Fundamentals of geometrical optics

Jacques SABATER

28.02.2023

Table des matières

I. Presentation 4

II. Course 5

1. Optical Paths 6
   - 1.1. Minimum or maximum 6
   - 1.2. Homogeneous mediums 7
   - 1.3. The Reversibility of Light 7
   - 1.4. Differential of the optical path 7
   - 1.5. Length alteration of a rectilinear path by means of displacement of its extremities 7
   - 1.6. Refraction laws 8
2. Optical systems 11
   - 2.1. Stigmatic image of an luminous point in an optical system 11
   - 2.2. Approached stigmatism 12
3. Dioptres 12
   - 3.1. The image of a luminous point in a diopter 12
   - 3.2. Paraxial approximation 14
   - 3.3. Paraxial conjugate formula 14
   - 3.4. Stigmatic points 14
   - 3.5. Focus, focal distance, refracting power 15
   - 3.6. Axial magnification gz 16
   - 3.7. Conjugate planes, focal planes 16
   - 3.8. Image size, transversal magnification, angular magnification 17
   - 3.9. Paraxial invariants, Lagrange-Helmhotz invariants 18
   - 3.10. Diopters planes, parallel plates 18
   - 3.11. Prisms 19
4. Mirrors 20
   - 4.1. Plane mirrors 20
   - 4.2. Translation and rotation of plane mirrors 21
   - 4.3. Spherical mirrors 22
   - 4.4. Focus, focal distance, focal plane 24
   - 4.5. Image construction in a mirror 24
5. Centered optical systems 25
   - 5.1. Lagrange-Helmoltz extended invariant 26
   - 5.2. Focus and principal planes of focal optical systems (with focus) 26
   - 5.3. The construction of images 28
   - 5.4. Conjugate equation at focus 28
   - 5.5. Refracting power, relationship between f and f' 28
   - 5.6. Descartes' conjugate equation, magnification 29
   - 5.7. Nodal points, antiprincipal points 30
   - 5.8. Image dimension of an object non punctual to infinity 30
   - 5.9. Optical systems refractive in the air 30
6. Association of systems 31
   - 6.1. Association of two optical systems 31
7. Lenses 31
   - 7.1. Thin lenses 32
   - 7.2. The simulation of a complex optical system with a thin lens 33
   - 7.3. Association of thin lenses 34
8. Afocal systems, magnification 34
   - 8.1. Afocal systems, magnification 34
   - 8.2. Focal visual optical systems magnification (magnifying glass, oculars, microscopes...) 36
9. Diaphragms, lenses and fields 36
   - 9.1. Examples 37

III. Case study : Calculating the optical characteristics of a two-lens system ***39***

1. Determining the position of image focus F' of S..***39***
2. Determining the position of H'..***39***
3. Determining the position of object focus F of S...***40***
4. Calculating the full light field...***40***

IV. Exercices ***42***

1. Exercice 1..***42***
2. Exercice 2..***42***
3. Exercice 3..***42***

Solution des exercices ***43***

I.Presentation

Module:

Instrumental optics and microscopy

Author(s):

Jacques SABATER - Institut d'Optique – Graduate School

Abstract:

The first part of this course establishes the fundamental basis of geometric optics from the Fermat's principle. The second part which is related to the Gauss's approximation, gives the general formulas of calculation of the positions of the images in the optical systems. The third part is about the formation of the image in the complex optical systems.

Keywords:

Fermat's principle, Optical path, Refraction, Reflexion, Stigmatism, Dioptre, Mirror, Combination, Foyer, Focal distance, Magnifying, Enlarging, Lens, Convergence, Diopter, Optical system, Afocal, Pupil, Field, Lucarne, Glasses, Lens, Ocular, Prism, Binoculars,

Prerequisites:

None

Learning outcomes:

Ce cours établit les bases de la propagation du rayonnement lumineux dans l'approximation de l'optique géométrique puis l'approximation de Gauss qui permettent un calcul simple des positions des images dans tous les systèmes optiques.

Course overview:

- Introduction
- Optical Paths
- Optical systems
- Dioptres
- Mirrors
- Centered optical systems
- Association of systems
- Lenses
- Afocal systems, magnification
- Diaphragms, lenses and fields

Design & production:

Le Mans Université

License:

1 - http://www.gnu.org/licenses/fdl.txt

II.Course

Light is composed of electromagnetic waves of which the propagation can be perfectly treated in accordance to Maxwell's equations. This is essential when optical elements modifying the propagation of luminous energy have very small dimensions, either close or inferior to the wavelength. Of course, it is not the case with the greatest majority of usual optical systems where one can consider the wave length to be infinitely small in comparison to their geometric dimensions.

The case is that which is concerned with *the approximation of geometric optics* where the undulatory nature of light is neglected, the interference, diffraction and polarization phenomena ignored. We demonstrate that the fundamental laws of geometric optics can be deducted from Maxwell's equations by shifting the wavelength towards 0.

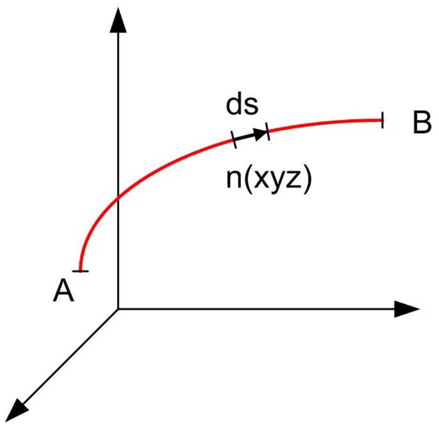
Fermat's principle

Enunciated by Pierre Fermat in 1657, **Fermat's principle** acts as a foundation to geometrical optics. The main propagation laws are derived from it. The terms by themselves cannot be any simpler : *light follows the shortest path in time*. In fact, it can be demonstrated that it is the longest in somes cases, but in any case the path is extremal.

Geometrical optics suppose that mediums are isotropic. A medium is isotropic when the index is independent from the propagation direction and from the direction of light polarization. The index (xyz) is therefore perfectly defined on every point in space. If v(xyz) is the local propagation speed in medium and c the propagation speed in the void, we have in every way : n=c/v.

Propagation speed depends only on the point considered, it is independent from the direction of propagation.

Let us calculate propagation time T_{AB} between two points in space A and B for a path effectively followed by light :



For a trajectory element of which the length is ds, propagation time dT is :

$$\mathrm{d}T = \frac{d\,\mathrm{s}}{v} = \frac{n \cdot ds}{c}$$

Total propagation time is therefore :

$$T_{\mathrm{AB}} = \frac{1}{c} \cdot \int_{\mathrm{A}}^{\mathrm{B}} n \cdot \mathrm{d}\,s \qquad (1)$$

The path followed by rays of light is such that T is minimum or maximum.

1. Optical Paths

Trajectory time for reasonable dimensions being extremely brief, it is best to multiply the two members of the expression (1) by c in order to obtain the length of the optical path L_{AB}rom A to B :

$$L_{AB}=\int_A^B n\cdot ds \qquad (2)$$

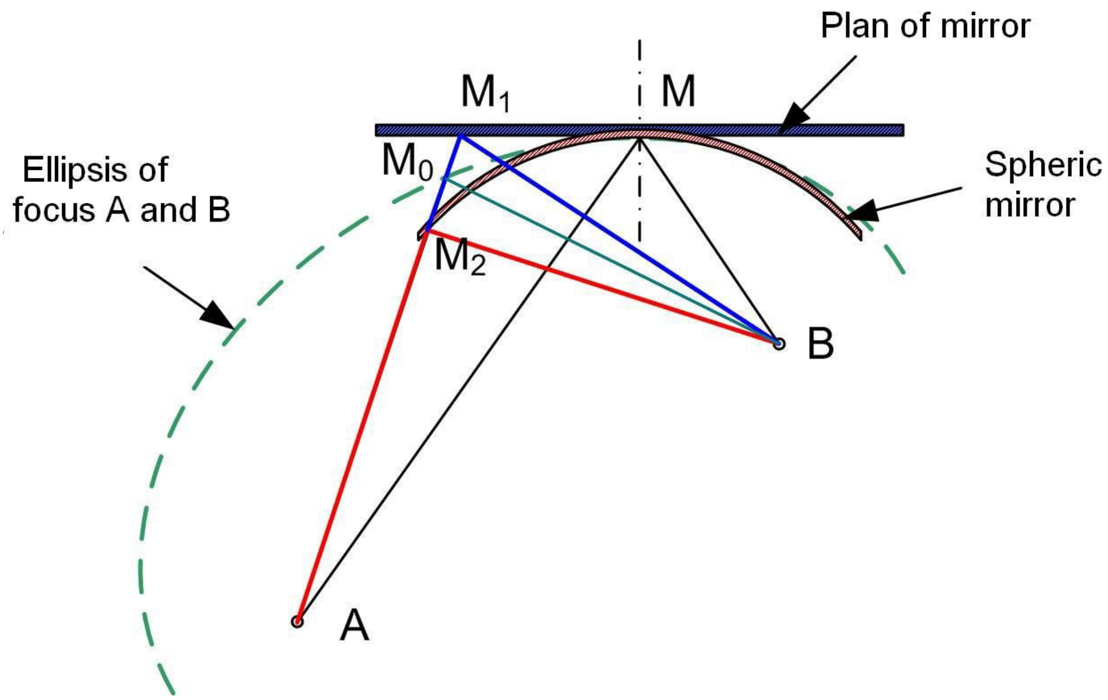
The optical path L_{AB}, being proportional toT_{AB}, has the same properties.

Fermat's principle therefore imposes that L_{AB} be extremal between A and B.

1.1. Minimum or maximum

Let us consider two points in space A and B in an homogeneous medium. Within a given point M, a plane surface, of which the normal is the bisector of the (AMB) angle, allows the AM ray to reflect towards MB, the light therefore follows the AMB path. **For a tangent spherical surface in M to the plane mirror the result is identical.** For these two surfaces, the ray reflecting itself in M alone passes through B.

The figure below is situated on the plane going through A, B and M. We traced the tangent ellipse in M to the plane mirror of which the focus are A and B. Any point M_0 of the ellipse is such that $L_0 = AM_0 + M_0B$is constant.



The optical paths for current points on the surfaces are :

Plane mirror : $L_1 = AM_1 + M_1B$

Spherical plane : $L_2 = AM_2 + M_2B$

The spherical plane having a beam curvature inferior to that of the ellipsis in M, it is evident that : $L_2 < L_0 < L_1$

For the plane mirror L is minimal. For the spherical mirror L is maximal.

1.2. Homogeneous mediums

A homogeneous medium is a medium where the index is identical at every point.

The integral (2) giving L_{AB} becomes :

$$L_{AB} = n \cdot \int_A^B ds$$

The straight line being the shortest path from A to B, propagation is rectilinear.

1.3. The Reversibility of Light

In the integral (2) ds is not signed. Claculation of B to A whilst taking the same path yields the same result.$L_{AB} = L_{BA}$

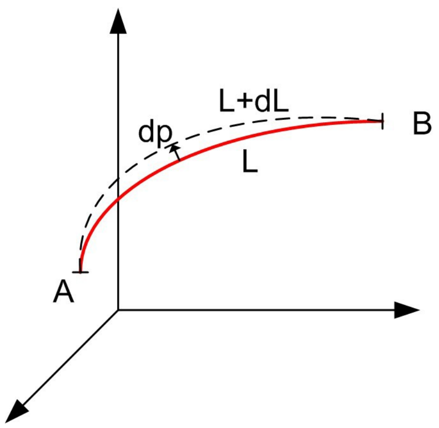
The extremal path will therefore be the same whatever the direction. The path taken by light from A to B or from B to A is identical.

1.4. Differential of the optical path

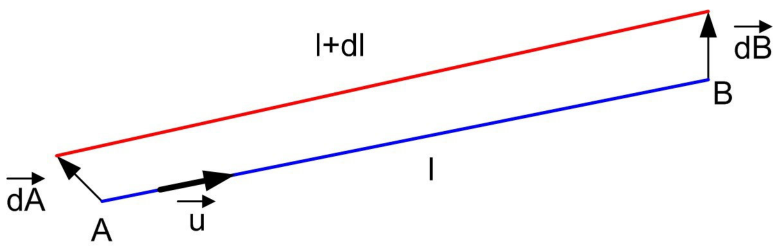
Let us consider two points A and B in space and the path taken by light from A to B. The length of the optical path from A to B is L.

Let p be a parameter defining the path from A to B with a small variation dp, A and B remaining the path extremes, and dL the variation induced from the optical path L.

Fermat's principle which imposes light to run an extremal to the optical path is expressed thus : $\frac{dL}{dp} = 0$



1.5. Length alteration of a rectilinear path by means of displacement of its extremities



The length l = AB can be expressed by the modulus of vector $\overrightarrow{AB}$:

$l = \overrightarrow{u} \cdot \overrightarrow{AB}$

(scalar product)

$\overrightarrow{u}$ is the unitary vector in the direction of AB. For small displacements of A and B denoted $\overrightarrow{dA}$ and $\overrightarrow{dB}$, we get :

$d(\overrightarrow{AB}) = \overrightarrow{dB} - \overrightarrow{dA}$

Then

$dl = \overrightarrow{du} \cdot \overrightarrow{AB} + \overrightarrow{u} \cdot d(\overrightarrow{AB})$

$\overrightarrow{du}$ is perpendicular to $\overrightarrow{AB}$ so $\overrightarrow{du} \cdot \overrightarrow{AB} = 0$, we can deduce :

$dl = \overrightarrow{u} \cdot (\overrightarrow{dB} - \overrightarrow{dA})$

If the index of the medium is n and $L = nAB$ the optical path (AB), the variation of the optical path dL is :

$dL = n\overrightarrow{u} \cdot (\overrightarrow{dB} - \overrightarrow{dA})$ (3)

1.6. Refraction laws

Let us consider surface S in a space, separating two medium of indexes $n_1$and n_2 respectively containing points A and B. The path effectively taken by light to go from A to B runs through Point A on the surface.

AI is the the incident ray, IB is the refracted ray.

$L = n_1 \cdot AI + n_2 \cdot IB$is the optical path (AIB).

Slight displacement dI of I causes a variation dL such that$dL/dI = 0$ according to Fermat's principle. The expression (3) applied to the trajectory AI and IB give us :

$dL_{AI} = n_1\overrightarrow{u_1} \cdot (\overrightarrow{dI} - \overrightarrow{dA}) = n_1\overrightarrow{u_1} \cdot \overrightarrow{dI}$

Similarly :

$dL_{IB} = n_2\overrightarrow{u_2} \cdot (\overrightarrow{dB} - \overrightarrow{dI}) = -n_2\overrightarrow{u_2} \cdot \overrightarrow{dI}$

Since $\overrightarrow{dA} = \overrightarrow{dB} = 0$, finally we get :

$dL_{AB} = -(n_2\overrightarrow{u_2} - n_1\overrightarrow{u_1}) \cdot \overrightarrow{dI}$

If $\overrightarrow{N}$ is the unitary vector of the normal, $\overrightarrow{V}$ is that in the $\overrightarrow{dI}$ direction $\overrightarrow{dI} = \overrightarrow{V} \cdot dI$ that is :

$dL_{AB} = -(n_2\overrightarrow{u_2} - n_1\overrightarrow{u_1}) \cdot \overrightarrow{V} \cdot dI$

For a trajectory effectively followed by light, Fermat's principle imposing,$dL/dI = 0$ for every $\overrightarrow{dI}$, $(n_2\overrightarrow{u_2} - n_1\overrightarrow{u_1})$ and $\overrightarrow{V}$ are perpendicular, therefore $(n_2\overrightarrow{u_2} - n_1\overrightarrow{u_1})$ and$\overrightarrow{N}$ are parallel.

