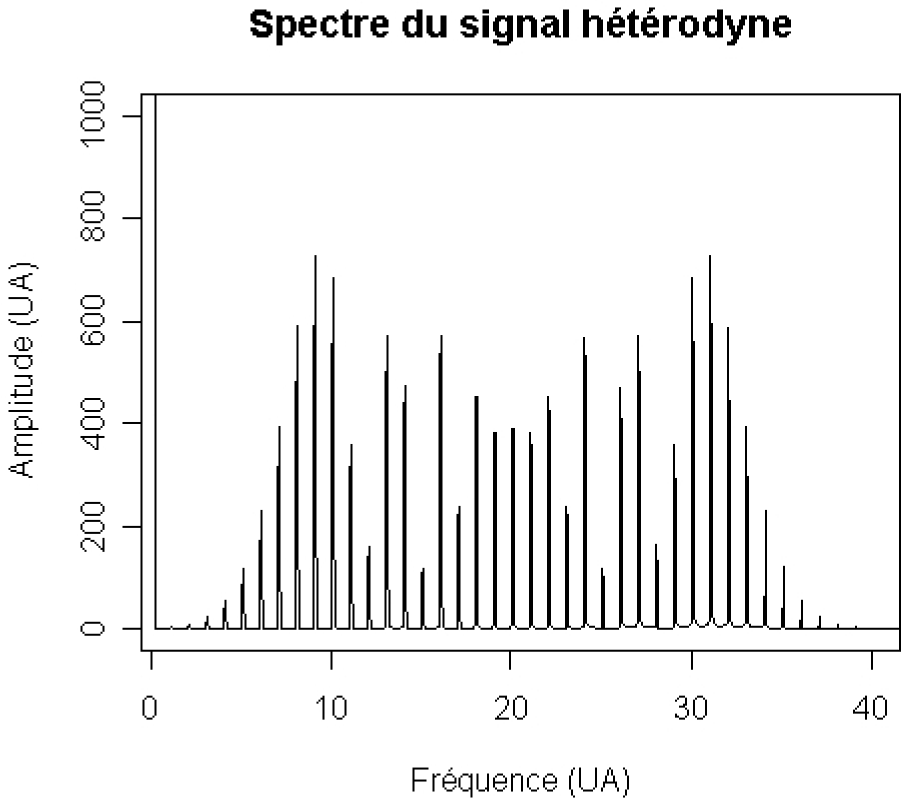



*Figure 4 - Spectre calculé par FFT sur le signal hétérodyne de la figure 3*

If the Bragg frequency is $\nu_B = 20\ \mathrm{MHz}$ , then the frequency of the sinusoidal vibration which is shown in Figure 3 is $1MHz$. The spectrum in Figure 4 is centered on the carrier frequency which is $20\ \mathrm{MHz}$ and which extends approximately $\pm 15\ \mathrm{MHz}$ around the central frequency. The optoelectronic detection circuit must therefore have a minimum bandwidth of $30\ \mathrm{MHz}$ to transmit without loss all the vibration information carried by the interference signal.

The $\Delta f$ bandwidth that the detection system must have can be calculated by applying Carlson's rule: $\Delta f = 2(\Delta f_{Dmax} + f_{smax})$, where $\Delta f_{Dmax}$is the maximum Doppler shift and $f_{smax}$is the maximum frequency of the vibrational velocity spectrum, which is assumed to be bounded.

From which we calculate a bandwidth $\Delta f = 2\left(\frac{\pi}{5}\nu_B + \frac{\nu_B}{20}\right) \approx 27\ textMHz$ , which agrees with the spectrum in Figure 4.

The heterodyne interferometer has limited bandwidth. To show this, suppose that we wish to measure a sinusoidal vibration of the same amplitude as in the previous example but of frequency equal to $2\ \mathrm{MHz}$. According to Carlson's rule, the $\Delta frequencies f_{Dmax}$and $f_{smax}$ will double. It will therefore be necessary to have a bandwidth of around $60\ \mathrm{MHz}$ to avoid any loss of information.

However, with a Bragg frequency $\nu_B = 20\ \mathrm{MHz}$ , the maximum bandwidth is between $0$ and $40\ \mathrm{MHz}$. The bandwidth of the heterodyne interferometer is limited by the choice of the Bragg frequency $\nu_B$. It is therefore necessary to adjust this Bragg frequency according to the frequencies and amplitudes of the vibrations to be measured.

The bandwidth of the detection electronics must be strictly limited to the useful band which is necessary for the measurement. Otherwise, the noise is increased unnecessarily, which has the effect of deteriorating the signal-to-noise ratio. The signal-to-noise ratio of a heterodyne interferometer can be compared to that of a homodyne interferometer having the same measurement range and same vibration measurement bandwidth (§ *Case study*).

## *Complément*

The change in frequency occurring in a **Bragg cell** is obtained by **acousto-optic interaction**. The light beam interacts with a progressive ultrasonic acoustic wave of frequency $\nu_b$ propagating in a transparent solid medium (often tellurium dioxide: $\mathrm{TeO_2}$). The acoustic wave crosses the optical wave at almost a right angle. The change in frequency experienced by the optical wave in Bragg diffraction is equal to the frequency of the acoustic wave. This change in frequency can be interpreted simply by considering that the acousto-optic interaction is an inelastic collision between an incident photon of energy $h\nu_0$and a phonon of energy $h\nu_b$ . The phonon is annihilated during the collision; its energy and momentum is transferred to the incident photon. The scattered photon having gained the energy and momentum of the phonon is deflected in the collision. The energy of the scattered photon is by virtue of the principle of conservation of energy: $h(\nu_0 + \nu_b)$ , hence the frequency increase $\nu_b$ of the frequency of the photon during the acousto-optic interaction. The beam deviation in Bragg diffraction can be interpreted from a classical point of view. A longitudinal acoustic wave creates, through the elasto-optical effect, a modulation of the refractive index of the medium with a period equal to the length of the acoustic wave $\lambda_b = c_a/\nu_b$, where $c_a$ is the speed of the acoustic wave $(c_a = 4200\ \mathrm{m/s} in \mathrm{TeO_2})$. The areas compressed by the elastic wave have a higher refractive index than the dilated areas. If the acoustic wave has a frequency $\nu_B = 80\ \mathrm{MHz}$, then its wavelength in the $\mathrm{TeO_2}$ is equal to $52.5\ \mu\mathrm{m}$ . The wavelength coincides with the pitch of the index grating induced by the acoustic wave. The diffraction of a light beam (refractive index grating created by an acoustic wave) is analogous to the Bragg diffraction of an x-ray beam by the atomic planes of a crystal. Applied to acousto-optic interaction, the Bragg relation is written: $2d.\sin(\theta) = m.\lambda$ , where $\theta$ is the angle of the beam with respect to the planes, $d$ is the grating period,$\lambda$ is the optical wavelength in the medium, and m is the diffraction order.

# IV.Measurements of several displacement or speed components

Using a single probe beam limits the displacement or velocity measurement to a single component. To measure a second component, it will necessarily be necessary to use a second beam and to measure three components, a third beam. We will first see the measurement of two components (§ 3.1 to 3.3), then of three components (§3.4).

## 1. "In-plane" and "out-of-plane" vibration measurements

### *Rappel*

Remember that measuring the movement or speed of a target can be done with a single optical beam provided that the direction of the sensitivity vector $\vec{s} = \vec{e}_r - \vec{e}_i$ is not perpendicular to the displacement $\vec{r}$ or to the speed $\vec{v}$. In the most common measurement configuration, the direction of the probe beam is normal to the target surface and the light is backscattered in the direction of the incident beam. Consequently, we only detect the out-of-plane component $\vec{v}_n$ because the direction of the sensitivity vector is: $\vec{s} = -2\vec{e}_i = 2\vec{e}_r$(figure 5).

