Specific applications of sensors for photonic and imaging

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28.04.2025

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I.Présentation

Module :

Semiconductor sensors and applications

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Résumé :

Once an object is placed in the beam of a laser diode, a part of the beam reflected by the latter is retro-injected in the active cavity of the diode and disrupts the physical properties of it. This disruption which is shown principally by a modification of the frequency of the emission power of the diode is known as the phenomenon of “self-mixing” Long considered as a parasitic phenomenon, notably in telecommunication by optical fibres, we will present its exploitation for metrological ends, such as the measurement of speed, of displacements, and of distances ;

Mots-clés :

Optical retro-injection, Self-mixing, laser diode, Displacement measurement, Speed measurement, Distance measurement

Pré-requis :

Interferences - Physics of semiconductor - transmitters semiconductor

Objectif(s) pédagogique(s) :

Understanding of meteorological applications of semiconductor lasers

Plan du cours :

- Introduction
- Single Laser Diode
- Laser Diode with Weak Optical feedback
- Conclusion

Conception & production :

Le Mans Université

Licence :

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II.Cours

First of all, we will describe the effects of optical feedback (that is, the phenomenon of “self-mixing ») by a simplified study of the laser emission of the diode, first without an object present, then with an object present in the trajectory of the light beam. The simplicity of this study rests in the fact that the energy provided by the spontaneous emission of photons will be neglected and, as a result, optical noise will be generated by the random and subtle nature of this type of emission. This frequently-employed simplification leads to a sufficiently precise description of the phenomenon of “self-mixing”, but does not allow for the introduction of the notion of the light beam's temporal coherence, which is the origin of the limited range of the metrological applications of this phenomenon.

Secondly, we will show how to take advantage of weak optical feedback for measuring speed, displacement, and distances.

1. Single Laser Diode

1.1. Laser Diode Principle

A laser diode is shaped like a plane-paralleled rectangle where the two faces, perpendicularly split at the plane where the emitting semi-conductors meet, form a Fabry-Pérot resonator. This resonator is the origin of the emission stimulated by characteristic light emission photons.

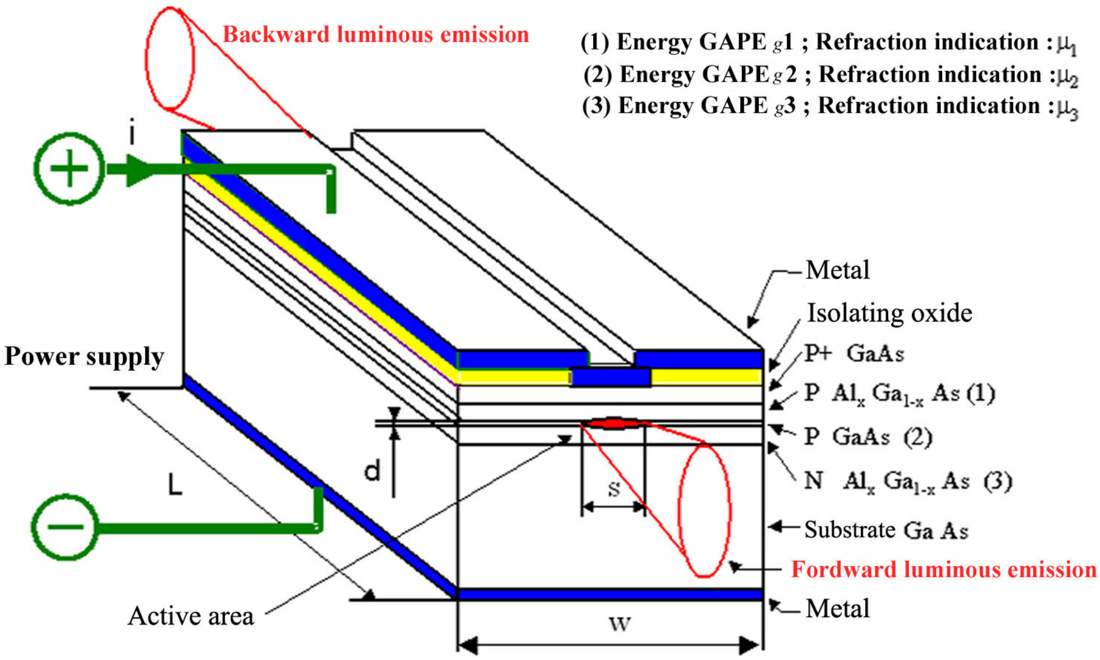

Figure 1 : DH-OS Laser Diode ("Double Heterostructure Oxie-isolated Stripe")

In order to obtain a continua land powerful laser emission at room temperature, the junction of semi-conductors must be as complex a structure as the double heterojunction of the preceding figure. This one presents the advantage of confining the carriers by a barrier of electro-static potential in a very reduced volume $V = Lxdxs$, called the active zone. This confinement makes it possible to obtain a higher density of carriers and, as a result, a higher gain for a weak injection current. In order to set the size scale of the active zone, remember that $L \sim 100$ à $500\ \mu\text{m}$, $d \sim 0,1$ à $0,5\ \mu\text{m}$, $s \sim 2$ à $5\ \mu\text{m}$.