$(n_2\overrightarrow{u_2} - n_1\overrightarrow{u_1}) = k\overrightarrow{N}$ shows that $\overrightarrow{u_1}, \overrightarrow{u_2}$ and $\overrightarrow{N}$ belong to the same plane P.

P is the incidence plane, it contains the incident ray, the refracted ray and the normal to the surface in I, i_1 and i_2 are, in this plane, the angles between the incident and refracted rays in relation to the normal. We deduct the vectorial refraction relationship :

$(n_2\overrightarrow{u_2} - n_1\overrightarrow{u_1}) = (n_2cosi_2 - n_1cosi_1) \cdot \overrightarrow{N}$ (4)

And by refraction in the surface plane :

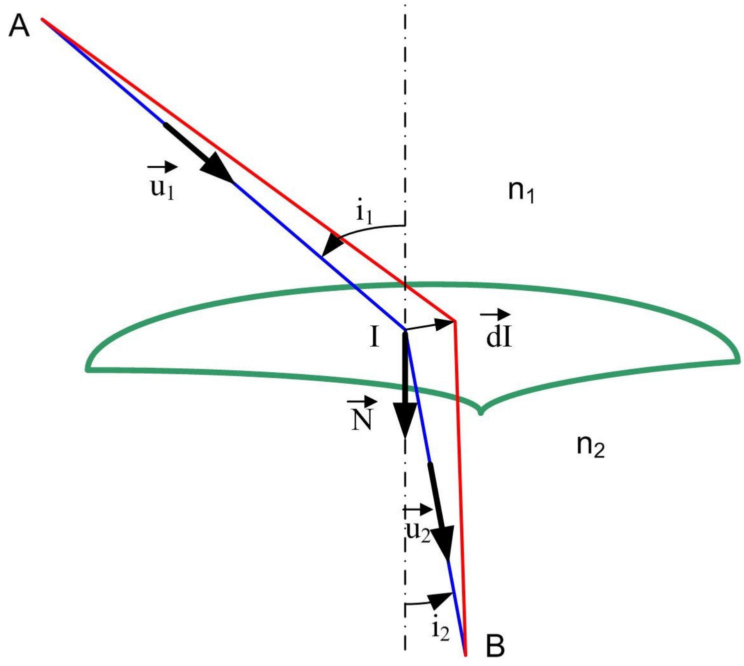
$n_1 \cdot sini_1 = n_2 \cdot sini_2$ (5)

Descartes' laws are deducted from the previous relationships :

Law 1 : **The refracted ray is in the incidence plan**

Law 3 : The angles i_1 et i_2 of incident and reflected rays are such that $n_1 \cdot \sin i_1 = n_2 \cdot \sin i_2$

La loi 2 is concerned with reflecting surfaces, for which $i_1 = -i_2$. We will see later that the formulas for refracting surfaces can be applied to reflecting surfaces with:$n_2 = -n_1$.

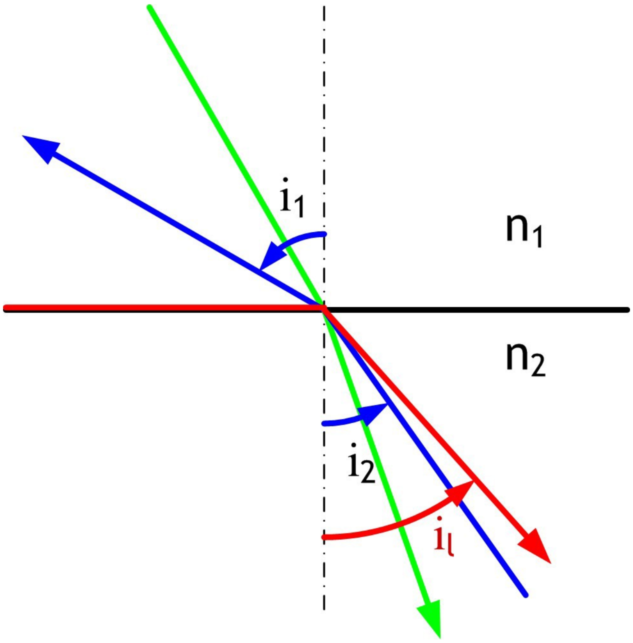


a) Refraction

Let us consider two medium of indexes n_1 et n_2, Descartes' third law gives us the relationship between the incidence and refraction angles in both **mediums** :

$n_1 \cdot sini_1 = n_2 \cdot sini_2$ (5)

This relationship is perfectly symmetrical, in accordance to the principle of reversibility of light. A ray arising from the second medium in an angle i_2 with the normal will form, after refraction from the surface, an angle i_1 in the first medium which meets with the relationship (5).



b) Limit angle

Limit angle

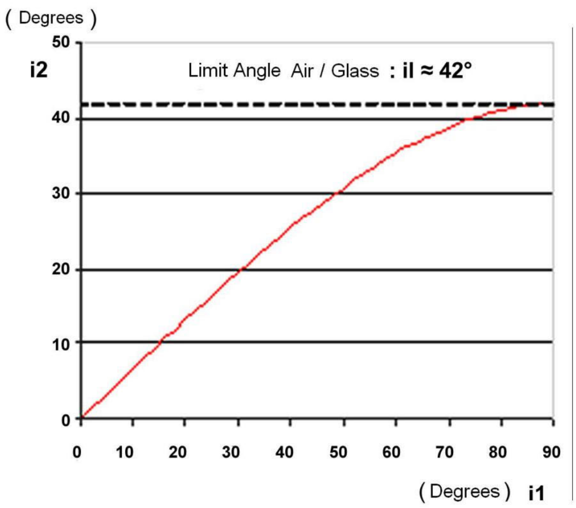
For $i_l = 90°$, a grazing incidence angle, the refracted ray forms, with the normal, the angle i_2 such that : $\sin i_2 = n_1/n_2$.

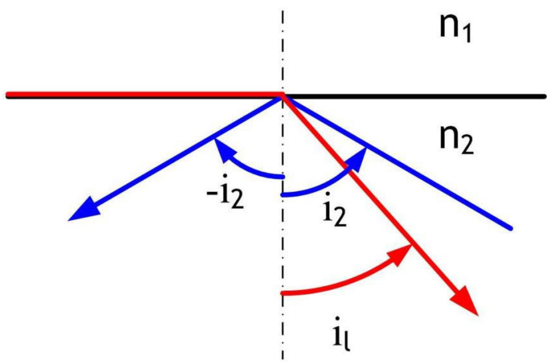
i_2 is thus called the limit angle i_1 .

$i_l = \arcsin(n_1/n_2)$ (6)

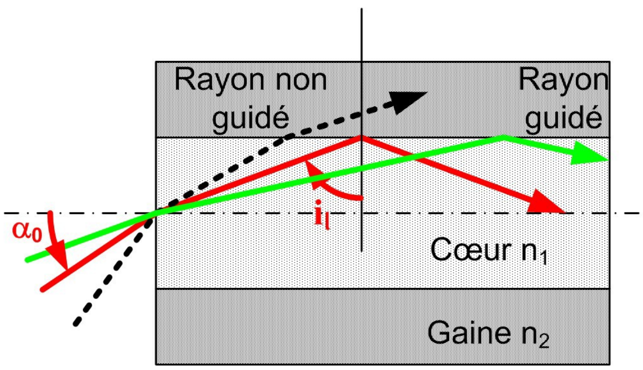
Any ray arising from the second medium of which the incidence angle i_2 is higher than i_l will undergo **total reflexion**. **Si le premier milieu est l'air ($n_1 = 1$), le tableau ci-dessous donne quelques valeurs de i_l : PAS TRADUIT**

Deuxième milieu	Indice pour λ = 587 nm	Angle limite i_l (deg)
Eau	1,333	48,6
Verre bas indice (BK7)	1,516	41,3
Verre haut indice (SF6)	1,805	33,6
Diamant	2,418	24,4





Total reflection in a multimodal optical fiber



The core index is n_1, the clad index is $n_2 < n_1$.

A light ray with an incidence over i_1 on the core-clad interface reflects itself entirely, it is guided.

Its incidence at fiber entry is lower than α_0.

α_0 is the half angle aperture of the fiber. The digital opening of the fiber is :

$$NA = \sin(\alpha_0) = \sqrt{n_1^2 - n_2^2}$$

2. Optical systems

An optical system is a set of surfaces which reflect (mirrors) or refract (diopter) light rays. A centered system have a symmetrical axis. Systems dotted with diopters only are called dioptrics (lenses, spectacles, microscopes). Systems composed of diopters and mirrors are said to be catadioptric (telescopes).

2.1. Stigmatic image of an luminous point in an optical system

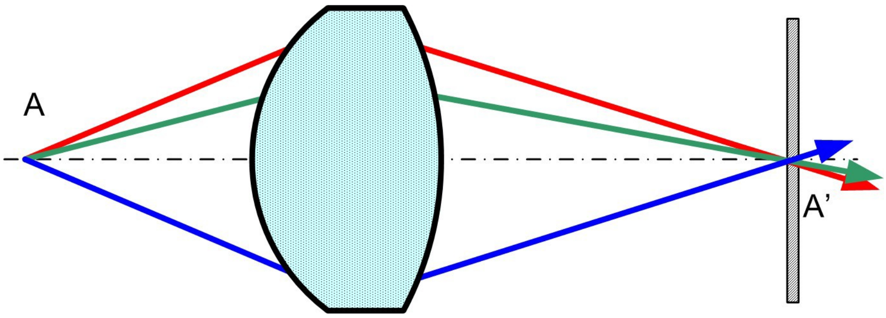
Let us consider a point A in a first space called **"object space"**. From A let us run a set of luminous rays going through the system. If all these rays converge to the same point A' of the **image space**, we can write :

- A' is the image of A through the system. It is also said that A' is the conjugate of A
- The system is said to be stigmatic for AA' conjugation

We demonstrate that stigmatism implies a constant value for the optical path (AA')

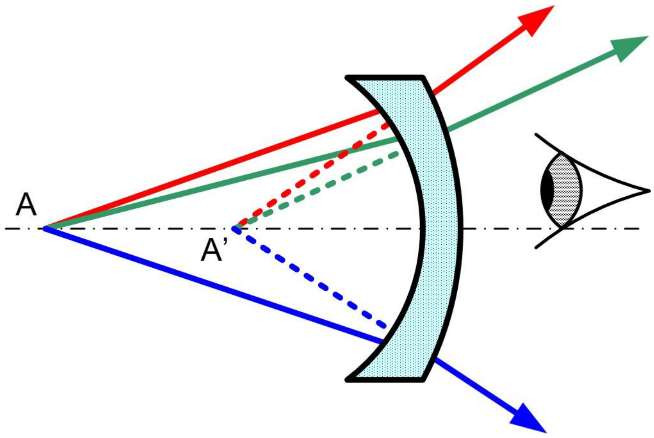
The case of a **real image** :

The image may be observed on a screen in the image space.



The case of a **virtual image** :

The image cannot be observed on a screen. It is nevertheless visible by an observer situated in the image space.



2.2. Approached stigmatism

Certain very simple optical systems are rigourously stigmatic :

- The parabolic mirror for an object point to infinity on the axis.
- An elliptic mirror for conjugation between geometrical focus.
- A stigmatic plane mirror pour all points in space.
- ...

These systems are not frequent and are not stigmatic for a unique object point. Generally, optical systems are not rigourously stigmatic.

In this case, the image of an object point on a screen or light receptor (film, CCD matrix) is a diffusion spot. If the dimension of the latter is lower than the film grain or the pixel size of the CDD matrix it will then be seen as quasi-ponctual, the optical system will be equivalent to a stigmatic system. It is said that there is an **approached stigmatism**.

Calculating for optimisation of optical systems consists in making these diffusion spots small enough on every point of the image.

3. Dioptres

A dopter is a surface separating two different medium indexes. Apart from those with mirrors or diffracting surfaces, usual optical systems (camera objective lenses, projection lenses, glasses, microscopes) are exclusively made of a number of diopters.

Optical systems generally have a revolution axis and diopters used are generally spherical or plane. The system axis is the line goring through the diopters centers of curvature, it is perpendicular to the diopters planes.

Some optical systems can have optical aspherical surfaces of revolution around the system axis. These asphérisation are necessary for the reduction of aberrations of which the study cannot be treated here. Such a surface will be assimilated to a spherical diopter of which the curvature ray is identical to that of the surface on the optical system axis.

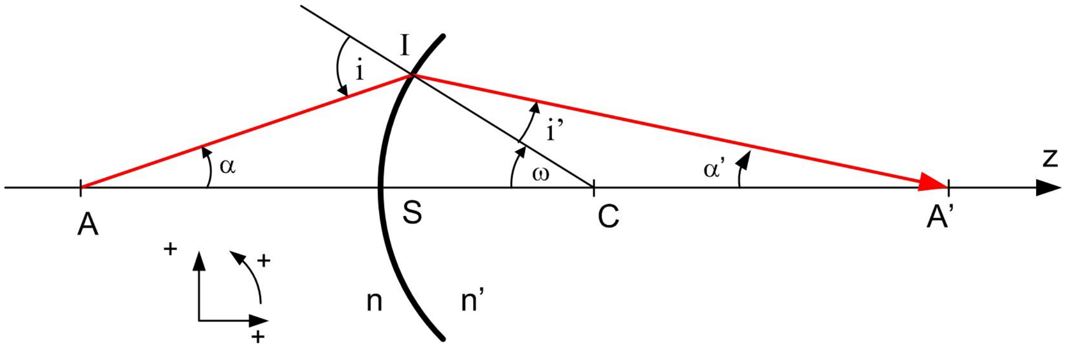
3.1. The image of a luminous point in a diopter

Let us consider a spherical diopter separating two mediums of indexes n and n', defined by its curvature center as C, its vertex as S, its curvature ray $R=\overline{\mathrm{SC}}$.

All lengths and angles are orientated in accordance with the trigonometry convention

A point A is situated on the object space on line SC. The ray arising from A through S is perpendicular to the diopter, it is not deviated. Another ray arising from A going through any point I from the diopter is subject to refraction, the ray arising cuts SC at a point A'.

Let us look for the position of A'. According to figure 12 :



i is the incidence angle of the ray on the dioptic.

i' is the refraction angle and, after (5), *n.sin(i) = n'.sin(i').*

An usual formula in the triangle CAI gives :

$$\frac{\overline{CA}}{\sin(\pi - i)} = \frac{\overline{CA}}{\sin(i)} = -\frac{\overline{IA}}{\sin(\omega)}$$

$$(\overline{CA} < 0,\ \overline{IA} < 0,\ \omega < 0,\ i > 0)$$

One can also write :

$$\frac{\overline{CA'}}{\sin(i')} = -\frac{\overline{IA'}}{\sin(\omega)}$$

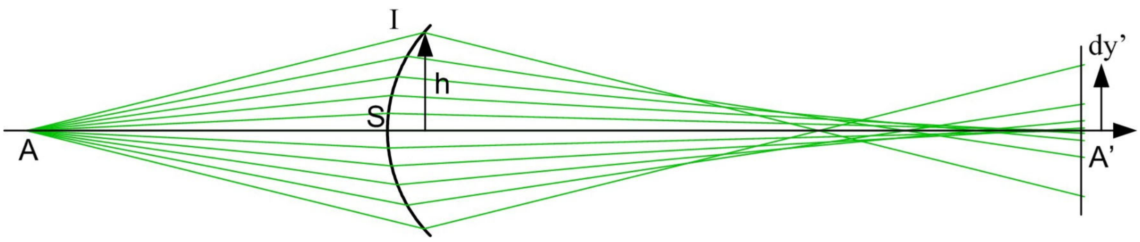
Therefore :

$$\frac{\overline{CA}}{\overline{CA'}} = \frac{\overline{IA}.\sin(i)}{\overline{IA'}.\sin(i')} = \frac{n.\overline{IA}}{n'.\overline{IA'}} \qquad (7)$$

Conjugate stigmatism would mean that A' does not depend on the position of I. It is thus necessary that CA' remains fixed, similarly for the IA/IA' ratio. This is obtained only in a particular position of A and is not achieved in general cases.