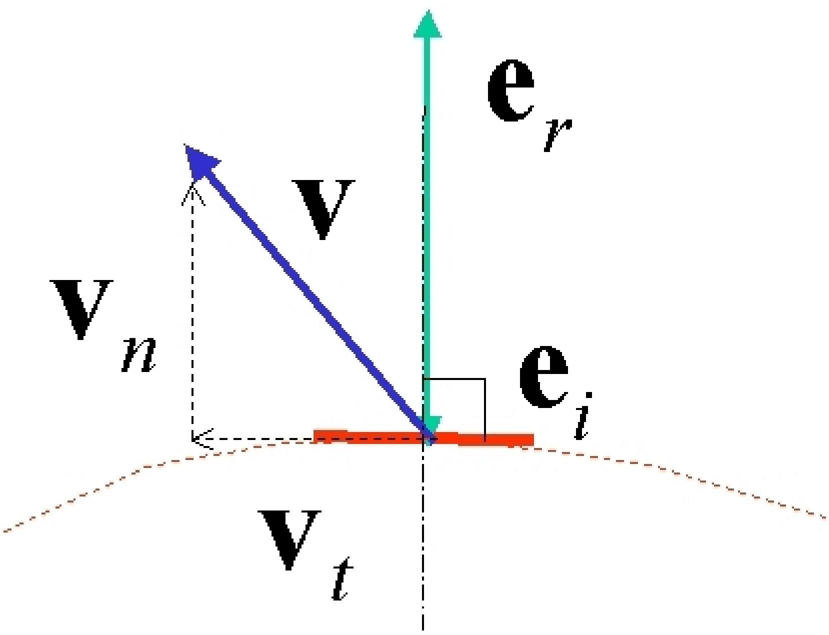



*Figure 5 - Composantes « dans-le-plan » et « hors-plan » de la vitesse de la cible*

When the surface is curved, the plane tangent to the surface of the target serves as a reference to define the notion of *out-of-plane* and *in-plane* component.

The measurement of the transverse component $\mathbf{v_t}$, called the in-plane component, is not accessible with a probe beam at normal incidence. To access the transverse component, the probe beam must necessarily be incident outside the normal. In this case, the Doppler shift is: $\delta\nu_D = (\vec{v}_n + \vec{v}_t) \cdot \vec{s}/\lambda$. There are two unknowns: the normal component $\mathbf{v_n}$ and the transverse component $\vec{v}_t$ of the speed. If the normal component is a priori negligible compared to the transverse component, a single measurement allows access to the component in the plane $\vec{v}_t$.

Otherwise, at least two measurements will have to be taken in different directions of the probe beam.

Let $\mathbf{s}_1$ and $\mathbf{s}_2$ be the sensitivity vectors corresponding to the two beam configurations. The two Doppler shifts are: $\delta\nu_{D1} = (\vec{v}_{\mathrm{n}} + \vec{v}_{\mathrm{t}}) \cdot \vec{s}_1/\lambda$ and $\delta\nu_{D2} = (\vec{v}_{\mathrm{n}} + \vec{v}_{\mathrm{t}}) \cdot \vec{s}_2/\lambda$. Let's choose two directions that are symmetrical about the normal. The sensitivity vectors are therefore: $\vec{s}_1 = \vec{s}_{\mathrm{n}} + \vec{s}_{\mathrm{t}} and \vec{s}_2 = \vec{s}_{\mathrm{n}} - \vec{s}_{\mathrm{t}}$ (Figure 6).

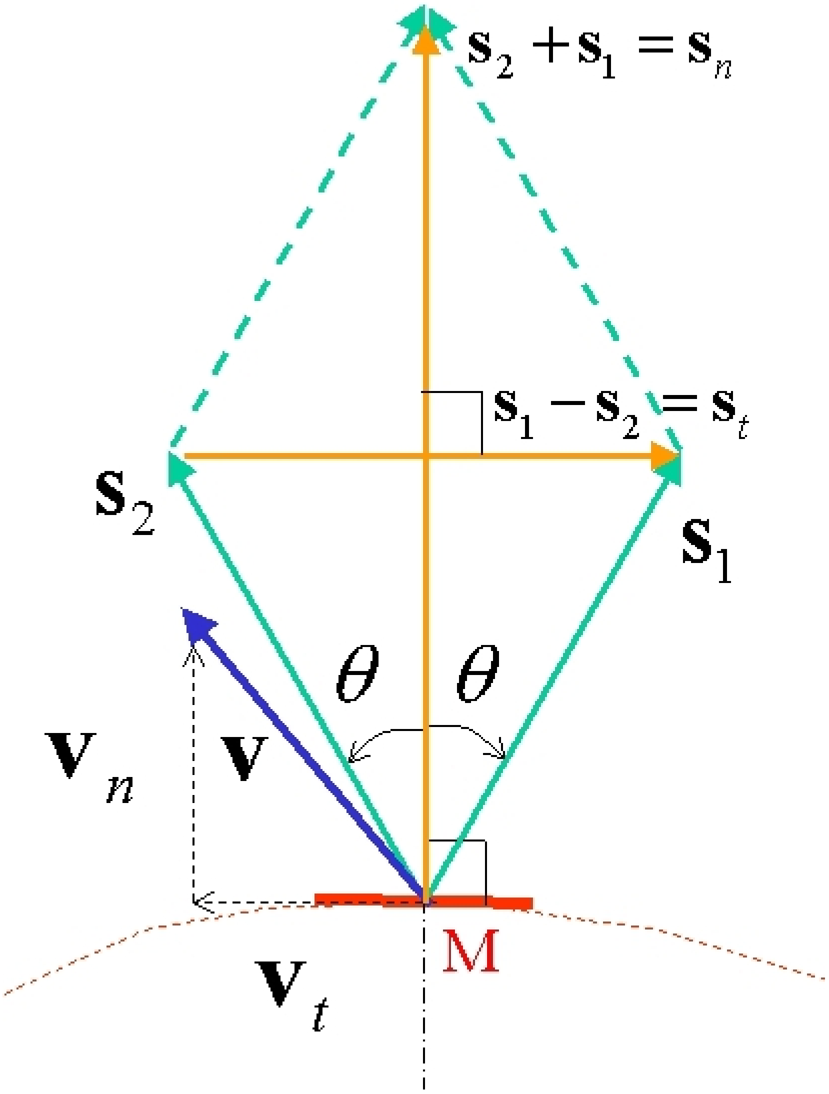



*Figure 6 - Mesure des composantes « dans le plan «  et « hors-plan » avec deux faisceaux sonde*

Subtracting the two Doppler shifts $1-2$ gives: $\delta\nu_{D_1} - \delta\nu_A = 2\vec{\mathrm{v}} \cdot \vec{s}_{\mathrm{t}}/\lambda$ . allows to extract the component in the plane while the sum of two Doppler shifts $\delta\nu_{D1} + \delta\nu_{D2} = 2\mathbf{v}\ .\mathbf{s}_{textbfn}/\lambda$ allows you to extract the out-of-plane component. If the phenomenon studied is repeatable, we can use the same laser vibrometer pointed successively at the same target point in both directions $\vec{s_1}$and $\vec{s_2}$ . A quicker solution is to use two laser vibrometers to have two simultaneous measurements.

## *Remarque*

We must not lose sight of the fact that the measurement of a transverse displacement cannot be carried out on a perfectly reflective surface because there would be no backscattered light. If the surface is shiny, we can apply to the surface, if possible, a paint with a strong retroreflective power or stick an adhesive which has the same properties to transform the surface into a **cooperative target**.

## 2. Surface velocimetry

When we are only interested in the transverse speed, the measurement configuration can be limited to two beams crossing at a point on the target and making an angle $\theta$between them. The plane of the two probe beams contains the normal to the surface passing through the point M of the target and it is assumed that the velocity vector $\mathbf{v}$ is in the plane of the beams. The scattered light is collected in the direction of the bisector of the two beams by a large numerical aperture objective (figure 7).