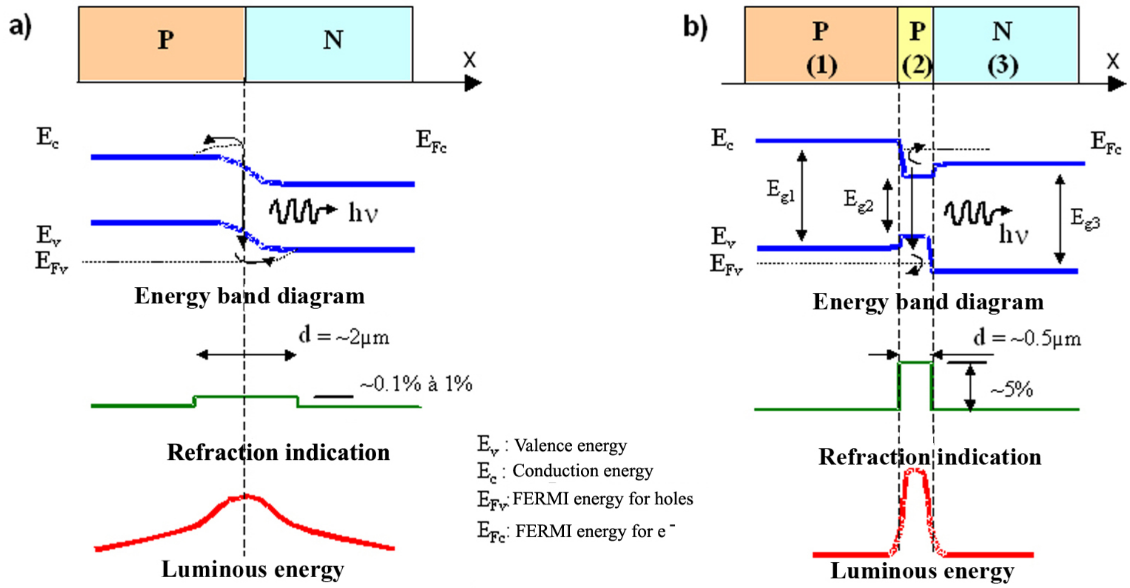

Figure 2 : Emitting Junctions a) Homojunction. b) Double-hétérojunction. In the case of a beterojunction, we obtain a better confinement of the photon carriers

Let's remember that a monomode laser diode is a transmitter of intense light (qqs mW to qqs $100\,\mathrm{mW}$), which is practically monochromatic (only one wavelength), and which is largely temporally coherent (interferences with the optical pathway difference of several meters).

1.2. Threshold Gain and Authorized Longitudinal Modes

A laser diode can be imitated by an active environment of length L defined by interfaces 1 and 2, which form the Fabry-Pérot cavity.

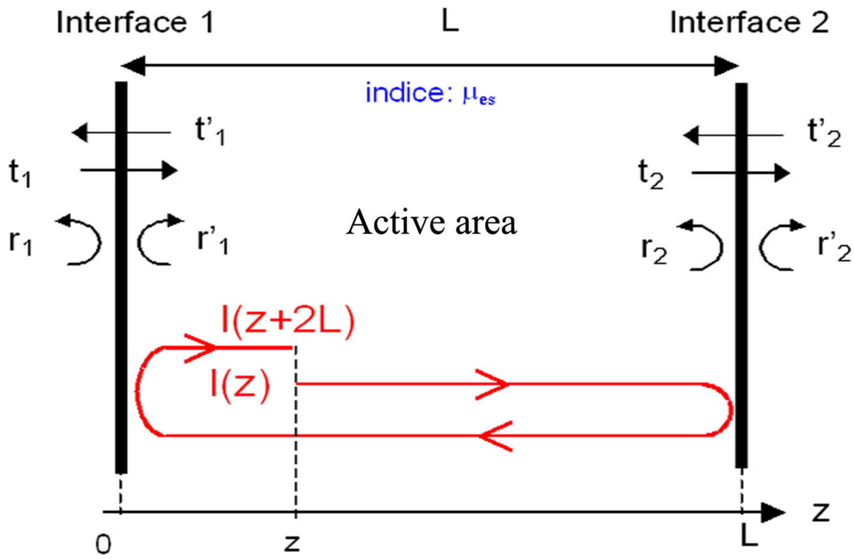

Figure 3 : Model of a Laser Diode

These interfaces are characterized by their reflection coefficients r and $\mathrm{r'}$ and their transmission coefficients t and $\mathrm{t'}$ in amplitude of the electric pathway. We will note μ_{es} as the effective sign of phase refraction without retro-injection.