Figure 13 concretely shows an example of ray tracing in a diopter, the aberation here is substantial. This tracing is obained by means of the free software Oslo-Edu1 which can be dowloaded from this address : *http://www.lambdares.com/downloads/index.phtml#osloedu*[2]

2 - http://www.lambdares.com/downloads/index.phtml#osloedu



3.2. Paraxial approximation

Figure 13 shows that the ray tranversal aberration increases with the height of incidence h of I on the diopter. Let us seek the limit A' of the intersection of the refracted ray when h leads to 0. When I leads towards S, the relationship (7) becomes :

$$\frac{\overline{CA}}{\overline{CA'}} = \frac{n \cdot \overline{SA}}{n' \cdot \overline{SA'}} \quad (8)$$

The study of aberrations shows that the distance gap dy' between the refracted ray and A' on a plane going through A' and perpendicular to the axis is approximately proportional to h^3.

For small values of h, dy' is very short, there is an **approximate stigmatism**.

In this case, rays incidences i and i' on the surface of the diopter are low, sinus and radian angles are very close, the relationship becoming : $n.i = n'.i'$

An incidence of $5°$ for which we have $[i - \sin(i)]/i \approx 1,2.10^{-3}$ is a good limit for this approximation. It is thus said that A' is the image of A in the paraxial approximation and is also called Gaussian approximation. **In paraxial approximation, all diopters, therefore all optical systems, are stigmatic.**

3.3. Paraxial conjugate formula

Let zz' be the optical system axis and S the coordinates origin, we obtain :

$\overline{SA} = z$

$\overline{SA'} = z'$

$\overline{SC} = R$

Therefore :

$\overline{CA} = z - R$

$\overline{CA'} = z' - R$

Once replaced by the following values and after simplification, the relationship becomes :

$$\frac{n'}{z'} = \frac{n}{z} + \frac{n' - n}{R} \quad (9)$$

3.4. Stigmatic points

Any point on the diopter is the image of itself, this image conjugate is of course stigmatic.

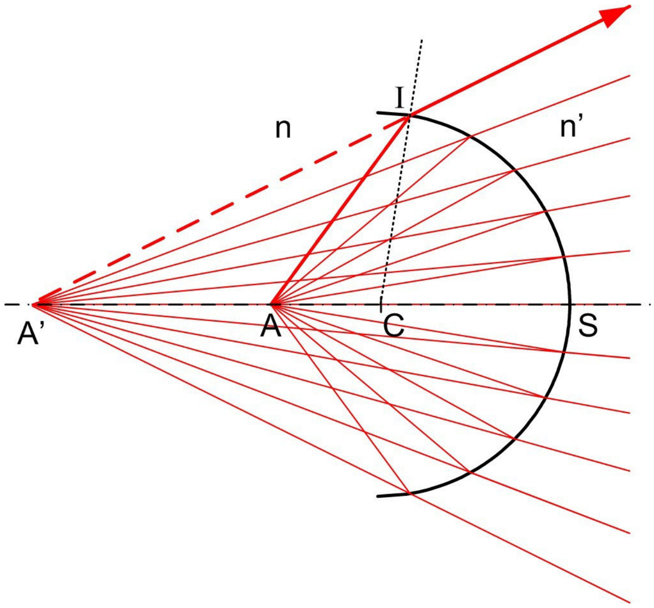
Any luminous ray arising from the curvature center C is perpendicular to the diopter and is not deviated. Therefore C is the image of itself and the image conjugate is stigmatic.

There is another diopter stigmatic conjugate for which the relationship (7) is rigorously verified. These points which are situated on the same side as the diopter rigorously satisfy the relationship : $n'.IA' = n.IA$

The optical path (AA'), $L = n \cdot \overline{AI} + n' \cdot \overline{IA'}$ is rigorously nil, this shows stigmatism. Figure 14 is an example of this conjugate image type ($n = 1,8$ et $n' = 1$). These particular points are **Young-Weierstrass' points**, they are often used in microscopy.

For these points :

$$\overline{SA} = z = \frac{(n+n')R}{n} \qquad \overline{SA'} = z' = \frac{(n+n')R}{n'}$$



3.5. Focus, focal distance, refracting power

The image focus is the image of the point towards infinity on the axis :

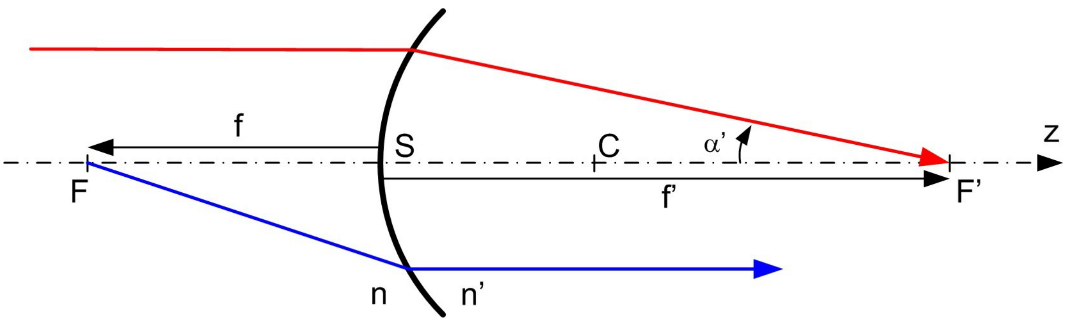
$$\frac{1}{z} = 0 \Rightarrow z' = f' = \frac{n'R}{n'-n}$$

F' is the image focus, $\overline{SF'} = f'$ is the image focal distance of the diopter.

The object focus is such that its image is to infinity on the axis :

$$\frac{1}{z'} = 0 \Rightarrow z = f = -\frac{nR}{n'-n}$$

F is the object focus $\overline{SF} = f$ is the object focal distance of the diopter.



f and f' and the diopter refracting power C_v are linked by the following relationship :

$$C_v = \frac{n'}{f'} = -\frac{n}{f} = \frac{n'-n}{R}$$

3.6. Axial magnification gz

Axial magnification pertains to small object and image displacements dz and dz'.

Differential equation of the conjugation gives us :

$$-\frac{n' dz'}{z'^2} = \frac{-ndz}{z^2}$$

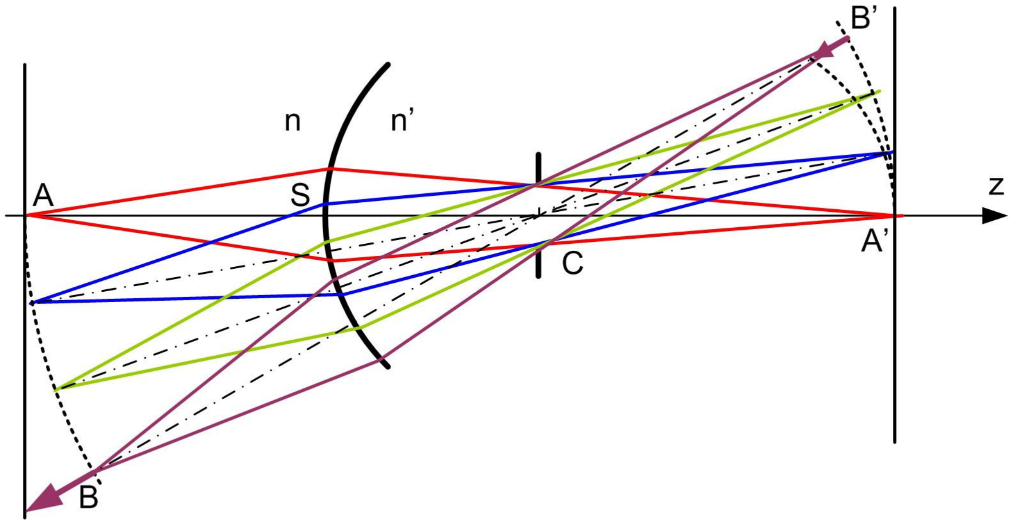
Hence :

$$g_z = \frac{dz'}{dz} = \frac{n}{n'} \cdot \frac{z'^2}{z^2} \quad (10)$$

g_z is always positive, small displacements of the object and of the image are always in the same direction.

3.7. Conjugate planes, focal planes

A set of object points situated on a sphere with centre C going through A have, as an image, some points situated on a sphere of which the centre is C going through A' with the axial beam revolving around C. The spherical object surface has for an image a curved image surface. The fact that the images are not formed on a plane is an aberration named "field curvature".



Following figure 16, if we displace an object point B situated outside the axis in the negative direction following the line BC through to the plane perpendicular to the axis going through A, its image B' moves in the same direction since g_z is positive. This displacement increases the curvature of the image plane, the image of a plane therefore is not a plane.

With respect to paraxial approximation, we will neglect these curvatures because the angles are small as well as objects and images distances to the axis. The gap between the real position of the image and the plane going through A' if of second order relatively to the distance to the axis. Generally, paraxial approximation only keeps the terms of the first order.

In Paraxial approximation the image of a plane, is a plane.

Any object point to infinity has an image in the **image focal plane**, plane perpendicular to the axis going through F'.

Any point of the **object focal plane**, plane perpendicular to the axis going through F, has an image to infinity.

3.8. Image size, transversal magnification, angular magnification

Let us consider an object AB situated at a distance z from the diopter, with a radius, R in a plane perpendicular to the axis. Its image is $A'B'$ at a distance z'. Let $y = \overline{AB}$ and $y' = \overline{A'B'}$

In the paraxial approximation, as in figure 17 :

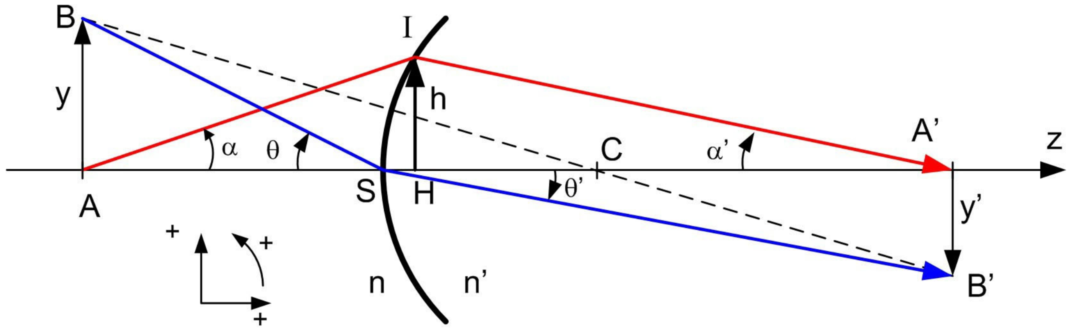
- $A'B'$ is perpendicular to the axis
- θ is the object field angle, θ being small, $\tan(\theta) = \theta = y/z$
- Similarly, *theta*' is the image field angle and $\theta' = y'/z'$
- Refraction of the ray in S going from B is such that :$n\theta = n'\theta'$

We deduct the dimension y' of the image :

$$y' = y \cdot \frac{nz'}{n'z}$$

And the transversal growth g_y :

$$g_y = \frac{\overline{A'B'}}{\overline{AB}} = \frac{y'}{y} = \frac{nz'}{n'z}\ (11)$$



For a given conjugation (AA') we define an angular growth $g_\alpha = \frac{\alpha'}{\alpha}$ between the angles relative to the axis of the two conjugated rays going through A and A'.

Following figure 17, I is the intersection of the rays with the diopter and $h = \overline{HI}$ the distance from I to the axis. In paraxial approximation, h is small, the diopter curvature is neglected and H is supposedly confounded with S. We have :

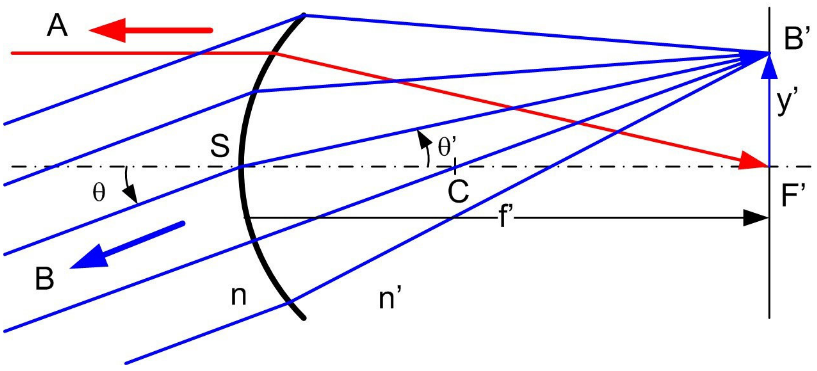
$$\alpha = -\frac{h}{z} \text{ et } \alpha' = -\frac{h}{z'}$$

We thus deduct :

$$g_\alpha = \frac{\alpha'}{\alpha} = \frac{z}{z'}\ (12)$$

In cases where the object AB is to infinity, its transversal dimension is given by its field angle θ. Following figure 18, A is on the axis, its image is F', B', the image of B, is in the image focal plan at a distance y' from the axis in such a way that :

$$y' = f' \cdot \theta' = \frac{n}{n'} f' \cdot \theta = -f \cdot \theta$$



3.9. Paraxial invariants, Lagrange-Helmhotz invariants

According to the conjugation relationship :

$$\frac{n'}{z'} - \frac{n}{z} = \frac{n' - n}{R}$$

We have the longitudinal invariant :

$$Q_z = n\left(\frac{1}{R} - \frac{1}{z}\right) = n'\left(\frac{1}{R} - \frac{1}{z'}\right)$$

The transversal magnification relationship :

$$G_y = \frac{y'}{y'} = \frac{n}{n'} \times \frac{z'}{z}$$

Gives us the transversal invariant :

$$G_y = \frac{ny}{z} = \frac{n'y'}{z'}$$

Carrying out the product g_α , we get :

$$g_y \cdot g_\alpha = \frac{y'\alpha'}{y\alpha} = \frac{n}{n'}$$

We deduct **the Lagrange-Helmhotz invariant** :

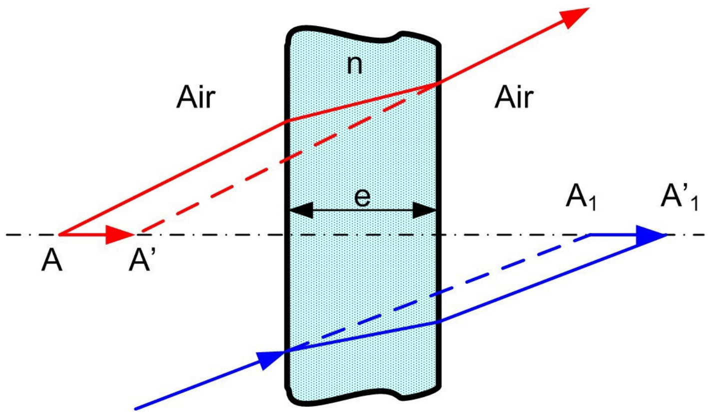
$$ny\alpha = n'y'\alpha' \ (13)$$

3.10. Diopters planes, parallel plates

They are characterised by $1/R = 0$. The formulas become :

$$\frac{z'}{z} = \frac{n'}{n} \text{ et } g_y = 1$$

A dioptric plane is perfectly stigmatic for any object point to infinity.