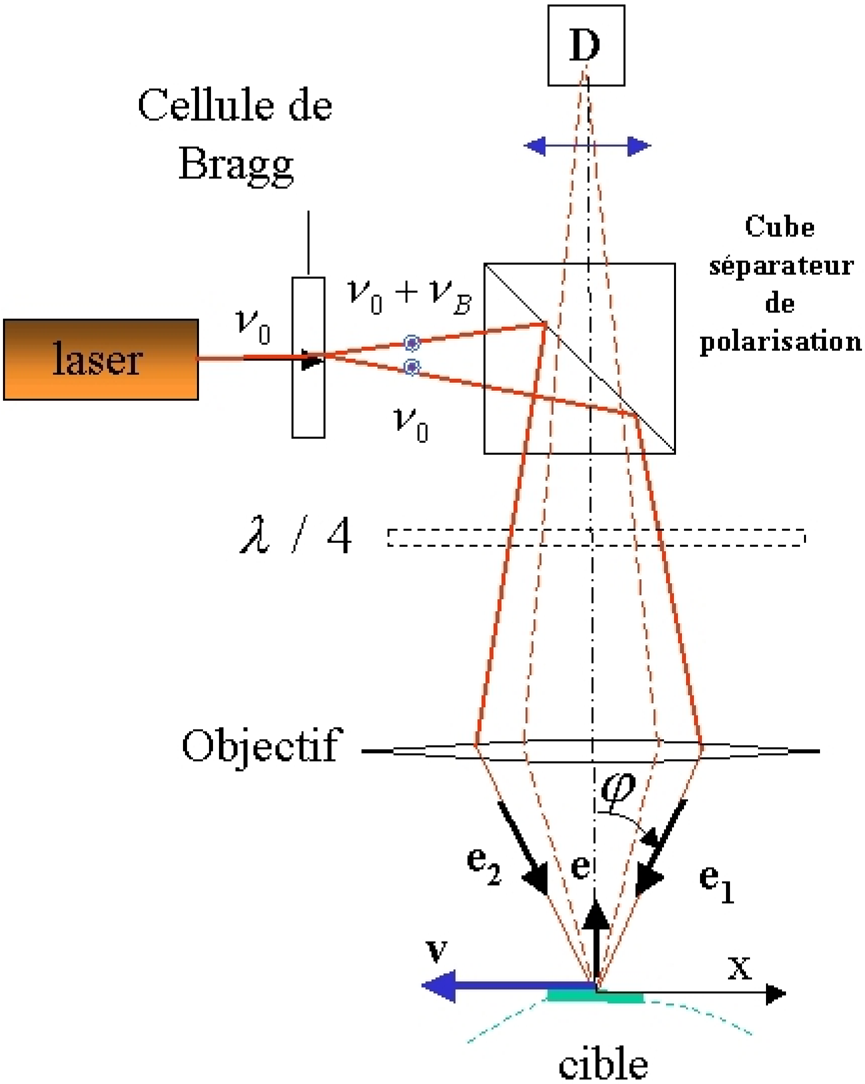



*Figure 7 - Interféromètre hétérodyne sensible à la composante « dans-le-plan » de la vitesse ou du déplacement*

The directions of incidence are given by the unit vectors $\vec{e_1}$ and $\vec{e_2}$ and the direction of observation of the scattered light is that of the unit vector*overrightarrowe* . The two beams scattered towards the photodetector have the frequencies $\nu_1 = \nu_0 + \delta\nu_{D1}$ and $\nu_2 = \nu_0 + \delta\nu_{D2}$, where $\nu_b$ is the frequency shift introduced by the Bragg cell on the diffracted beam at order 1 and $\delta\nu_{D1}$ and $\delta\nu_{D2}$ are the Doppler shifts $\delta\nu_{D1} = \vec{v} \cdot (\vec{e} - \vec{e_1})/\lambda$ *and* $\delta\nu_{D2} = \vec{v} \cdot (\vec{e} - \vec{e_2})/\lambda$. The two scattered beams interfere at the photodetector giving a frequency modulation signal

$f_1 = \nu_b + \vec{v} \cdot (\vec{e_2} - \vec{e_1})/\lambda = \nu_b + 2v \sin\varphi/\lambda$ , with a carrier frequency equal to $\nu_b$ . This type of interferometer is heterodyne. Frequency demodulation of the interference signal gives a signal proportional to the transverse speed of the target.

# 3. Laser Doppler Anemometry (LDA) et Laser Doppler Velocimetry (LDV)

Laser Doppler Anemometry is a technique for measuring gas flows. It is analogous to the technique of measuring transverse displacements of a solid surface. The gas being transparent, it must be seeded with small particles of micrometric or submicrometric dimensions which are carried by the gas and which therefore have the same speed as it. The particles behave like light-scattering targets. The speed measuring instrument must not obstruct the gas flow, there is therefore no other solution than to arrange the probe beams near the direction perpendicular to the flow (figure 8) .

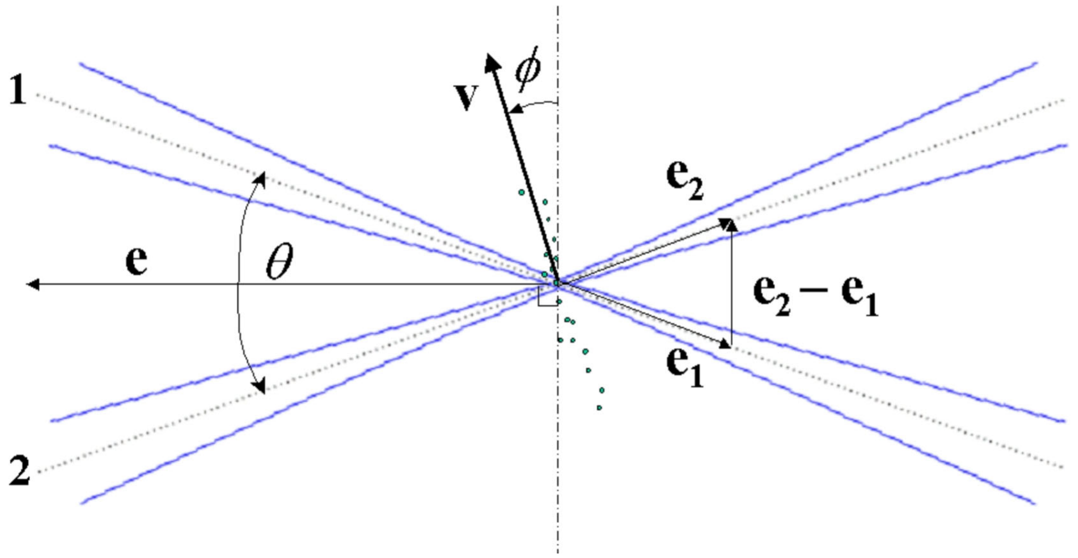



*Figure 8 - Configuration des faisceaux utilisés en anémométrie Laser Doppler (ALD)*

The configuration of the probe beams is the same as that allowing transverse velocity measurements. The two beams are focused into a small target area called the measurement volume (figure 9).

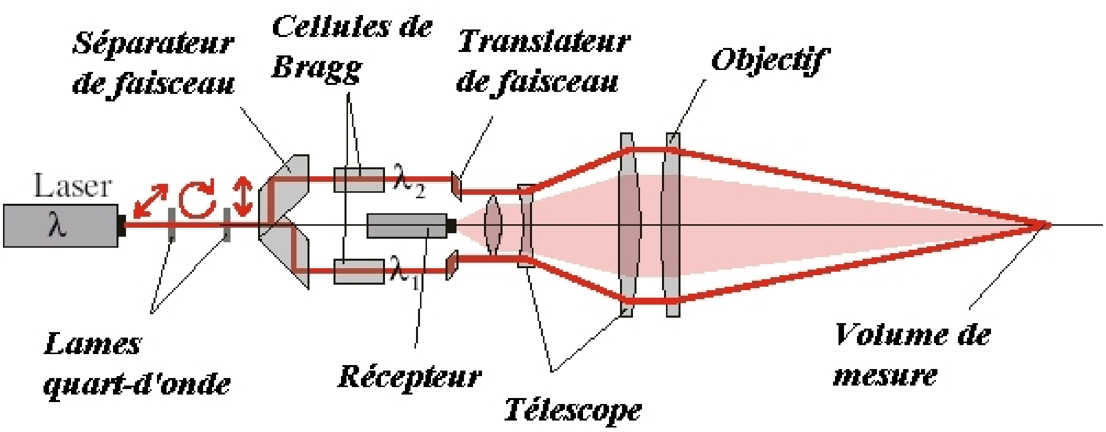



*Figure 9 - Configuration générale d'un anémomètre laser Doppler*

The measurement volume coincides with the minimum diameter zone of the laser beams.

A laser beam has a **Gaussian** transverse intensity profile: $lI(r) = I_0 \exp(-\frac{2r^2}{w^2})$ , where r is the distance of a point from the axis of the beam and w is the Gaussian radius of the beam, defined as the radius of the circle where the illuminance is $1/e^2 \approx 0.135$ of the value in the center$r = 0$. The minimum value of the beam radius at focus is the belt radius: $w_0 = (\lambda f)/(\pi w)$ , where f is the focal length of the lens and w is the front beam radius the objective.