The optical intensity flux $\mathrm{I(z)}$ follows an exponential law in $-\gamma z$ où $\gamma = -\mathrm{g} + \alpha_p$ is the absorption coefficient of the active environment, g is the gain of this environment and α_p is the loss coefficient owed principally to the absorption by free carriers (i.e. the Auger effect.)

The condition of laser emission is obtained as soon as the electric pathway associated with this flux is stabilized in its amplitude and phase by a return-trip to the active cavity where :

$$r'_1 r_2 \exp[-j\frac{4\pi\mu_{es}\nu_s}{c} L + (g_{ths} - \alpha_p).L] = 1$$

Where g_{ths} is the threshold gain without feedback ν_s is the emission frequency without feedback, and c is the velocity of the light in the void.

The resolution of this equation in module and phase brings us to the threshold gain and to the frequency of authorized emissions.

$$\boldsymbol{g_{ths} = \alpha_p + \frac{1}{L}\ln\frac{1}{r'_1 r_2}} \text{ and } \boldsymbol{\nu_s = u\frac{c}{2.L.\mu_{es}}} \text{ where u is an integer}$$

A distinct authorized frequency corresponds to each value for $u$which is therefore a distinct longitudinal mode of possible laser emission. Careful, this study predicts nothing of the optical energy associated with each of these modes, which are theoretically infinite in number (numerous laser diodes are monomode).

Attention

A distinction should be made between the sign of the phase refraction μ_{es} and the sign of group refraction

$$\bar{\mu}_{es} = \mu_{es} + \nu_s \frac{\partial \mu_e}{\partial \nu}$$

given that the environment sign is dependent upon the wavelength. So, we say that it is dispersive. Here is the difference in emission frequency between two authorized modes :

$$\nu_{s.u+1} - \nu_{s.u} = \frac{c}{2.L.\bar{\mu}_{es}}$$

1.3. Emission Frequency

In the case of a slow modulation and a weak current injection amplitude, noted as i surrounding a medium injection current, noted as i_0, the influence of the termal inertia of the environment is negligible. Thus, the modulation of the refraction sign and of the cavity length induced by thermal variation of the active environment via the injection current can be expressed by the following :

$$(\mu_{es} L - \mu_{es0} L_0) = \varepsilon_i (i - i_0)$$

Where L_0 is the cavity length for the current i_0, μ_{es0} the effective phase sign associates with i_0 and ε_i a (positive) proportionality coefficient transforming the optical extension of the cavity by an increase in injection current via the temperature.

Let's consider a monomode laser diode emitting a frequency ν_{s0} associated with the injection current i_0. The differential of the relationship which gives the authorized emission frequency surround the state corresponding to the current i_0, brings us to the expression of emission frequency without feedback upon the optical length of the cavity. Using the preceding relationship, we express it based on the injection current by :

$$\nu_s = \nu_{s0} + \frac{\partial \nu_s}{\partial i}(i - i_0) \quad \text{with} \quad \frac{\partial \nu_s}{\partial i} = -\frac{\varepsilon_i \nu_{s0}.}{\mu_{es0} L_0}$$

It appears only in respecting the hypothesis of slow modulation and weak injection current amplitude, the emission frequency without feedback is linearly proportional to the injection current. The typical values for $\partial \nu_s / \partial i$ range from -300 MHz/mA to some $-$GHz/mA.

1.4. Emission Power

In the active zone of a laser diode, since the mobility of the electrons (n) is much higher than that of the holes (p), the total current density is nearly equal to that of the electrons following :

$$J = J_n + J_p \simeq J_n \qquad [\mathrm{A.m^{-2}}]$$

On the other hand, the variation in the electrons' density in comparison with the time is equal to the difference between the electrons' contribution

$$\frac{1}{q}\frac{\partial J_n}{\partial x} = \frac{1}{q}\frac{\partial J}{\partial x} \qquad [\mathrm{s^{-1} m^{-3}}]$$

and the rate of the recombining of the electrons $r' = r_{st} + r_{nr.sp}$ with q being the electrons' charge, $r_{st}(\text{resp.} r_{nr,sp})$ being the rate of the recombing of the electrons by the stimulated emission of photons (resp. non-radiating and photon spontaneity).