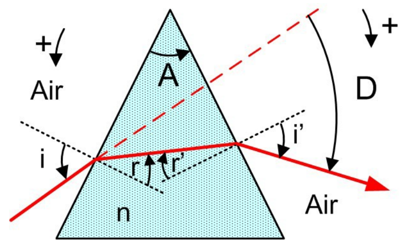
The parallel plate is composed of two distant dioptric planes of e, n is the medium index. An object point A has, as an image, a point A' situated on the perpendicular led from A to the plates sides.

We show that :

$$\overline{AA'} = e\frac{n-1}{n} \quad (14)$$

This is true for any point A in space, real or virtual. For a given object this displacement is independent from the plate positionning.

3.11. Prisms



A prism of index n is composed of two dioptric planes forming an angle A. Following figure 20, a luminous ray enters from side 1 under incidence i and comes out of side 2 under incidence *i'*, the corresponding refraction angles in the prism are *r* and *r'*, D is the deviation from the ray provoqued by the prism. The angular sign convention is normal for side 1 and inversed for side 2.

We have the following form :

sin(i) = n.sin(r)

sin(i') = n.sin(r')

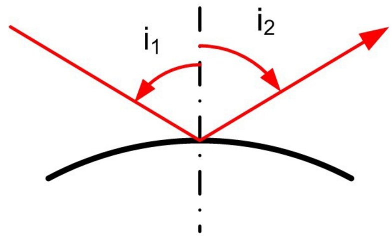
A = r + r'

D = i + i' - A

At the minimum of deviation: i = i' et r = r', we obtain a relationship between n, A and D, allowing index measures of optical material :

$$n = \frac{\sin[(A+D)/2]}{\sin(A/2)} \quad (15)$$

4. Mirrors



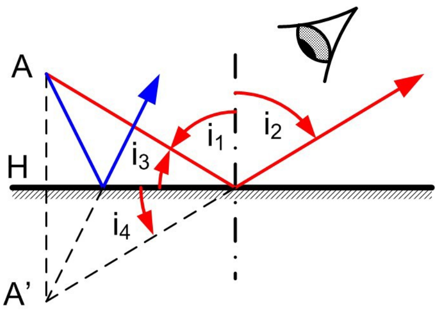
Mirrors of optical quality are generally made of a surface of plane polished glass, convex or concave on which is disposed a reflecting coating.

A luminous ray reflecting itself on a mirror follows Descartes' second law :

If i_1 eand i_2 are the angles for the incident ray and for the reflected ray in relation to the normal to the surface, we have :

$$i_2 = -i_1 \quad (16)$$

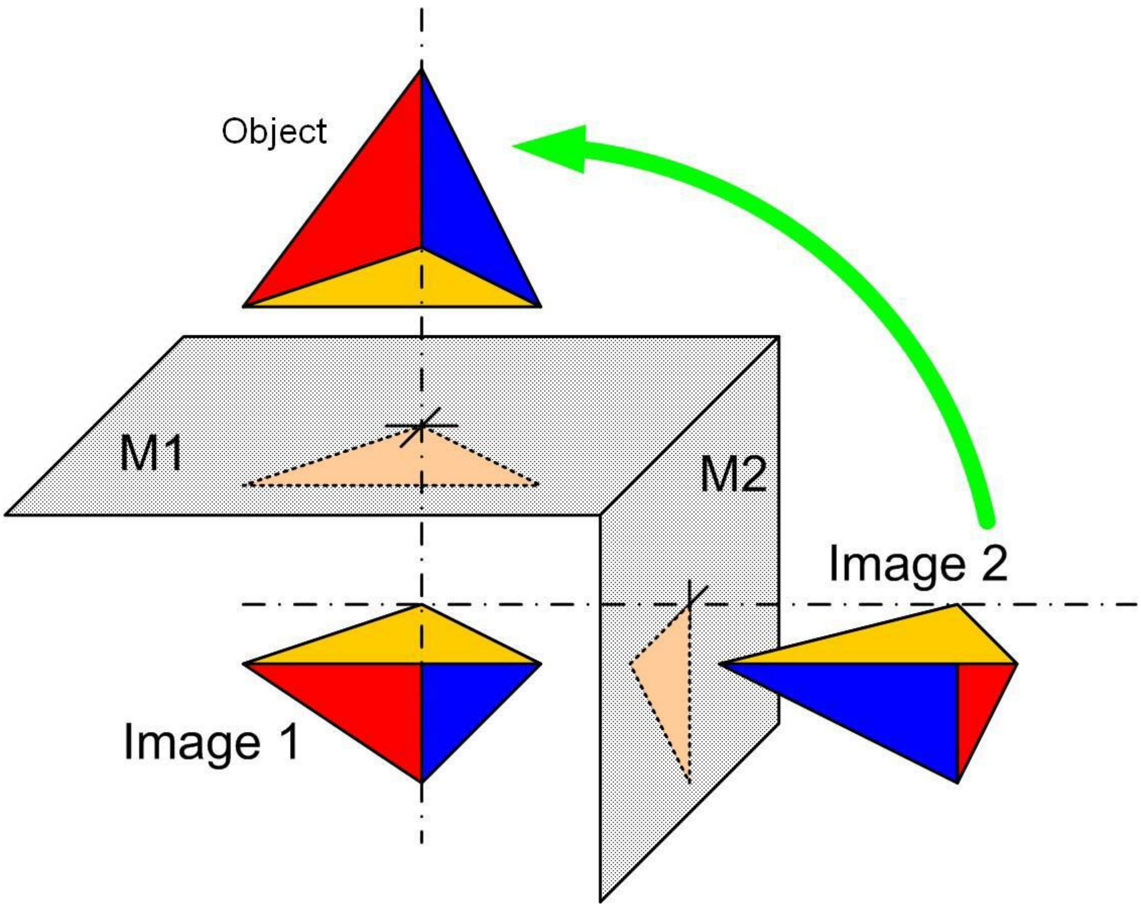
4.1. Plane mirrors



Let an object point be A and its projection on the mirror H. Any ray out of A follows the law (16). The prolongation of the reflected ray cuts the line AH at a point AH'. On figure 22 we easily demonstrate that if the absolute values of i_1 and i_2 are equal, those of i_3 and i_4 also are and $HA = HA'$.

Point A' is the symetry of A in relation to the mirror.

This is true for all luminous rays issued of A, the image is stigmatic, figure 22 show a real object and a virtual image. If we inverse the direction of the luminous rays, the object becomes A', virtual, and the image becomes A, real.



The image of an object in volume by a plane mirror has identical dimentions to the object but is not superimposable to it. It is said that it is a **left** handed image. A second image obtained from the first by a second mirror becomes superimposable to the object, by translation or rotation as in figure 23, it is said that it is a **right** handed image.

In such visual intruments as binoculars and spectacles, a paired number of reflexions ensures a right handed image.

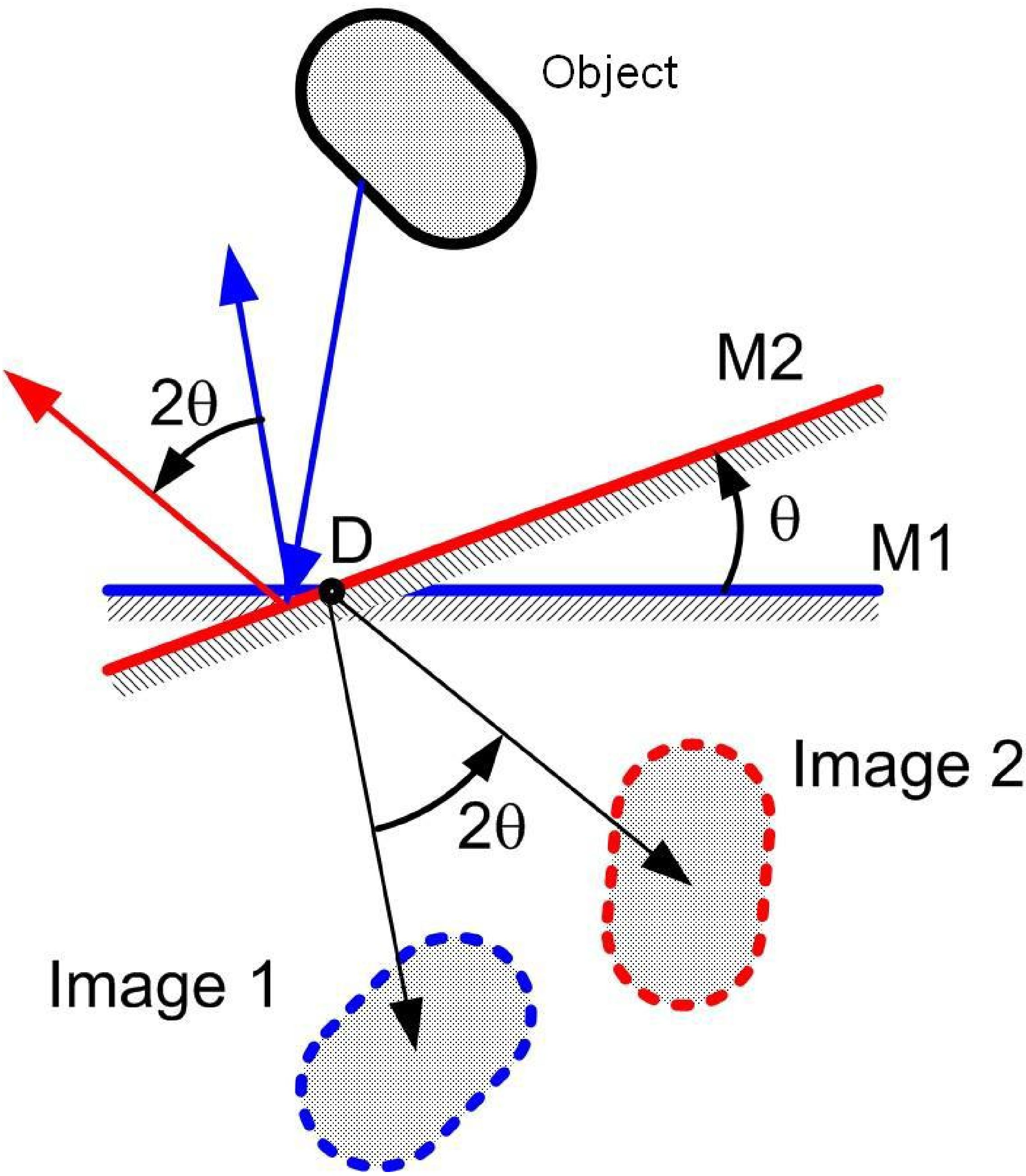
4.2. Translation and rotation of plane mirrors

A translation of a plane mirror of perpendicular vector $\vec{v}$ yields an image translation of vector $2\vec{v}$.

A rotation of a plane mirror M_1 with an angle θ around any axis parrallel to M_1 yields a mirror plane M_2 which cuts through M_1 following a line D parrallel to the rotation axis.

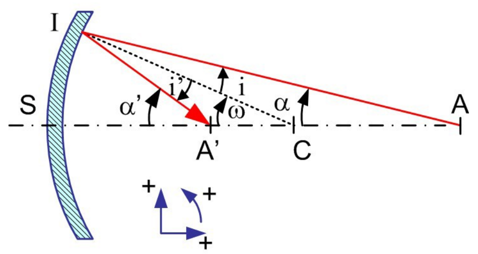
The image of an object in M_2 is obtained from the image in M_1 by a rotation of axis D and angle 2θ.

This is true for a luminous ray or for a rays beam.



4.3. Spherical mirrors

A spherical mirror is a concave or convex reflecting surface defined by the centre of curvature C and a vertex S located on the surface. The curvature ray is $R=\overline{SC}$.



Let us consider a point A of the line SC. A luminous ray arising from A reflects itself on a point I of the mirror et cuts its line SC in A'. If we carry out the same operation as for the diopters, we have :

$$\frac{\overline{CA}}{\sin(i)} = -\frac{\overline{IA}}{\sin(\omega)}$$

And

$$\overline{FA} = z - f$$

Or sin(i) = $-\sin(i')$ therefore :

$$\frac{\overline{CA}}{\overline{CA'}} = \frac{\overline{IA} \cdot \sin(i)}{\overline{iA'} \cdot \sin(i')} = -\frac{\overline{IA}}{\overline{IA'}}$$

The expression below is analogeous to the diopters' by replacing n' by $-n$. We therefore have, as for the diopters, $\overline{SA} = z \quad \overline{SA'} = z' \quad \overline{SC} = R$:

$$\frac{1}{z'} + \frac{1}{z} = \frac{2}{R} \ (17)$$

$$g_y = \frac{\overline{A'B'}}{\overline{AB}} = \frac{y'}{y} = -\frac{z'}{z} \ (18)$$

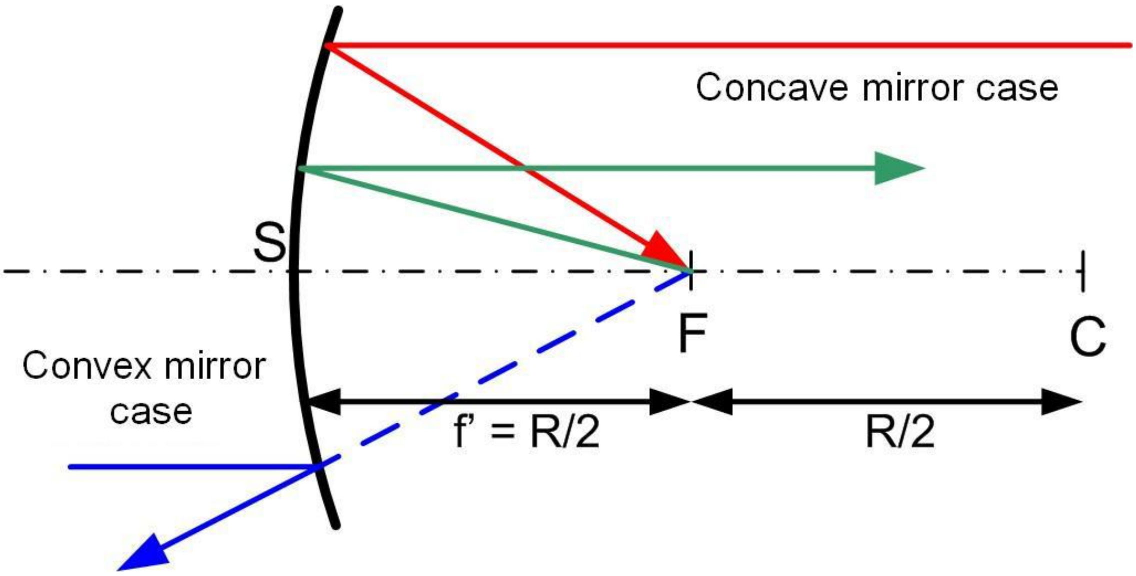
$$g_\alpha = \frac{\alpha'}{\alpha} = \frac{z}{z'}$$

$$g_z = \frac{dz'}{dz} = -\frac{z'^2}{z^2} \ (19)$$

These formula are identical whether the mirror is concave or convex.