Particles passing through the measurement volume scatter light in all directions with non-uniform intensity. Light is observed in the direction of the unit vector $\overrightarrow{e}$. Before diffusion the beams have respectively the frequencies $\nu_1 = \nu_0 + \nu_{b1}$and $\nu_2 = \nu_0 + \nu_{b2}$, where $\nu_{b1}$and $\nu_{b2}$ are the Bragg frequencies and $\nu_0$ is the laser frequency. The scattered beam 1 having undergone the Doppler shift $\delta\nu_{D1}$ has a frequency $\nu_1 = \nu_0 + \nu_{b1} + \delta\nu_{D1}$; it interferes with the scattered beam 2 which has undergone a Doppler shift $\delta\nu_{D1}$ of frequency $\nu_2 = \nu_0 + \nu_{b2} + \delta\nu_{D2}$ . The interference signal has an instantaneous frequency equal to the difference of the two frequencies: $\nu_1 - \nu_2 = (\nu_{b1} - \nu_{b2}) + (\delta\nu_{D1} - \delta\nu_{D2})$.

The Doppler shifts on beams 1 and 2 are respectively: $\delta\nu_{D1} = \overrightarrow{v} \cdot (\overrightarrow{e} - \overrightarrow{e_1})/\lambda$ and $\delta\ nu_{D2} = \overrightarrow{v} \cdot (\overrightarrow{e} - \overrightarrow{e_2})/\lambda$ , where $\overrightarrow{e_1}$ and $\overrightarrow{e_2}$ are the unit vectors of the two beams incident on the measurement volume. The difference of the two Doppler shifts is $\delta\nu_D = \delta\nu_{D1} - \delta\nu_{D2} = \overrightarrow{v} \cdot (\overrightarrow{e_2} - \overrightarrow{e_1})/\lambda$.

The interference signal is a frequency modulated signal: $f_I = \nu_b + \overrightarrow{v} \cdot (\overrightarrow{e_2} - \overrightarrow{e_1})/\lambda$ with carrier frequency $\nu_b = \nu_{b1} - \nu_{b2}$ . Demodulation of the signal allows access to the term $\delta\nu_D = \overrightarrow{v} \cdot (\overrightarrow{e_2} - \overrightarrow{e_1})/\lambda$ which depends on the angle $\phi$ between the speed and the vector $\overrightarrow{e_2} - \overrightarrow{e_1}$ and on the angle $\theta$ between the directions of the two unit vectors $\overrightarrow{e_1}$ and $\overrightarrow{e_2}$ . According to Figure 9, we have $\delta\nu_D = 2\mathrm{v} \cdot \cos(\phi)\sin(\theta/2)/\lambda$ .

The measuring principle that is used in **Laser Doppler Anemometry** can be transposed to measure liquid flow velocities. The measurement technique analogous to LDA in liquids is called **Laser Doppler Velocimetry** (LDV). The liquid should entrain small light-scattering particles. This is how the speed of blood flow can be measured through the diffusion of suspended components in the blood, such as blood cells and platelets.

## 4. 3D measurements

It should be noted, however, that measurements in two directions coplanar with the normal only give that part of the transverse component which is in the plane of the beams. A third measurement direction is therefore necessarily needed to access the three components of the speed. The use of three laser vibrometers has been implemented in the PSV-400-3D system from Polytec®. It allows the measurement of the three Cartesian components of speed or displacement with automatic scanning of the surface. (see *https://www.dbkes.com.tr/brosur/psv_400_3D.pdf*[1]) (consulted on12/12/2024)

We will show in this paragraph that measuring the three components of the speed of a point requires only three probe beams. Let $\mathbf{e_1}$, $\mathbf{e_2}$and $\mathbf{e_3}$ be three unit vectors defining three non-coplanar directions whose components are denoted: $\mathbf{e_i}\begin{pmatrix}\alpha_i\\ \beta_i\\ \gamma_i\end{pmatrix}$ in an Oxyz marker in the laboratory (Figure 10).

1 - http://www.polytec.com

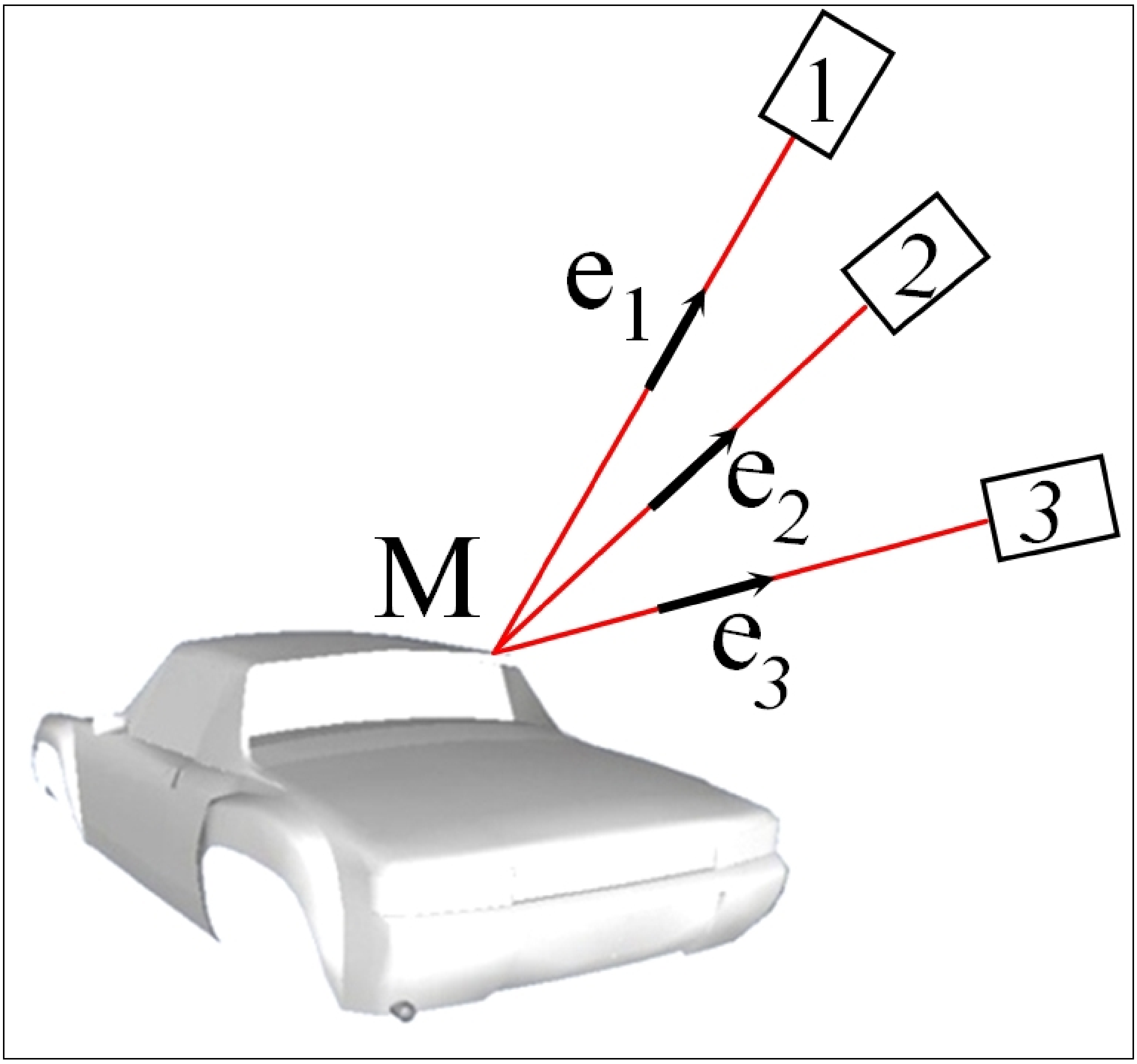



*Figure 10 - Configuration à trois vibromètres laser permettant la mesure des trois composantes cartésiennes de la vitesse ou du déplacement d'un point*

The probe beams of the three velocimeters (or vibrometers) are focused at a point M whose speed we want to measure $\vec{\mathbf{V}}\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$. Each velocimeter measures a speed signal which is proportional to the Doppler shift $\delta\nu_{Di}$ experienced by the probe beam backscattered by the target M in the direction $\vec{e_i}$ .