The conjugation formula is symetrical in z and z'. If A' is image of A, A is image of A'.

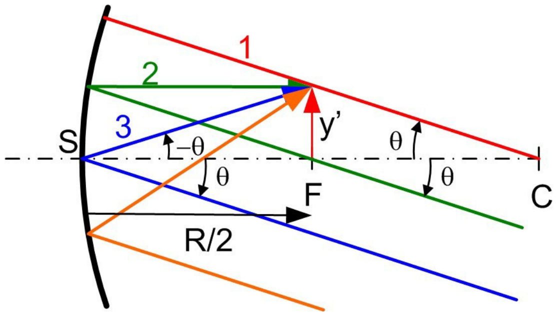
4.4. Focus, focal distance, focal plane



If the object is to infinity ($1/z = 0$), according to (17), the image, in F', is such that $z' = f' = R/2$.

Similarly for the object focus F, we have $1/z' = 0$ and $z = f = R/2$.

F and F' are confounded and are at the centre of segment SC.



The focal plane contains the images of the points to infinity.

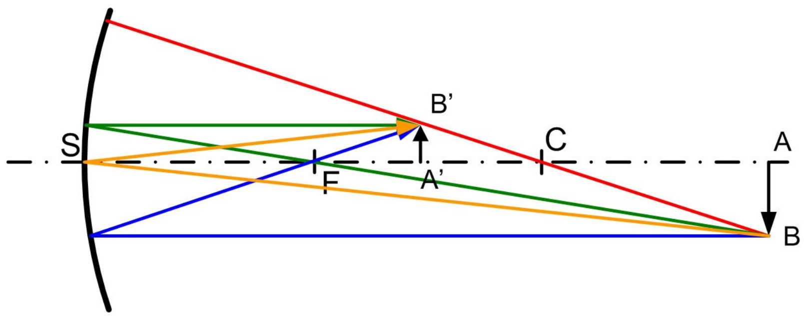
An object to infinity is caracterised by a beam of parallel rays forming an angle θ with the optical axis. Following figure 27, the ray l going through C reflects to itself, ray 2 going through F reflects itself in parallel to the axis. Ray 3 going through S reflects itself symetrically in relation to the axis.

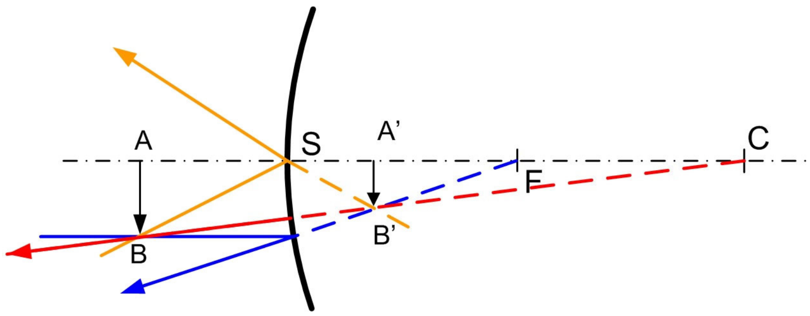
The dimension y' of the image is : $y' = -\theta \cdot R/2$

4.5. Image construction in a mirror

From point B of object AB, the ray going through C (Red) reflects to itself, the ray going through F (Green) reflects itself in parallel to the axis, the ray parallel to the axis (Blue) reflects itself by going through F and the ray going through S (Orange) reflects itself symetrically in relation to the axis.

B' is at the intersection of emerging rays, A' is on the **projection** of B' on the axis.



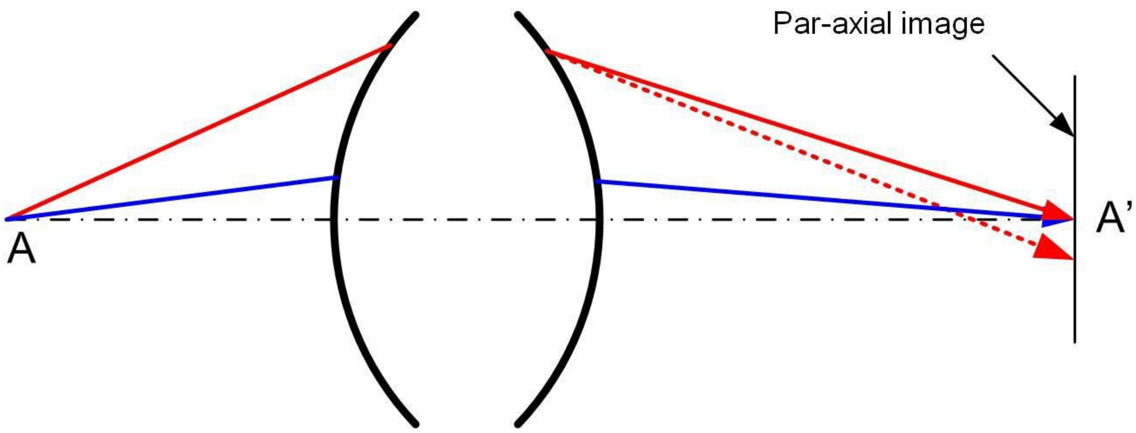


5. Centered optical systems

A centered optical system is compsed of a succession of diopters or mirrors. It has a symetrical axis of revolution. We have seen that generally stigmatism does not exist. Paraxial approximation helps to disregard stigmatic imperfections and to suppose that these systems are perfect. This is essential in order to calculate the position of the images.

Moreover, optical systems are generally spared from stigmatisms leading to **approximate stigmatism** The position of the images are thus that of paraxial imagery, even if incidence angles on optical surfaces are well above paraxial approximation.

In fact stigmatism implies that all luminous rays converge to the same point.



Paraxial stigmatism presents a convergence position which then becomes that of the totality of luminous rays emerging from the system. Following figure 30, a marginal ray must converge on the paraxial image so that stigmatism is ensured.

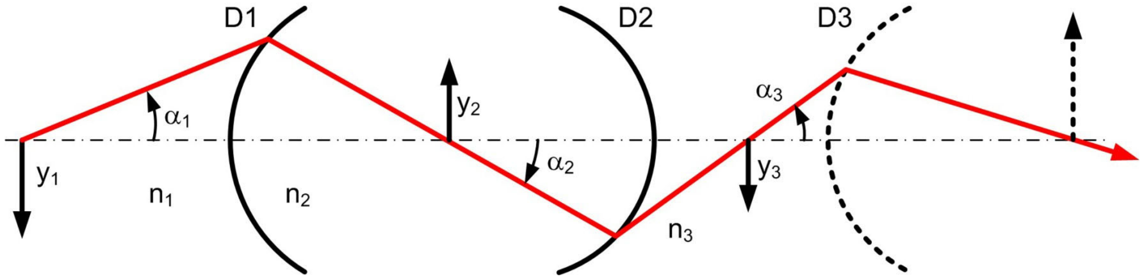
Moreover, aberrations correction yields an absence of field curvature and a constant magnification, which is also one of the caracteristics of paraxial conjugations.

Any centered optical system free from aberrations has an imagery perfectly determined by paraxial imagery.

5.1. Lagrange-Helmoltz extended invariant

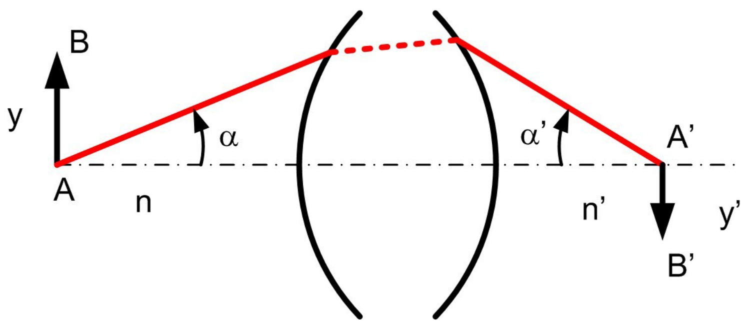
An optical system is composed of several diopters $D1, D2, D3$ ...

Following (13) we have, for each diopter : $n_1 y_1 \alpha_1 = n_2 y_2 \alpha_2 = n_3 y_3 \alpha_3 ...$



Consequently, for any optical system S, for any object AB of dimension y having for image in $S\, A'B'$ of dimension y', and a luminous ray going from A forming and angle α with the axis, arriving as A' under the angle α', we have :

$n y \alpha = n' y' \alpha'$ (20)

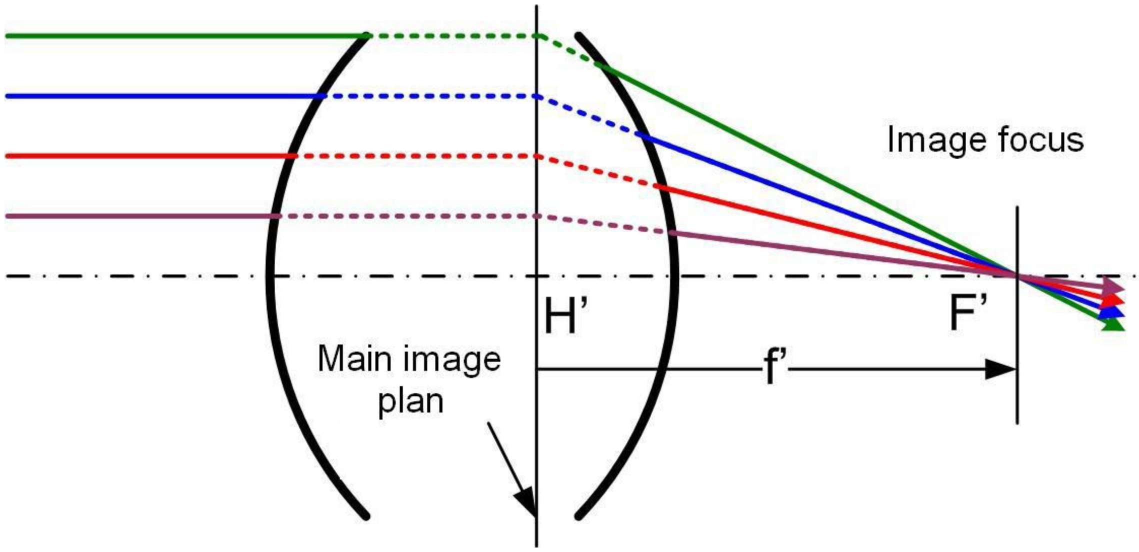


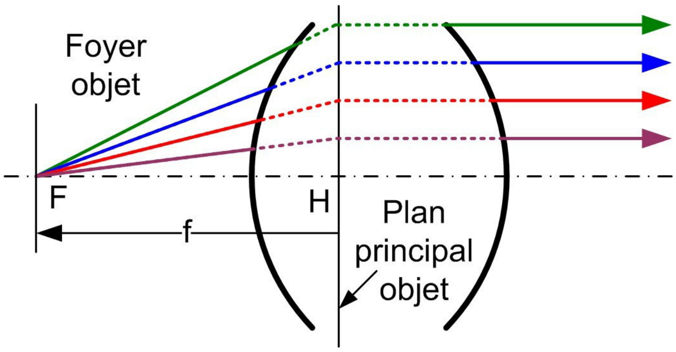
5.2. Focus and principal planes of focal optical systems (with focus)

By definition, the image focus F' of an optical system is the image of the infinite point on the axis. The beam issued out of this point is made of parallel rays to the axis.

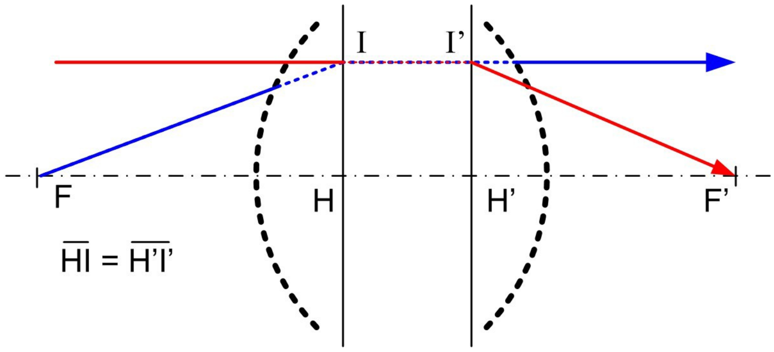
These rays focalise in F' after crossing the system. The location of the intersection points of each incident ray with its corresponding image ray is, in paraxial approximation, a plane which shall be called **image principal plane** of the optical system.

This plane cuts the axis in H', $f' = \overline{H'F'}$ is the focal image of the optical system. Distance . H' is the image principal point. We proceed in the same way as for the object focus F, the object principal plane, the object focal distance $f = \overline{HF}$.





Following figure 35, any luminous ray issued from F cust the object principal plane in I and comes out parallel to the axis, it cuts the image principal plane in I'. An incident ray parallel to the axis groing through I, also goes through I' the converges in F'. These two rays cross each other in I in the object space then in I' in the image space I and I' are therefore conjugated.



The principal planes are conjugated with an associated transversal magnification equal to 1.

F, F', H et H' are the **cardinal points** of the optical system.

5.3. The construction of images

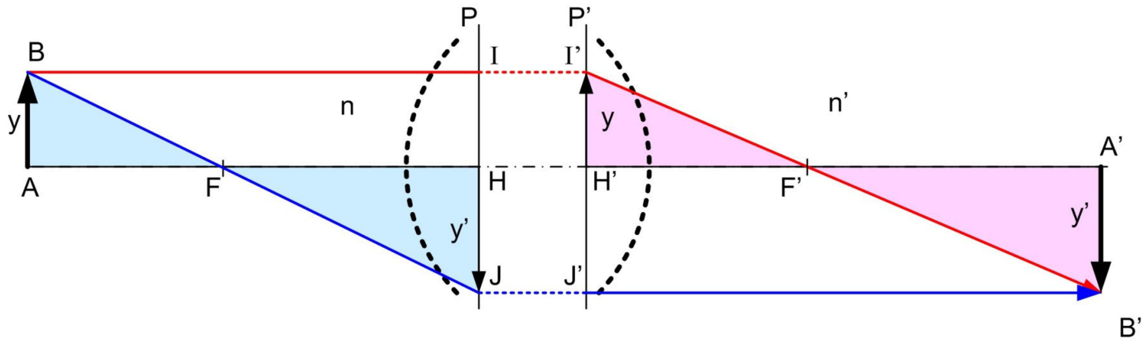
Let us consider an optical system, its focus F and F', its principal points H abd H', its principal planes P and P', an object AB of dimension y. So as to build the image B' from B, let us run from B two luminous rays.

Ray 1 (Red) : Parallel to the axis, cuts P in I. The image ray goes through I' (image of I) and F' (Incident parallel to the axis), we have :

$\overline{HI} = \overline{H'I'} = y$

Ray 2 (Blue) : Goes through F, cuts P in J. The image ray goes through J' (image of J) and comes out parallel to the axis (Issued of F), we have :

$\overline{HJ} = \overline{H'J'} = y'$



These rays intersect in B', image of B. Paraxial stigmatism causes that any other ray issued from B crossing the optical system goes through B'.

B' is perfectly defined by the position of the object B and the position of the 4 points (H, H', F, F').

The optical system is perfectly defined by the cardinal points (H, H', F, F').

Point A', image of A, is on the **projection**, of B' on the axis.