It is assumed that three probes have the same wavelength $\lambda$. Therefore, the Doppler shift on the ith beam is: $\delta\nu_{Di} = \dfrac{2\vec{\mathbf{V}} \cdot \vec{\mathbf{e_i}}}{\lambda}$

The component of the velocity in the direction of $\vec{\mathbf{e_i}}$ *is* : $S_i = \dfrac{\lambda}{2}\delta\nu_{Di} = \vec{\mathbf{V}} \cdot \vec{\mathbf{e_i}} = \alpha_i V_x + \beta_i V_y + \gamma_i V_z$

We can summarize the three linear relations in matrix form: $\mathbf{M} = \begin{pmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{pmatrix}$, *where* $\vec{\mathbf{V}} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$, and $\vec{\mathbf{S}} = \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$.

The determinant of the matrix **M** is non-zero because the unit vectors $\vec{e_1}, \vec{e_2}, \vec{e_3}$ are linearly independent. We can obtain the components of the velocity in the Oxyz frame by the relation $\mathbf{V=M^{-1}.S}$ , where $\mathbf{V=M^{-1}.S}$ is the inverse of the matrix **M** .

# V.Rotational measurements

In certain situations it is necessary to measure the rotation speed of a solid object. We will see in § 4.1 the principle of measuring a component of the rotation vector $\vec{\Omega}$ of a solid. In § 4.2, we will describe the principle of measuring a laser gyrometer. It is an on-board precision instrument, based on a Sagnac interferometer, which makes it possible to measure the three components of the rotation speed.

## 1. Rotational velocimeter

Suppose we want to measure the angular velocity $\vec{\Omega}$ of a cylinder rotating around an axis. We can consider using a single-beam velocimeter to detect rotation. The probe beam is focused at a point M on the surface whose linear velocity $\vec{\mathbf{v}}$ has a component in the direction of the probe beam (figure 11).

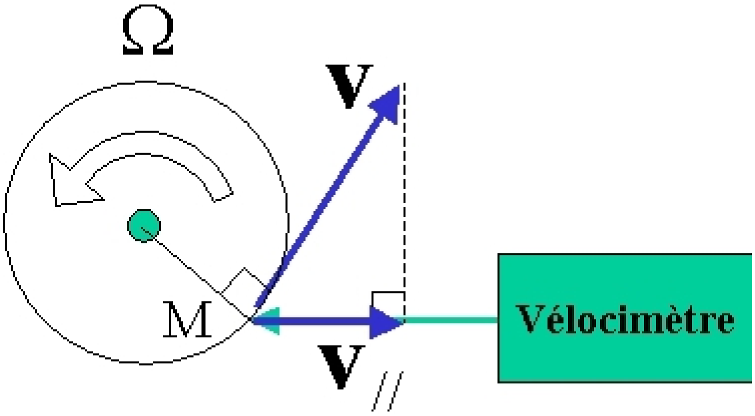



*Figure 11 - Mesure de la vitesse angulaire d'un cylindre tournant par mesure d'une composante de la vitesse*

The surface of the cylinder must diffuse light well to obtain the best possible signal-to-noise ratio. The use of a single probe beam requires calibrating the velocimeter in the measurement configuration to establish the proportionality relationship between the angular velocity and the component of the linear velocity that is measured.

If the surface of the rotating object does not have revolution symmetry around the axis of rotation, measuring the instantaneous angular velocity with the previous method will not be possible due to the influence of the shape of the surface on the speed signal (figure 12).

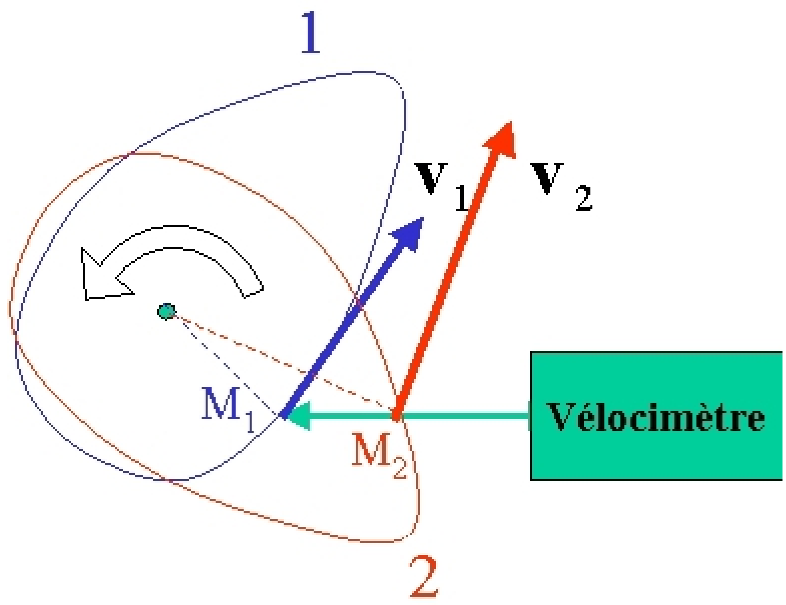



*Figure 12 - Influence de la forme de l'objet en rotation sur la mesure de la vitesse lorsqu'on n'utilise qu'un seul faisceau sonde*

The point of intersection of the probe beam with the surface being variable during rotation, the speed signal will be variable even if the rotation speed is constant.

To measure an angular rotation speed that is independent of the dimensions and shape of the object, a configuration with two parallel probe beams is required. An interferometer is matched to each beam (figure 13).

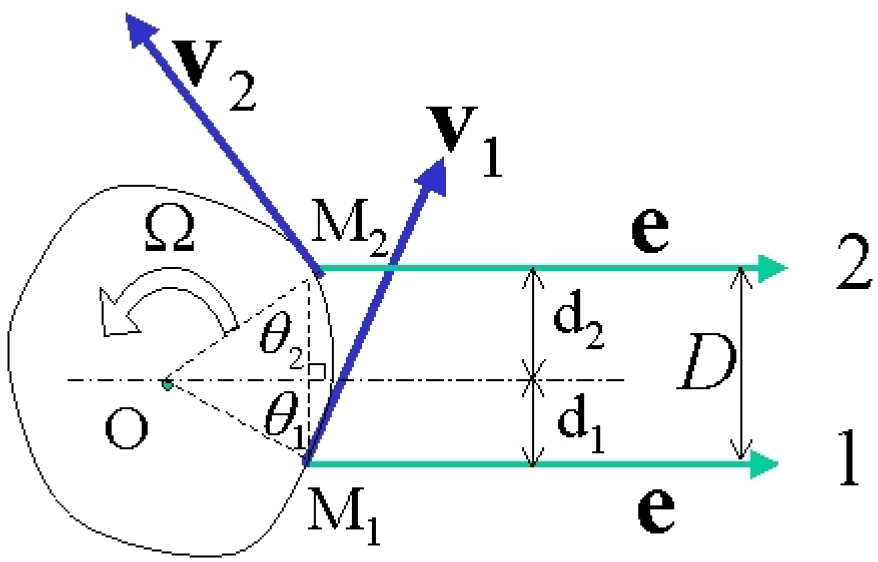



*Figure 13 - Principe d'un vélocimètre rotationnel à deux faisceaux parallèles*

We will show that the angular velocity signal can be obtained from the difference of the two Doppler shifts experienced by the two probe beams. Let $\vec{\mathbf{e}}$ be the unit vector indicating the backscattering direction of the two probe beams. The two Doppler shifts after backscattering on the points $M_1$ and $M_2$are respectively: $\delta \nu_{D1} = 2\vec{\mathbf{v}_1} \cdot \vec{\mathbf{e}} / \lambda and \delta \nu_{D2} = 2\vec{\mathbf{v}_2}.\vec{\mathbf{e}} / \lambda.$

The difference of the two offsets is:

$$\delta\nu = 2\,(\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1)\cdot\vec{\mathbf{e}}/\lambda = 2\,(\vec{\mathbf{v}}_O + \vec{\Omega}\times\overrightarrow{\mathbf{OM}_2} - \vec{\mathbf{v}}_O + \vec{\Omega}\times\overrightarrow{\mathbf{OM}_1})\cdot\mathbf{e}/\lambda$$

We used the formulas of solid kinematics: $\vec{\mathbf{v}}_1 = \vec{\mathbf{v}}_O + \vec{\Omega}\times\overrightarrow{\mathbf{OM}_1}$ and $\vec{\mathbf{v}}_2 = \vec{\mathbf{v}}_0 + \vec{\Omega}\times\overrightarrow{\mathbf{OM}_2}$, where $\vec{\mathbf{v}}_0$ is the translation speed of the axis. Using the permutation property of the mixed product, we obtain:

$$\delta\nu = \left(\frac{2D}{\lambda}\right)\vec{\Omega}\cdot\vec{\mathbf{e}}_2$$

where $\vec{e}_z = \vec{e}_x \times \vec{e}_y$ is the unit vector perpendicular to the plane of the two beams. This result shows that we measure the component of the rotation vector which is in the direction perpendicular to the plane of the two beams. The measurement is not influenced by a possible translation of the axis of rotation. The sensitivity of the measurement increases linearly with the distance *D* between the two beams.