5.4. Conjugate equation at focus

On the previous figure, triangles (FAB) and (FHJ) as well as $(F'H'I')$ and $(F'A'B')$ are similar, we can therefore write, knowing that $\overline{HJ} = y'$ and $\overline{H'I'} = y$:

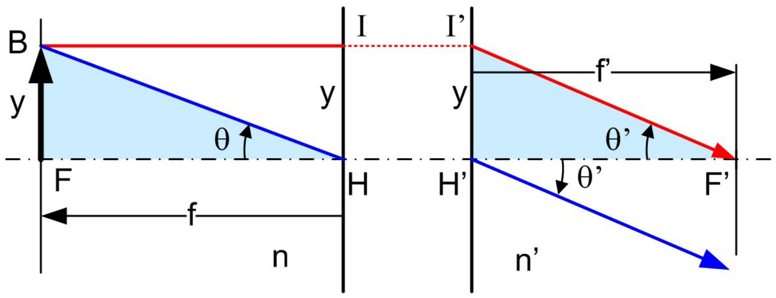
$$\frac{y'}{y} = \frac{\overline{FH}}{\overline{FA}} = \frac{\overline{F'A'}}{\overline{F'H'}}$$

From there we can deduct a conjugate relationship, said to be **Newton's** :

$\overline{FA} \cdot \overline{F'A'} = f \cdot f'$ (21)

5.5. Refracting power, relationship between f and f'

The optical system is defined by (H, H', F, F').



Let us consider an object point B situated on the object focal plane. Its image B' is to infinity. Foloowing figure 37, the ray BI parallel to the axis (red) has for an image $I'F'$ forming the angle θ' with the axis as all emerging rays since image B' is to infinity.

Ray BH (blue) has for image a ray issued from H' (image of H) and forming an angleθ'. The relationship (20) applied to the conjugation (HH') gives us : $n\theta = n'\theta'$ or $\theta = y/f$ and $\theta' = -y/f'$ From this we deduct Cv being the system refracting power :

$$Cv = \frac{n'}{f'} = -\frac{n}{f} \ (22)$$

5.6. Descartes' conjugate equation, magnification

Principal points H and H' are taken to have for origin :

$\overline{HA} = z$

And

$\overline{H'A'} = z'$

So that

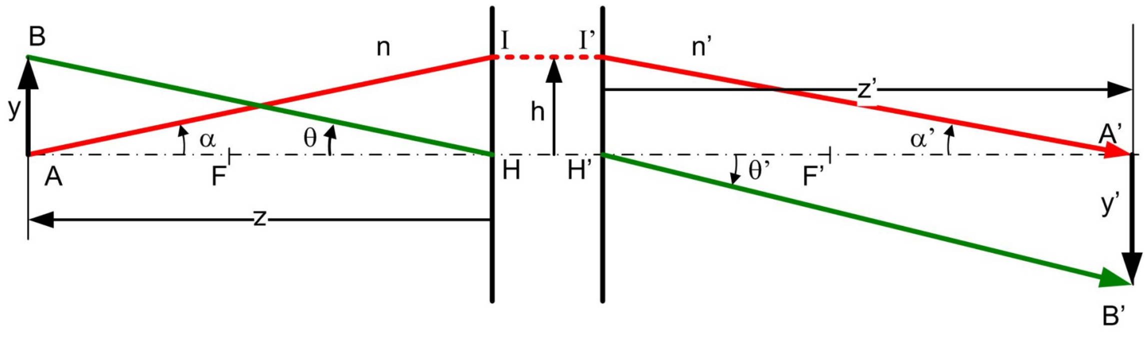
$\overline{FA} = z - f$

And

$\overline{F'A'} = z' - f'$

Replacing in (21), the conjugate equation becomes :

$$\frac{n'}{z'} = \frac{n}{z} + \frac{n'}{f'} = \frac{n}{z} + Cv \ (23)$$



Following figure 38, the ray (green) issued from B and going through H forms an angle with the axis. As previsously shown, its image going from H' forms an angleθ' such that $n\theta = n'\theta'$. Since $\theta = y/z$ and$\theta' = y'/z'$, we deduct from it the transversal magnification g_y :

$$g_y = \frac{\overline{A'B'}}{\overline{AB}} = \frac{y'}{y} = \frac{nz'}{n'z} \ (24)$$

With (22) and (23) :

$$g_y = \frac{f}{f-z} = 1 - \frac{z'}{f'} \text{ (25)}$$

To one sole value of z (or of z') corresponds only one value of g_y and conversely. These formula are identical to that of diopters'. For one diopter, the principal planes are confounded and situated on the dioopter surface.

The same kind of reasonning leads to an angular magnification g_α:

$$g_\alpha = \frac{\alpha'}{\alpha} = \frac{z}{z'} \text{ (26)}$$

And to an axial magnification g_z :

$$g_z = \frac{dz'}{dz} = \frac{n}{n'} \cdot \frac{z'^2}{z^2} \text{ (27)}$$

5.7. Nodal points, antiprincipal points

Nodal points N and N' are conjugate points so that $g_\alpha = 1$, we thus deduct :

$\overline{HN} = \overline{H'N'} = f + f'$

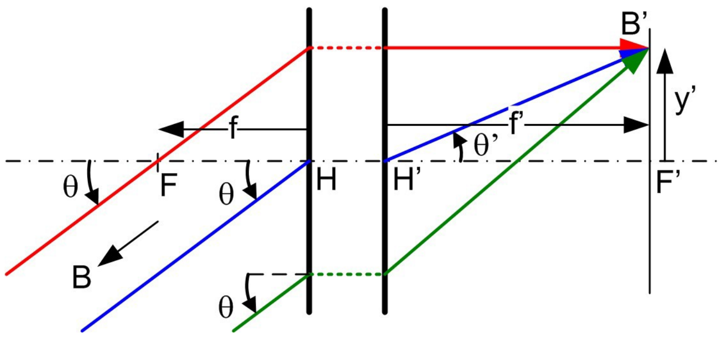
Antiprincipal points K and K' are the same as $g_y = -1$, according to (25) we thus deduct :

$\overline{HK} = 2f$

And

$\overline{H'K'} = 2f'$

5.8. Image dimension of an object non punctual to infinity



An object AB of dimension θ o infinity has an image FB' in the image focal plane of which the dimension is :

$y' = -f \cdot \theta$ (28)

As for $n\theta = n'\theta'$ we also have : $y' = f'\theta' = nf'\theta/n'$

5.9. Optical systems refractive in the air

They represent the majority of existing optical systems (lenses, camera lenses, eye-pieces...)

In the air, $n = n' = 1$

- Refracting power :

$Cv = 1/f' = -1/f$ et $f' = -f$ (29)

- Newton's conjugate equation :

$\overline{FA} \cdot \overline{F'A'} = -f'^2$ (30)

- Descartes' conjugate equation :

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f'} \quad (31)$$

- Magnification :

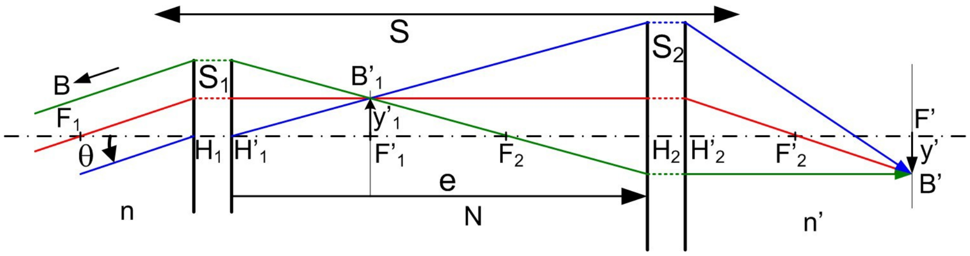
$$g_y = \frac{z'}{z} \quad g_\alpha = \frac{z}{z'} \quad g_z = g_y^2 = \frac{z'^2}{z^2} \quad (32)$$

- N and N' are respectiively confounded with H and $H'-N$.

6. Association of systems

6.1. Association of two optical systems

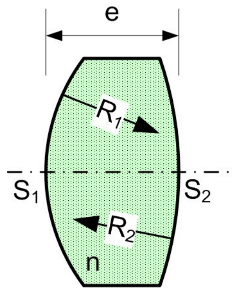
Let an optical system S be made of two sub systems $S1(F_1F'_1H_1H'_1)$ and $S2(F_2F'_2H_2H'_2)$ with a focal distance f'_1 et f'_2. Let us look for the optical properties of S.
The entrance index is n, the intermediate index N, the exist index n'.
An object to infinity AB with angular dimension θ yields, in the image focal plane $F'_1B'_1$ of S_1, an image with a dimension $y'_1 = -f_1 \cdot \theta$ following (28). The system S_2 renders a definitive image $F'B'$ in the image focal plane of S of which the dimension y' can be expressed by :
The magnification g_{y2} of the conjugation $F'_1 \rightarrow F'$:
$y' = y'_1 \cdot g_{y2} = -f_1 \cdot \theta \cdot g_{y2}$
The relationship (28) applied to S,f being the object focal distance of S :
$y' = -f \cdot \theta$
We deduct the object focal distance of S :
$f = f_1 \cdot g_{y2}$ (33)



Gullstrand's formula give a simple relationship beween the convergences $Cv, Cv_1, Cv_2 de S, S_1, S_2$, distance $\overline{H'_1H_2} = e$ and the intermediate index N :
$Cv = Cv_1 + Cv_2 - e \cdot Cv_1 \cdot Cv_2 / N$ (34)

7. Lenses

They are made of transparent medium of index n between two diopters of radius R_1 et R_2. The lens axis goes through the curvature centres of the sides. This axis cuts the diopters at their vertex S_1 and S_2. e = S_1S_2 is the thicken at the vertex of the lens.



Diopters refracting power vare respectively :

$Cv_1 = (n\text{-}1)/R_1$ et $Cv_2 = (1\text{-}n)/R_2$

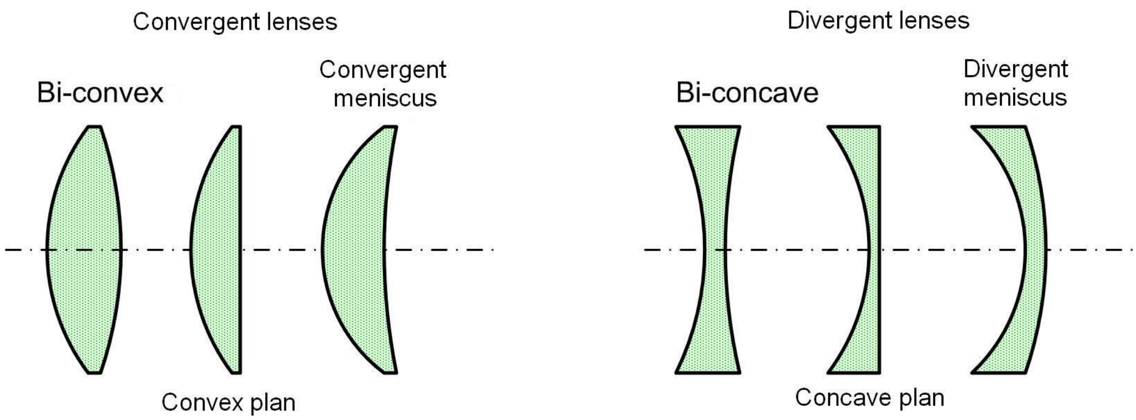
The lens is in the air, following (29) the lens refracting power Cv is :

$Cv = 1/f' = -1/f$.

For certain types of lenses (ophthalmic lenses), Cv is expressed in dioptrie rated δ (1.δ = 1.m^{-1}).

Gullstrand's formula gives the refracting power Cv of a lens as the associaiton of two diopters. We get :

$$\frac{1}{f'} = \mathrm{Cv} = (n-1)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - \frac{e.(n-1)^2}{R_1 R_2} \qquad (35)$$



Convergent lenses are thicker at their centre than on their sides, inversely divergent lenses are thicker on their sides.

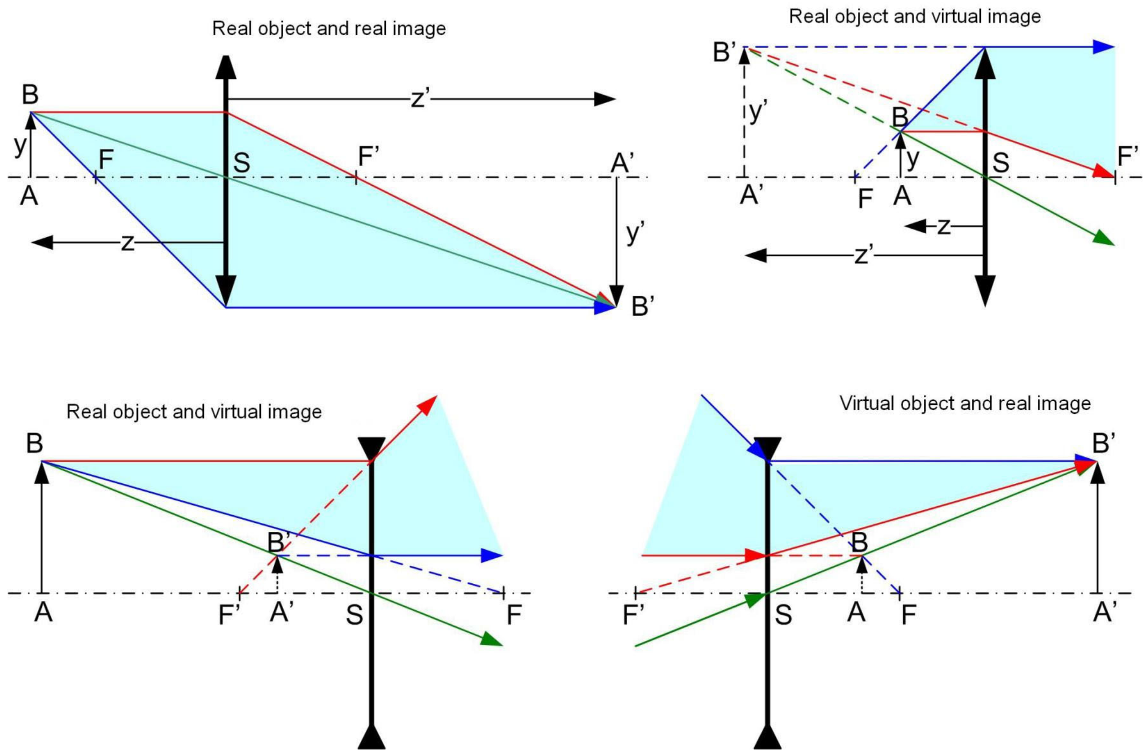
7.1. Thin lenses

A lens is considered thin if e is small in comparison to R_1 and R_2. The term $\frac{e\cdot(n-1)^2}{R_1 R_2}$ is négligeable and the focal distance f' is such that :

$$\frac{1}{f'} = (n-1)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (36)$$

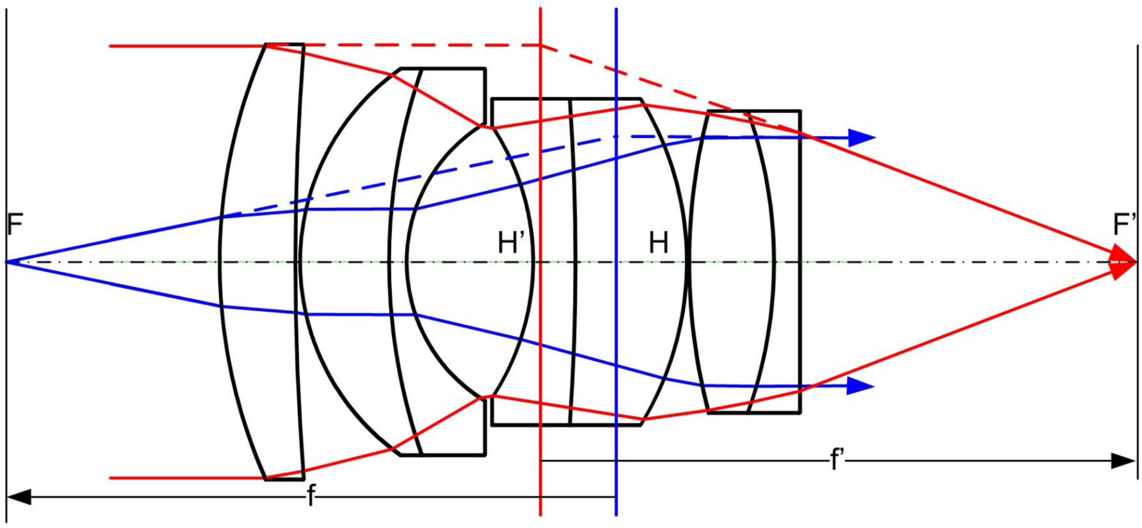
Specific caracteristics of thin lenses :

Rayon issu de B (Rayon objet)	**Rayon réfracté allant en B' (Rayon image)**	**Couleur**
Parallèle à l'axe	Passe par F'	Rouge
Passe par F	Parallèle à l'axe	Bleu
Passe par S	N'est pas dévié	Vert



7.2. The simulation of a complex optical system with a thin lens

The example in figure 45 shows an objective lens for 7 lens field of view, with focus positions and principal planes positions.