### *Remarque*

To obtain optimal detection, the shape of the surface is not important provided you are probing a non-deformable solid. A translation of the velocimeter in the plane of the beams has no influence on the measurement. However, there is a device position that optimizes the signal-to-noise ratio.

## 2. Fiber laser gyroscope or “gyrofiber”

We consider a Sagnac interferometer made up of an optical fiber coil of total length *L* and radius *R*. (figure 14)

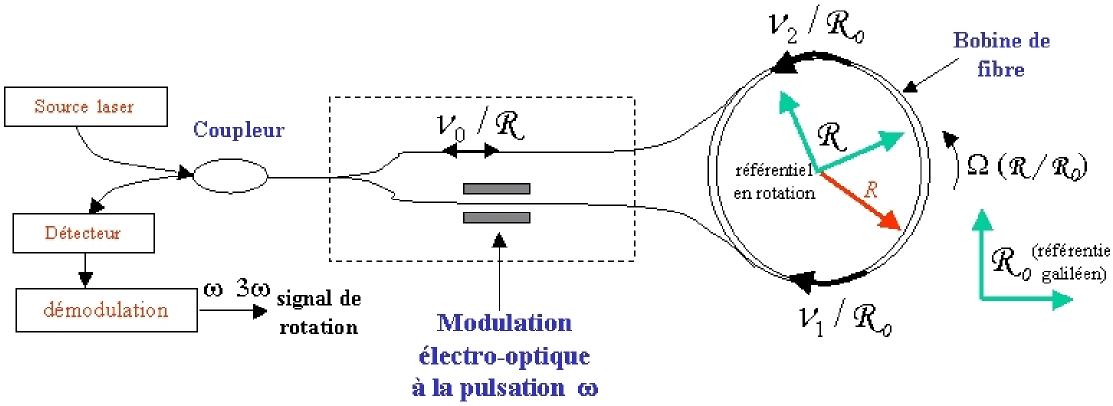



*Figure 14 - Interféromètre de Sagnac à fibre*

A laser gyrometer is an instrument capable of measuring very low rotation speeds. It is part of the navigation instruments on board planes and ships. The entire device: laser source and optical fiber coil forms an integral block which defines a reference frame $\mathfrak{R}$. Let us assume that the rotation of the gyrometer takes place in the plane of the turns with an angular speed $\Omega(\mathfrak{R}/\mathfrak{R}_0)$, defined in relation to the terrestrial frame of reference $\mathfrak{R}_0$ assumed to be Galilean. We will subsequently show that after a propagation length **L**, the phase shift between the two counter-propagating waves is:

$$\Phi = \frac{4\pi \mathrm{L}}{c}\frac{R\Omega}{\lambda}$$

The high sensitivity of a fiber gyrometer comes essentially from the long fiber length **L** of the interferometer, typically several hundred meters. The signal-to-noise ratio of the instrument is increased by using sinusoidal phase modulation on both waves, of pulsation \omega and amplitude $\Phi_0$, by means of an electro-optical device.

The total phase shift, of the form $\Phi = \frac{4\pi L}{c}\frac{R\Omega}{\lambda} + \Phi_0 \cos \omega t$, gives an answer interference $P = P_0(1 + m \cos \Phi)$ non-linear. Therefore, the $\omega$ sinusoidal phase modulation causes the harmonics to appear in the interference signal: $\omega$,$2\omega$ , $3\omega$, etc.

**Synchronous detection** makes it possible to measure the amplitude of the harmonic components: $\omega$, $2\omega$, $3\omega$, ...

It provides a very sensitive means of measuring angular velocity $\Omega$. The absence of a $\omega$ component corresponds to the absence of rotation of the gyrometer (Figure 15).

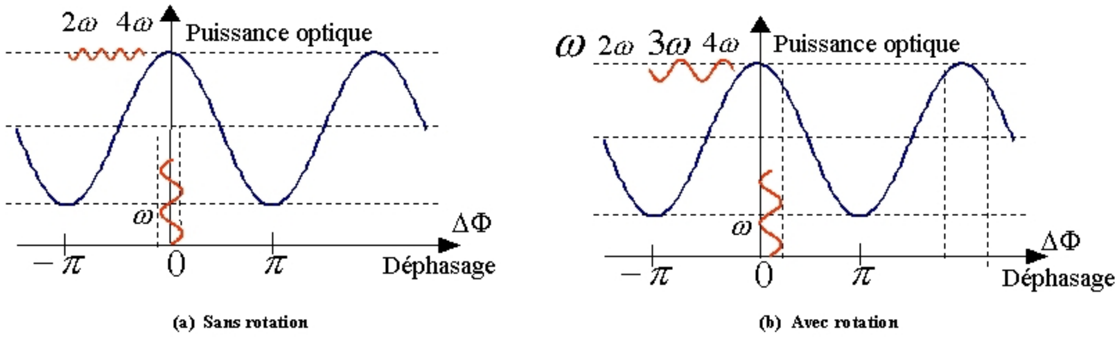



*Figure 15 - Utilisation d'une modulation pour porter le signal de rotation d'un interféromètre de Sagnac*

The resolution of the instrument is determined by the signal-to-noise ratio of the demodulated signal, which depends on the signal-to-noise ratio of the interference signal. There is a fiber length that optimizes this ratio. Fiber attenuation includes macro-bend and micro-bend losses and intrinsic fiber attenuation.

When we wish to measure the three components of the rotation vector $\vec{\Omega}$, we must integrate three Sagnac interferometers whose coils are mutually perpendicular. The integration of angular velocities gives the absolute orientation of the vessel in space.

## *Complément*

The Sagnac effect can be considered to arise from the Doppler effect. The laser source has a frequency $\nu_0$ in the reference frame $\Re$ linked to the fiber and its wavelength in vacuum $\lambda = c/\nu_0$ . The propagation of the waves in the two opposite directions is symmetrical in the $\Re$ \Re reference frame; they therefore have the same frequency $\nu_0$ .

On the other hand, in the Galilean frame of reference $\Re_0$ of the laboratory, there is an asymmetry between the two directions of propagation and the frequencies of the two waves, shifted by the Doppler effect are $\nu_1 = \nu_0(1 - \text{v}/c)$ and $\nu_2 = \nu_0(1 + \text{v}/c)$ , with $\text{v} = R\Omega$ , the linear speed of a fiber element in the repository $\Re_0$. The two counter-propagating waves arrive at the photodetector having traveled the same optical path $L_0$ in $\Re_0$ .

The difference between the proper length L of the fiber in $\Re$ and the improper length $L_0$ in $\Re_0$ is of second order in \text{v}/c and can therefore be neglected because $R\Omega\, llc$ . The phase variations of the waves on the optical path L are$\Delta\varphi_1 = 2\pi\nu_1 L/c$ and $\Delta\varphi_2 = 2\pi\nu_2 L/c$, hence the corresponding phase shift is:

$$\Phi = \Delta\,\varphi_2 - \Delta\,\varphi_1 = \frac{4\pi \mathrm{L}}{c}\frac{R\Omega}{\lambda}$$

This last formula gives the phase shift between the two waves which expresses the Sagnac effect.

## *Remarque*

If we carry out the preceding reasoning in the rotating frame of reference, that is to say non-Galilean, we conclude that there is no Sagnac effect. This paradox is only resolved if we reason in the non-Galilean frame of reference by taking into account the theory of general relativity. The theory of special relativity should only be applied to Galilean frames of reference.