H, H', F and F' suffice to perfectly define the system in the paraxial approximation.

The translation of the entire image space by vector $\overrightarrow{H'H}$ which do not modify the conjugate formulae in relation to H and H', as well as magnification, this complex objective can be perfectly simulated with a thin lens situated on H and H' and with the same focal distance.

Most complex optical systems in the air, made of thick lenses or of multi-lens optical sub sets can be studied in this way by replacing these elements with thin lenses.

7.3. Association of thin lenses

Each lens i is caracterised by its location and its convergence $Cv_i = 1/f'_i$.

To determine convergence $Cv = 1/f'$ of the association of two lenses we apply Gullstrand's formula (34) with $N = 1$:

- The case of two lenses at distance e : $Cv = Cv_1 + Cv_2 - e \cdot Cv_1 \cdot Cv_2$ being :

$$\frac{1}{f'} = \frac{1}{f'_1} + \frac{1}{f'_2} - \frac{e}{f'_1 \cdot f'_2}$$

We thus deduct :

$$f' = \frac{f'_1 f'_2}{f'_1 + f'_2 - e} \quad (37)$$

- The case of two joined lenses(Cv_1 et Cv_2) : $e = 0$ hence $Cv = Cv_1 + Cv_2$

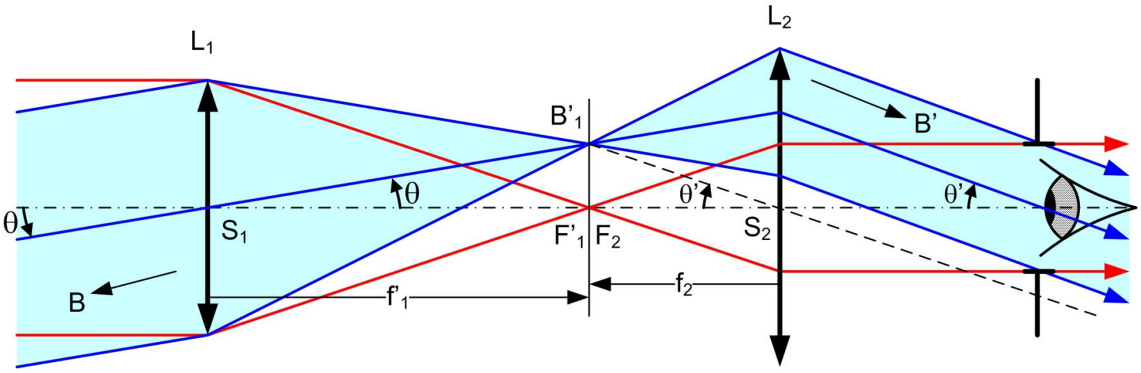
8. Afocal systems, magnification

8.1. Afocal systems, magnification

In formula (37), we notice that in cases where distance e between two lenses is equal to $f'_1 + f'_2, f'$ is infinite, Cv is null. The point at infinity is the image to itself. The system has no focus. The image focus of the first lens is confounded with the object focus of the second.

In the case of a complex system S in the air composed of two sub systems, as soon as the image focus of the first is confounded with the object focus of the second, the system is afocal.

Such is the case for visual observation systems as glasses, binoculars, telescopes.

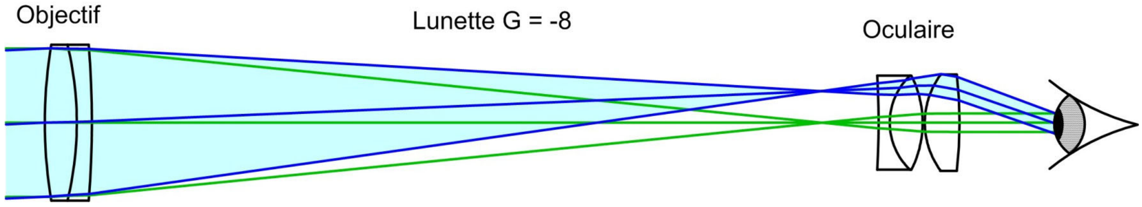


The above figure shows such a system, composed of two sub systems which are assimilated to thin lenses L_1 and L_2. Object B is to infinity in the direction θ. The main properties found in these afocal systems are :

- F'_1 confounded with F_2
- The object point at infinity is the image to itself.

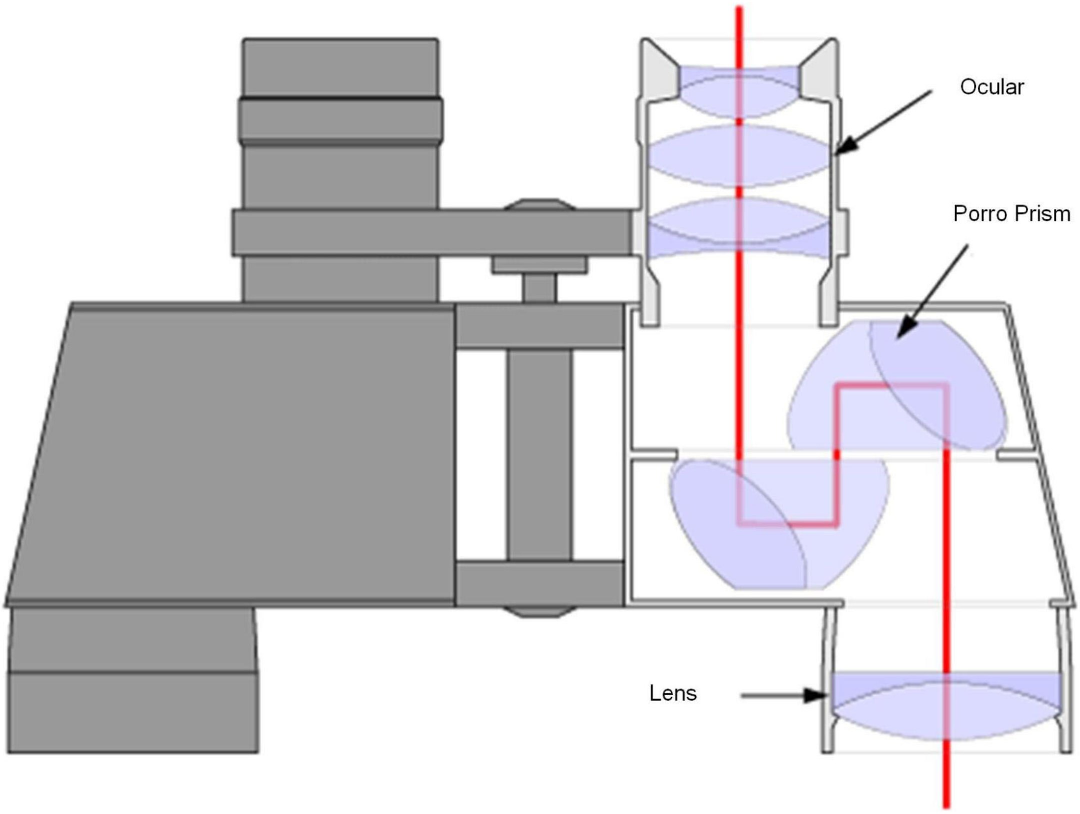
- Transversal magnification is constant : $g_y = \frac{y'}{y} = -\frac{f'_2}{f'_1}$
- Angular **magnification** G is constant : $G = g_\theta = \frac{\theta'}{\theta} = -\frac{f'_1}{f'_2}$

An astronomical telescope is an afocal instrument composed of a head system called the objective which gives an intermediate image. This image is observed with an ocular which, from the intermediate image, gives an image situated at infinity observable by the eye. The magnification is negative, images are inverted.

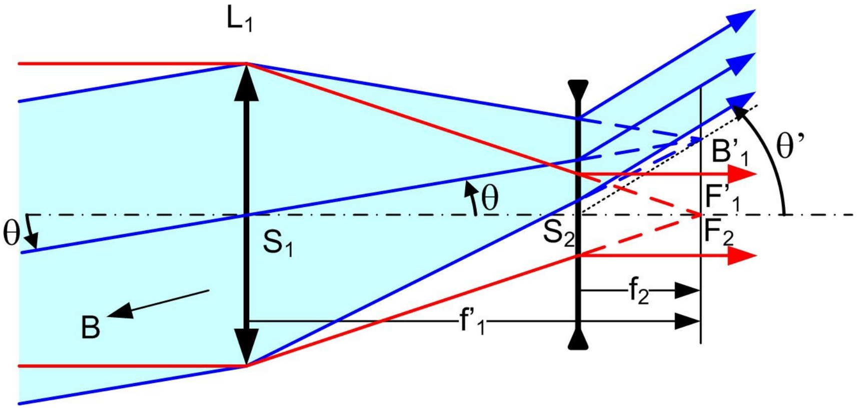


The terrestrial visual afocal instruments (glasses, binoculars) have a reversing device, generally with prisms, between the lens and the ocular, which gives a positive enlarging.

In a binoculars body, a set of two Porro's prisms enables a $180°$ rotation of the image around the optical axis and thereby straightens the images.

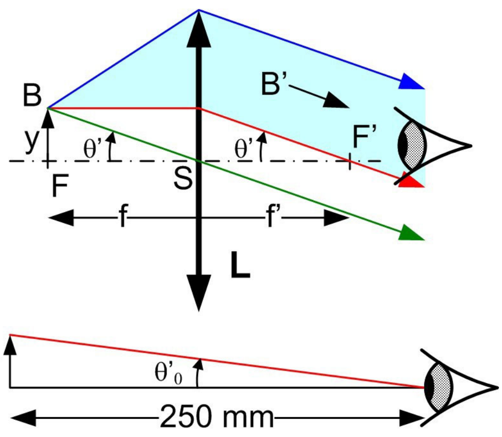


Galilee's binoculars: It is characterized by a second divergent system, the magnification is then positive, the images are straight but the field is much reduced. We use them with weak magnification such as theatre binoculars.



8.2. Focal visual optical systems magnification (magnifying glass, oculars, microscopes...)

These optical systems are used to allow an observer to obtain, a long distance image from an finite distance object, visible with eye without any accommodation.



If we admit that we assimilate such a system to a thin lens L, figure 50 represents an object FB of dimension y in the focal object image of L. The image B' of B is at the infinity in the direction $\theta' = y/f = -y/f'$.

The *Gc* magnification said « commercial » of these systems is the ratio between θ' and θ'_0, angle under which we directly see the same object with the eye at a distance of $250mm$, thus:

$G_c = \dfrac{\theta'}{\theta'_0}$ we can deduce $G_c = \dfrac{250}{f'}, f'$ in mm

A $\times 5$ magnifying glass has a $50mm$ focal distance. An $\times 20$ ocular has a $12,5mm$ focal distance.

9. Diaphragms, lenses and fields

An optical system is transversely restricted geometrically by the frames of the lenses or mirros and other various mechanical diaphragms.

For a given combination, **the pupil** is the diaphragm which limits the luminous beam coming from the object. This diaphragm is generally put inside the optical system. We define the exit pupil of the system by the image of this diaphragm through the lenses (or mirrors) which follow it. It is also the image of the entrance pupil through the lenses (or mirrors) which precedes it.

The exit pupil is the image if the entrance pupil in the optical system. **The field object** is the maximum lateral dimension of the object.

When the object is infinite, the object field expresses in an angular way. The field is usually restricted by a special diaphragm called: **"field diaphragm"**.

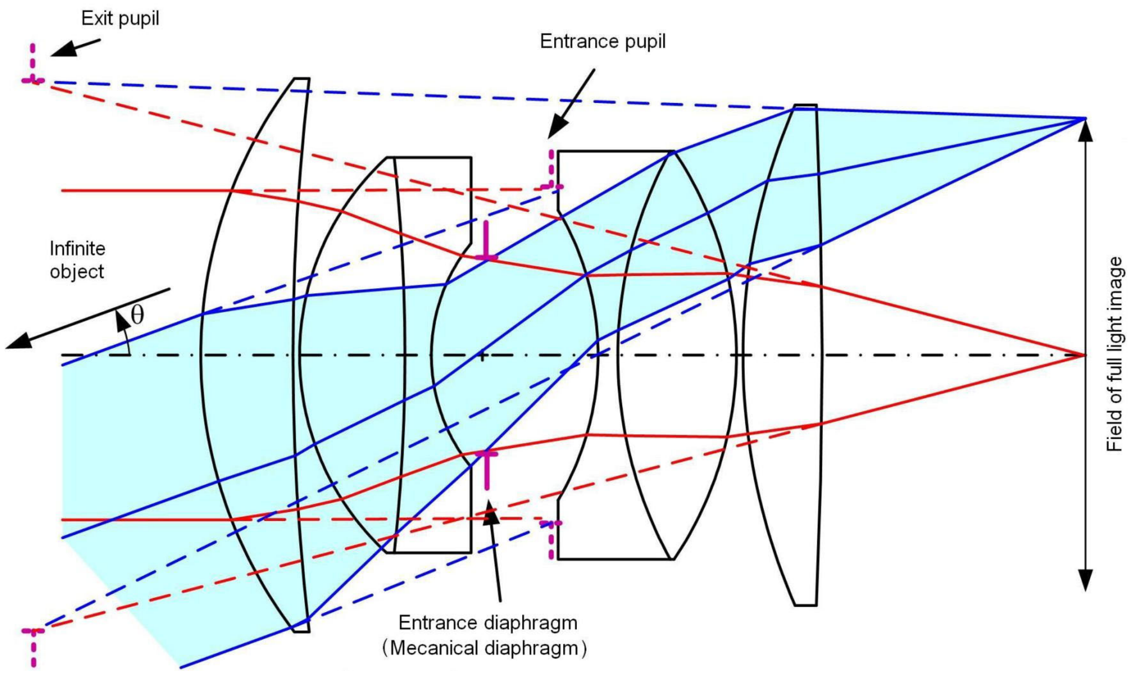
The full light field object is defined by the set of points of the objects for whose the luminous beam crossing the system are restricted only by the opening diaphragm, D.O. It provides a quasi uniform lightning to the full light field. Beyond the full light field, the diaphgragmations provoked by the frames of the lenses extensively reduce the lightning of the image. This reduction of the lightning at the field edge is called **vigneting**.

9.1. Examples

Exemple : 1 :

We take into account a 6 lenses shooting objective, the object is at the infinity. An iris put in the middle of the objective is used as an opening diaphragm for the system. The entrance and exit pupils are virtual.

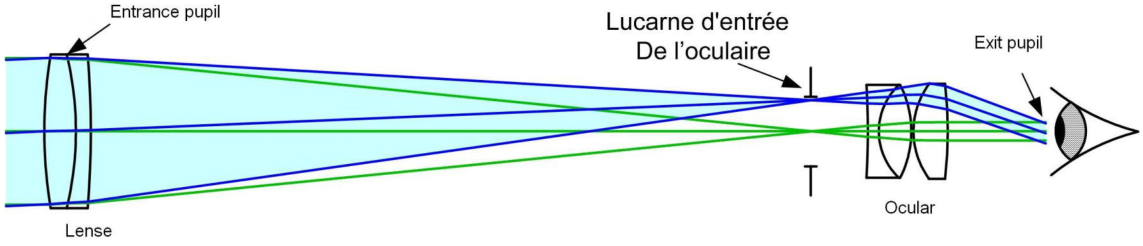
The full light field points are lighted up by the D.O. Figure 52 shows the edge of the full light field image, for a point beyond, the beam of light will be limited, in part, by the frames of lenses.



Exemple : 2 :

Refracting telescope: the object is at the infinity; the objective of the refracting telescope makes the beams limitation, it is D.O and entrance pupil, its image by the ocular is the exit pupil which is the place where the eye pupil of the observer has to set itself.

The entrance and exit pupils are real. A diaphragm situated in the focal plan of the ocular executes the field limitation. The image full light field is infinite; it is confounded with the infinite image of the ocular diaphragm of the ocular field.

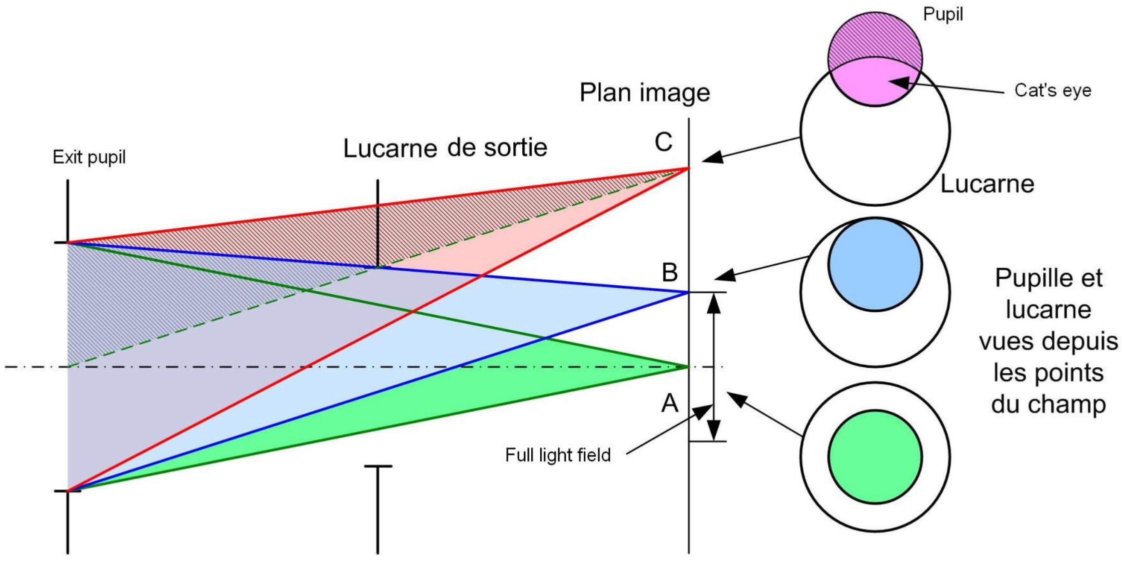


Second diaphragm : Combined object or image of the field diaphragm

In an optical system with two dominant diaphragms (we do suppose that other **diaphragmations** do not play a part), we calculate the full light field in the following way :

- Diaphragms are brought back to the same space, for instance image space.
- The pupil is the object conjugate or D.O. image seen under the smallest angle from the centre of the image fiel (point A).
- The secondary stop is the object or image conjugate of the field diaphragm.
- The side of the full light field (point B) can be obtained by seeking the intersection with the image field - the closest to the centre of the field - of the ray which joins the side of the pupil and the side of the **secondary stop** (in the same space : object, image or intermediate).

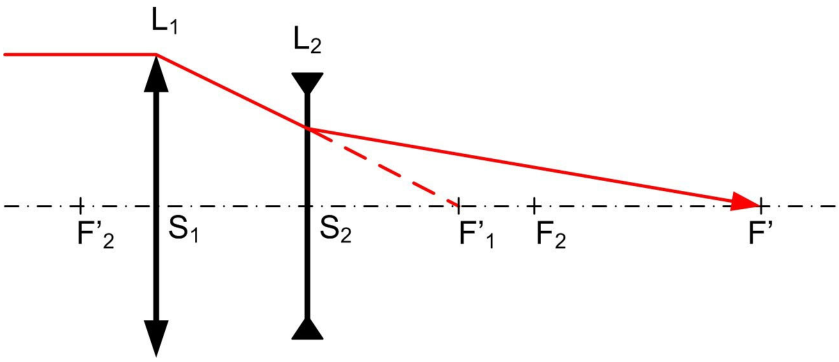
For a point outside the full light field (point C for instance), rays issued out of the pupil converging in C are partly obstructed by the secondary stop, diaphragmation is said to be "cat"s eyed".



III.Case study : Calculating the optical characteristics of a two-lens system

An optical system is made of a first convergent optical system equivalent to a convergent thin lens L_1 with focal distance $f'_1 = -40\text{mm}$, and a diameter of 40 mm and a second optical system equivalent to a divergent thin lens L_2 with focal distance $f'_2 = -30\text{mm}$ and a diameter of 30 mm. Lenses L_1 et L_2 are distant by 20 mm.

$\overline{S_1F'_1} = f'_1 = -40\text{mm}$ and $\overline{S_2F'_2} = f'_2 = -30\text{mm}$.



1. Determining the position of image focus F' of S

F' is the image in the system of the point at infinity on the axis.

The object being infinite on the axis, L_1 gives it an image in its image focus F'_1.

Of F'_1 , L_2 gives a definitive image which is the image focus F' de S.

F' is the conjugate of F'_1 in lens L_2.

To determine the position of F' let us apply the conjugate formula (31) $\frac{1}{z'} = \frac{1}{z} + \frac{1}{f'}$

For the conjugation being considered : $\frac{1}{\overline{S_2F'}} = \frac{1}{\overline{S_2F'_1}} + \frac{1}{f'_2} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60} \Rightarrow \overline{S_2F'} = 60mm$

2. Determining the position of H'

We know that $f' = \overline{H'F'}$, f' enables us to calculate the position of H'.

Calculating f' :

Méthode : Method 1

The magnification of conjugaison $F'_1 \Rightarrow F'$ in L_2 is $z'/z = 3$

The formula (33) gives us $f = 3f_1$ and therefore $f' = 3f'_1 = 120mm$

Méthode : Method 2

$$1/f' = Cv = 1/f_1' + 1/f_2' - S_1S_2/f_1'/f_2' = 1/40 - 1/30 + 20/(20*30) = 1/120$$

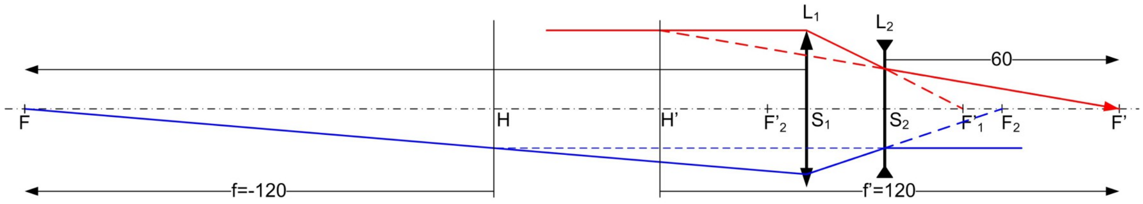
3. Determining the position of object focus F of S

We calculate in the same way by inverse return of light, F is the image of F_2 in L_1 by inverse return, or F_2 the image of F by direct trajectory, the equation is written in this way :

$$\frac{1}{\overline{S_1F_2}} = \frac{1}{S_1F} + \frac{1}{f_1'} \Rightarrow \frac{1}{\overline{S_1F}} = \frac{1}{S_1F_2} - \frac{1}{f_1'} = \frac{1}{50} - \frac{1}{40} = -\frac{1}{200} \Rightarrow \overline{S_1F} = -200mm$$

H is such that $\overline{FH} = -f = f' = 120mm$

Figure EC 2 shows the system diagram and the cardinal points.

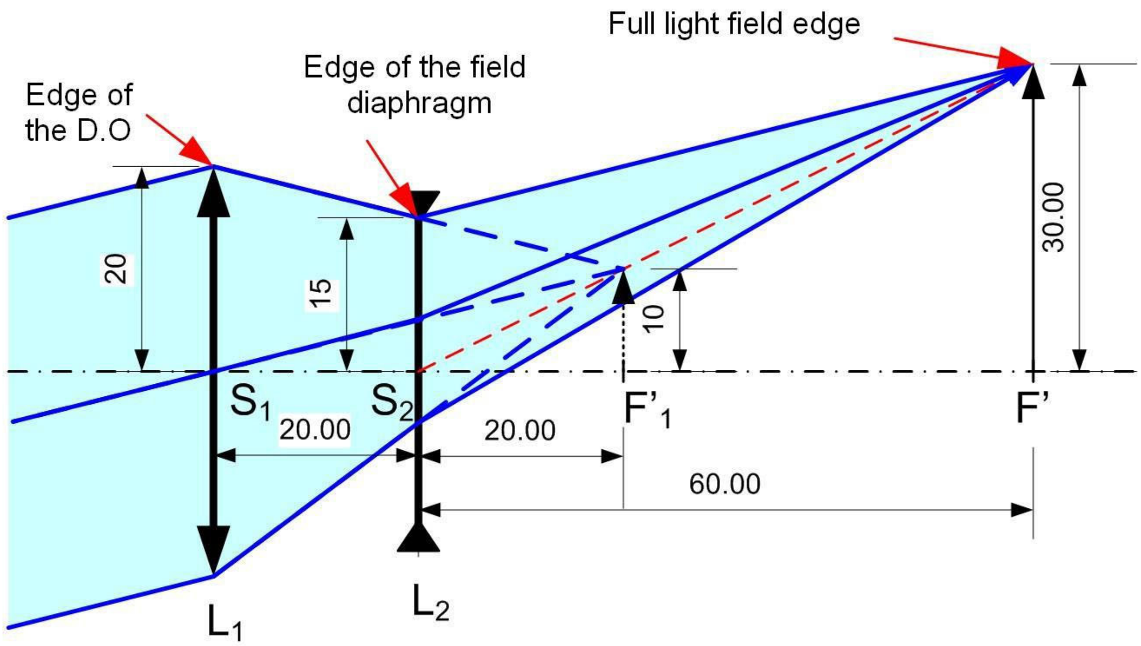


4. Calculating the full light field

Figure EC2 shows that L_1 is pupil and L_2 secondary stop because **the farthest ray** for one point on the axis goes through the side of L_1. EC3 shows a ray beam arising from the edge of the full light field.

The **farthest ray beam** goes through the pupil side and the edge of the secondary stop. It cuts the intermediate field situated in the focal plane of L_1 to $10mm$ from the axis.

The magnification of the conjugation $F_1' \rightarrow F'$ of L_2 being 3, the full light field edge is at $30mm$ from the axis. Therefore the total dimension of the full light field is $60mm$.


Full light field edge
Edge of the field diaphragm
Edge of the D.O
20
15
10
30.00
S1
S2
F'1
F'
20.00
20.00
60.00
L1
L2

IV.Exercices

1. Exercice 1

A telescope has a concave mirror with a curvature radius of $2000mm$.

Question 1

[Solution n°1 p 43]

To which distance from the vertex of the mirror is situated the image of a star ?

Question 2

[Solution n°2 p 43]

What is the dimension of the moon of which the visible diameter is 30 arc minutes ?

2. Exercice 2

We use a magnifying glass which commercial magnification *Gc* is equal to 5 to produce an enlarged image of a luminous object on a screen. The magnifying glass is assimilated to a thin lens. The distance of the object on the screen is 360 mm.

$$G_c = \frac{P}{4} \Rightarrow P = 4 - G_c$$

And

$$P = \frac{1}{f'}$$

Question

[Solution n°3 p 43]

What is the distance of the image to the lens ? What is the image magnification ? What is the direction of the image in relation to the object ?

3. Exercice 3

A Huygens ocular has two lenses $L1$ (objective) and $L2$ (ocular) supposed thin with focal distances respectively of $f'1 = 30mm$ and $f'2 = 10mm$. The distance between the two lenses is $S_1S_2 = 20mm$.

Question

[Solution n°4 p 43]

Determine the ocular focal distance and place the cardinal points.

Solution des exercices

>Solution n°1 (exercice p. 42)

The mirror has a ray curvature of $2000mm$ its focal distance f' is $1000mm$**.** The image of a star is at $1000mm$ from the vertex of the mirror.

>Solution n°2 (exercice p. 42)

The moon has a visible diameter of 30 arc minutes, such that :

$$\theta = \frac{30 \times \pi}{180 \times 60} RAD = 8.7.10^{-3} RAD$$

The image dimension in the focal plane is :

$$y' = f' \cdot \theta = 8.7.10^{-3} \times 1000 = 8.7mm$$

>Solution n°3 (exercice p. 42)

A magnifying glass of commercial magnification 5 has a focal distance f' of $250/5 = 50mm$. The lens is convergent. The object and the image are real and from each side of the lens. z (object) is negative and z' (image) positive. We can write the two equation system with two unknown quantities :

Conjugate relationship :

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f'}$$

$$z' - z = 360$$

After simplification, we obtain the second degree equation in z' *:*

$$z^2 - 360z' + 18000 = 0$$

Roots are 30 and 300.

For $z' = 60$, $z = -300$ and $gy = z'/z = -1/5$ the image is smaller.

For $z' = 300$,$z = 60$ and $gy = -5$ The image is greater. This is the solution.

Negative magnification means that **the image is inversed in relation to the object.**

>Solution n°4 (exercice p. 42)

Ocular focal distance is given by Gullstrand's formula (34) :

$$\frac{1}{f'} = \frac{1}{f'_1} + \frac{1}{f'_2} - \frac{e}{f'_1 \cdot f'_2} = \frac{1}{30} + \frac{1}{10} - \frac{20}{30 * 10} = \frac{1}{15}$$

$$f' = 15mm$$

The ocular image focus (from the eye) is the image focus $F'1$ of lens $L1$ in $L2$. The conjugation equation :

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f'}$$

Is written :

$$\frac{1}{\overline{S_2F'}} = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$$

F' is $5mm$ away from $L2$

H' position :

H' **is in the middle of segment** S_1S_2

By inverse return of light, we get :
$\overline{S_1F} = 15mm$ and $\overline{S_1H} = 30mm$