# VI.Case study: heterodyne velocimeter

The table below shows the specifications of an industrial vibrometer that integrates a heterodyne interferometer with a Bragg cell at frequency $\nu_B = 40$ MHz.

| | |
|---|---|
| Grandeur mesurée | Vitesse |
| Traitement du signal | Numérique |
| Domaine de fréquences | 0 – 22kHz |
| Gammes de mesures | 3 |
| Vitesse crête (mm/s) | $\pm 20$ / $\pm 100$ / $\pm 500$ |
| Résolution (1) (µm/s RMS) | $< 0{,}05$ / $< 0{,}1$ / $< 0{,}3$ |
| Distance de travail | 0,2 à 30m |
| Sécurité laser | He-Ne de classe II (sécurité oculaire garantie) |
| **Sortie analogique** du signal de vitesse | BNC, $50\,\Omega$, $\pm 4\,V$, CNA 24 bits |
| Dynamique | > 90dB |
| Précision d'étalonnage | $\pm 1\,\%$ (20Hz - 22kHz) |
| Domaine de fréquences | 0,5Hz - 22kHz |
| **Sortie numérique** du signal de vitesse | 24 bits, 48kS/s de 0- 22kHz |
| Filtres passe- bas numérique (FIR) | 1kHz, 5kHz, 22kHz (-0,1dB), 120dB /dec (ordre 6) |
| Filtres passe- haut analogique | 100Hz (-3dB), 60dB / dec (ordre 3) |

**Spécifications du vibromètre**

* (1) Resolution is defined as the signal amplitude (RMS) for which the signal-to-noise ratio is $0\ dB$ for a spectral width of $10$ Hz; the amplitude being measured on retro-reflective adhesive.

The analog output signal is obtained by an analog digital conversion on 24 bits of resolution, i.e. digitization on $17 \times 10_6$ levels. In the measuring range $\pm 500$ mm/s, the ratio $0.3 \mu m/s$ between the highest measurable speed and the resolution is $(500\text{mm/s})(0,3\ \mu\text{m/s}) \approx 1.7 \times 10^6$. The large measurement dynamic justifies the 24-bit resolution for digital output and digital-to-analog conversion.

Consider a target that oscillates at a frequency of $20$ kHzwith a speed amplitude of $20$ mm/s, which allows us to use the caliber $\pm 20$ mm/s. The maximum Doppler shift is $\delta\nu_D = 2 \times 20 \times 10^{-3}/633 \times 10^{-9} = 63$ kHz . The maximum displacement is $\delta x = \text{v}/(2\pi f) \approx 160$ nm , which corresponds to a maximum phase shift $\Phi = \frac{4\pi}{\lambda}\, deltax \approx \pi$.

With the maximum Doppler shift, this phase shift is obtained in a duration $\Delta t$such that $2\pi\delta\nu_D\Delta t = \pi$ . Hence $\Delta t = 1/(2\delta\nu_D) \approx 8\mu$s , corresponding to $N_p = \nu_B\Delta t \approx 320$ oscillations of interference signal at $40$ MHz (carrier frequency at Bragg frequency). In principle, a minimum duration of $8\ \mu$s is sufficient to carry out a speed measurement. This means that the $20kHz$ sinusoidal velocity signal can be sampled at 6 points per period.

The spectrum of the interference signal for a vibration of amplitude $20$ mm/s at a frequency of $20$ kHz is shown in Figure 16.

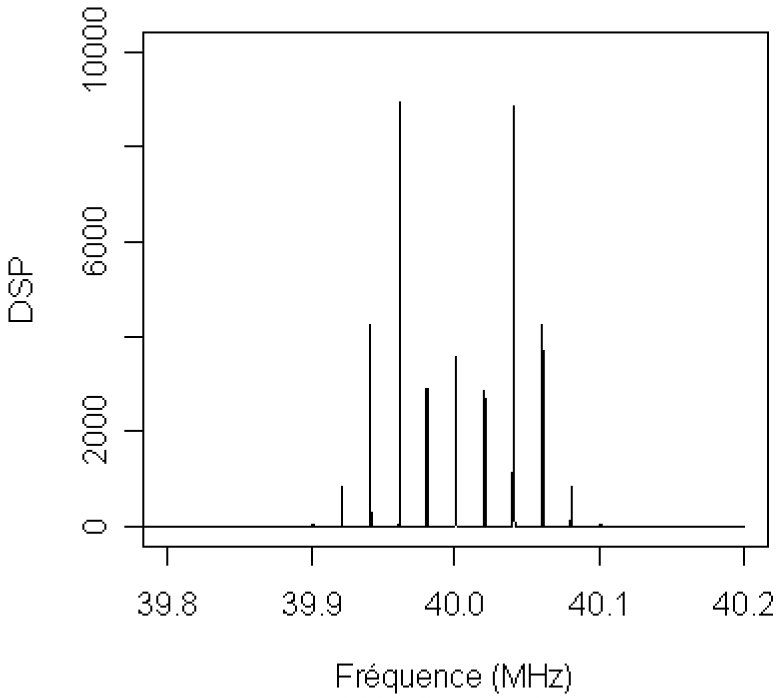



*Figure 16 - Spectre du signal hétérodyne*

It is spread over the band $40\ \mathrm{MHz} \pm 100\ \mathrm{kHz}$ around the carrier. This can be obtained by Carlson's rule. To limit the noise in the measurement chain, it will be necessary to filter the interference signal from the input by a bandpass centered on the frequency of $40\ \mathrm{MHz}$ and of width $200\ \mathrm{kHz}$. For larger speed ratings, the bandwidth must be widened.

According to the specifications, the resolution of the $\pm 20\ \mathrm{mm/s}$ caliber corresponds to the maximum RMS noise of $50\ \mathrm{nm/s}$ for a bandwidth of $10\ \mathrm{Hz}$. In the $22\ \mathrm{kHz}$ bandwidth, the RMS value of noise is:

$$v_{RMS} = 50\ \mathrm{nm/s}\sqrt{(\frac{22 \times 10^3}{10})} \approx 2,35 \mu m/s$$

The Doppler shift corresponding to this speed is : $\frac{2v_{RMS}}{\lambda} \approx 7,5\ \mathrm{Hz}$.

The relative uncertainty in measuring the modulation frequency must be $7.5\ \mathrm{Hz}/40\ \mathrm{MHz} = 1.85 \times 10^{-7}$, or $\approx 0.1\ textppm$ for clock precision. To precisely measure a frequency close to $40\ \mathrm{MHz}$ to within $7.5\ \mathrm{Hz}$, it is necessary to precisely measure the duration of the 320 oscillations of the signal at $40\ \mathrm{MHz}$, i.e. an approximate duration of $8\ \mu\mathrm{s}$, with an uncertainty less than $1.5\ \mathrm{ps}$!

Or an uncertainty of 1 ps in determining the start or end of the oscillations. The digital solution is possible with modern acquisition systems present in certain oscilloscopes which have very high clock precision, a sampling frequency which can reach $500 MEch./s$ with a vertical resolution of 12 bits (quantization $1/2^{12} = 2.5.10^{-4}$). Digital signal processing could allow the determination of the duration of $N_p$ periods with an uncertainty of the order of $1\ \mathrm{ps}$.

For comparison, let us calculate the noise equivalent displacement of a homodyne interferometer which would have a bandwidth of $20\,\mathrm{kHz}$ and a displacement measurement range of $\pm 50\,\mathrm{nm}$. Let us assume that the average power received by the photodetector is $P_0 = 100\,\mu\mathrm{W}$at the wavelength of $633\mathrm{nm}$, with quantum efficiency $\eta = 1$ and a contrast $m = 0.5$. We calculate:

$$\frac{\delta \mathrm{x}}{\sqrt{\Delta f}} = \frac{1}{2\pi \mathrm{pi}} \times \sqrt{\frac{\mathrm{hc}\lambda}{2n\mathrm{P}_0}} = \frac{1}{2\pi \times 0,5} \times \sqrt{\frac{6,62 \times 10^{-34} \times 3 \times 10^{8} \times 633 \times 10^{-9}}{2 \times 1 \times 100 \times 10^{-}}} =$$

$8 \times 10^{-1}\ m\sqrt{Hz}$

For a bandwidth of $20kHz, \delta x = 1.12\ \mathrm{pm}$ and the noise equivalent velocity amplitude at $20\,\mathrm{kHz}$ is $\delta \mathrm{v} = 2\pi f \delta x = 2\pi \times 20 \times 10^{3} \times 1,12\ \mathrm{pm} \approx 140\ \mathrm{nm/s}$. We can conclude that the performances in terms of resolution are the same, if we take into account the moving measurement dynamics of the homodyne interferometer limited in linear regime to $\pm 50\,\mathrm{nm}$, i.e. 3 three less than for the heterodyne velocimeter with $\pm 160\ \mathrm{nm}$ at $20\ \mathrm{kHz}$ .

# VII.Exercice : Questions

Consider the case study velocimeter.

## Question 1

[Solution n°1 p 28]

Calculate the minimum bandwidth of the analog bandpass filter of caliber $\pm 500\text{mm/s}$ which filters the interference signal.

## Question 2

[Solution n°2 p 28]

The two laser beams of a surface velocimeter are focused on a diffusing target with an objective of focal length $f = 20\text{ cm}$ and numerical aperture $NA = 0.1$ . Gaussian laser beams have Gaussian radii $w = 1\text{ mm}$ before the lens. How many speckle grains cover the objective field on average?

# VIII.Exercice : Exercice

We seek to measure the rotation vector $\vec{\Omega}\begin{pmatrix}\Omega_x\\ \Omega_y\\ \Omega_z\end{pmatrix}$ of a solid on the principle of the rotational velocimeter shown in § 4.1. We plan to build an instrument comprising three parallel probe beams arranged in a triangle and focused on the solid object. An interferometer is associated with each beam, whose direction is that of the unit vector $\vec{\mathbf{e}}$ . Each interferometer measures the Doppler shift that the light wave undergoes when backscattering onto the target. With beams 1 and 2, we define the unit vector $\vec{\mathbf{e'}}_3 = \vec{\mathbf{e}} \times \vec{\mathbf{e}}_{12}$ , where $\vec{\mathbf{e}}_{12}$ is the unit vector perpendicular to $\mathbf{e}$ lying in the plane of beams 1 and 2 and oriented from 1 to 2. In the same way, with beams 2 and 3, we define the vectors $\vec{\mathbf{e}}_{23}$ and $\vec{\mathbf{e'}}_1 = \vec{\mathbf{e}} \times \vec{\mathbf{e}}_{23}$ .

Show that the pair of interferometers 1 and 2 makes it possible to obtain a signal proportional to $\vec{\Omega} \cdot \vec{\mathbf{e'}}_3$, that the pair of interferometers 2 and 3 make it possible to obtain a signal proportional to $\vec{\Omega} \cdot \vec{\mathbf{e'}}_1$ .

We pose $\vec{\mathbf{e'}}_i\begin{pmatrix}e_{ix}\\ e_{iy}\\ e_{iz}\end{pmatrix}$ and $R_i = \vec{\Omega} \cdot \vec{\mathbf{e'}}_i$.

Write the matrix relation between $\vec{\Omega}\begin{pmatrix}\Omega_x\\ \Omega_y\\ \Omega_z\end{pmatrix}$ and the vector $\vec{R}\begin{pmatrix}R_1\\ R_2\\ R_3\end{pmatrix}$.

## Question

[*Solution n°3 p 29*]

Can we determine the rotation vector $\vec{\Omega}$ ?

# Solution des exercices

## >Solution n°1 (exercice p. 26)

It is necessary to calculate the maximum Doppler shift which is $(\delta \nu_D)_{max} = 2 \times 0.500 / 633 \times 10^{-9} = 1.6\ \text{MHz}$.

The order of magnitude of the bandwidth is therefore $40\ \text{MHz} \pm 2\ \text{MHz}$. For a harmonic oscillation at $20\ \text{kHz}$, with a speed amplitude of $0.5\ \text{m/s}$, the spectrum of the frequency modulation signal spreads practically over the $40\ \text{MHz}\ band.pm 1.6\ \text{MHz}$ (figure 17).

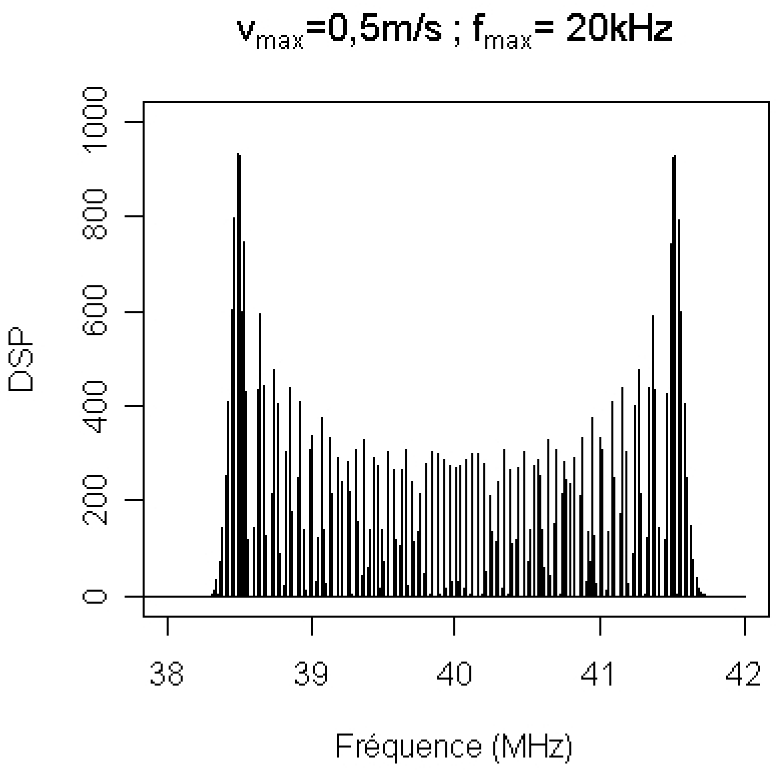



*Figure 17 - Spectre du signal de modulation de fréquence d'un interféromètre hétérodyne sur une cible en oscillation sinusoïdale à la fréquence de 20kHz et une amplitude de vitesse est de 500mm/s. La porteuse a une fréquence de 40MHz.*

## >Solution n°2 (exercice p. 26)

The divergence of a focused beam is $\theta = w/f$ and covers a solid angle $\delta\omega = \pi\theta^2$ ; it is the average solid angle of a speckle grain in the position focus of the target. The solid angle subtended by the objective is $\Omega = \pi NA^2$. The number of speckle grains is:

$$N = \Omega/\delta\omega = \text{NA}^2/\theta^2 = \left(\frac{\text{NA}f}{w}\right)^2 \approx 400$$

## >Solution n°3 *(exercice p. 27)*

The matrix relationship is:

$$\begin{pmatrix} e_{1x} & e_{1y} & e_{1z} \\ e_{2x} & e_{2y} & e_{2z} \\ e_{3x} & e_{3y} & e_{3z} \end{pmatrix} . \begin{pmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{pmatrix} = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix}$$

$$\begin{pmatrix} e_{1x} & e_{1y} & e_{1z} \\ e_{2x} & e_{2y} & e_{2z} \\ e_{3x} & e_{3y} & e_{3z} \end{pmatrix} \cdot \begin{pmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{pmatrix} = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix}$$

The matrix $\begin{pmatrix} e_{1x} & e_{1y} & e_{1z} \\ e_{2x} & e_{2y} & e_{2z} \\ e_{3x} & e_{3y} & e_{3z} \end{pmatrix}$ has a zero determinant because the vectors $\overrightarrow{\mathbf{e'_1}}$ , $\overrightarrow{textbfe'_1} = \overrightarrow{\mathbf{e}} \times \overrightarrow{\mathbf{e_{23}}}$ and $\overrightarrow{\mathbf{e'_3}}$ are coplanar. It is not possible to simultaneously determine the three components of the rotation vector with this device.

It is necessary to know one of the three components a priori to determine the other two. For example, one can determine the $\mathbf{\Omega_z}$ component with a suitably oriented two-beam rotational velocimeter.

# Bibliographie

**[Génération et détection optiques d'ondes élastiques]** ROYER DANIEL, *Génération et détection optiques d'ondes élastiques*, Techniques de l'ingénieur, ---.

**[Laser Ultrasonics]** SCRUBY C.B., DRAIN L.E., *Laser Ultrasonics*, Adam Hilger, Bristol, Philadelphia and New York, 1990.

**[Optical Remote measurement of Ultrasound]** DEWHURST R.J., SHAN Q., *Optical Remote measurement of Ultrasound*, Meas. Sci. Technol., 1999--, n° 10.

# Webographie

**[]** *http://www.faro.com* (consultation 29 06 2007).

**[]** *http://www.polytec-pi.fr/* (consultation 29 06 2007).