In hypothesizing that the active zone is homogenous and that all of the electrons recombine there, the electron contribution becomes simply :

$$\frac{1}{q}\frac{\partial J}{\partial x} = \frac{J}{qd}$$

Where d is the width of the active zone.

The evolution of the density of extra electrons in the active zone of a monomode laser diode is thus given by the "continuity equation".

$$\frac{dn_s}{dt} = \frac{J}{qd} - r_{st} - r_{nr.sp} \quad \text{with} \quad \begin{cases} r_{st} = \dfrac{dN_s}{dt} = R_{st}(n_s) \cdot N_s \\ r_{nr.sp} = \dfrac{n_s}{T_n} \end{cases}$$

Where n_s is the density of electrons supplementary to the diode without feedback, N_s is the density of photons of the diode without feedback produced by stimulated emission, τ_n is the life time of an electron, and $R_{st}(n_s)$ is the coefficient of stimulated emission dependent upon n_s and accounting for the optical confinement factor.

The evolution of luminous flux $I(z)$ in the interior of the laser diode, which is proportional to the density of photons, is given by : $dI(z)/dz = (g - \alpha_p).I(z)$ In noticing that

$$dI/dz = \frac{\tau_L}{2.L} dI/dt$$

with τ_L being the time for a return-trip to the interior of the laser cavity, one obtains for the written expression of the stimulated emission coefficient :

$$R_{st}(n_s) = v_g . g(n_s) \text{ with } v_g = \frac{c}{\overline{\mu}_{es}} = \frac{2L}{\tau_L}$$

Where v_g is the group speed in the active environment of the laser diode.

Under the conditions of spontaneous emission, since the density of photons could be considered as negligible, the rise of the injection current allows for the build-up of the density of electrons and thus of the gain $g(n_s)$. When for a threshold current density without feedback J_{ths}, this gain reaches the threshold value of g_{ths}, the laser emission condition is satisfied. The stimulated emission thus becomes predominant to the spontaneous emission and the density of photons is no longer negligible. For a current density J superior to the threshold value, the laser emission condition must always be verified, meaning that the gain and, as a result, the density of electrons no longer differ in their threshold values g_{ths} and n_{ths} demonstrating the relationship $g(n_{ths}) = g_{ths}$. Here's how the continuity equation allows us to express the density of photons contingent upon the current densities J and J_{ths} :

$$N_s = \frac{1}{q.d.v_g.g_{ths}} (J - J_{ths})$$

The emission power without feedback P_s being proportional to the density of photons in the cavity, is thus expressed according to the injection current $i = J.L.s$ and to the injection current without feedback $i_{ths}$by the relationship :

$$\boldsymbol{P_s = \eta.(i - i_{ths})} \text{ with } \eta = \frac{\kappa}{q.d.v_g.g_{ths}}$$

Where κ is a proportionality coefficient between the density of photons and the luminous emission power, $V = L.s.d$ the active volume, and η the external quantum yield called the slope efficiency by laser diode manufactures.

This relationship demonstrates a linear variation of the emission power in the case of a slow modulation of the injection current above the threshold current.

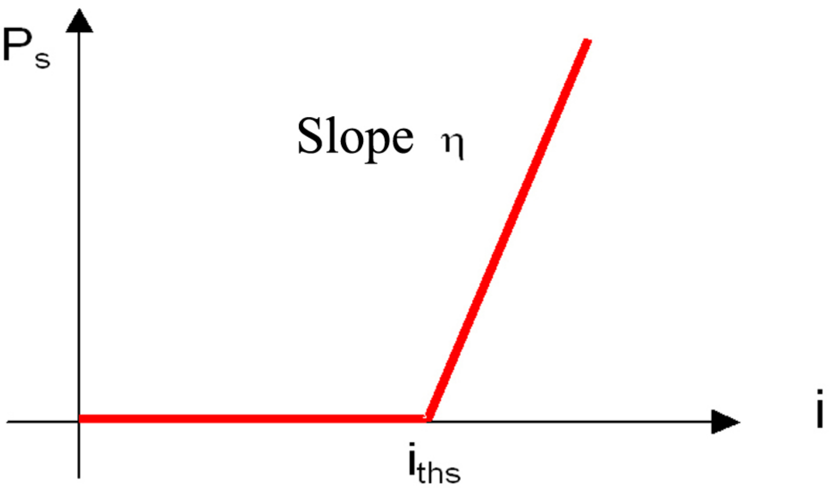

Figure 4 : Emission Power of a Diode without Retro-Injection

2. Laser Diode with Weak Optical feedback

2.1. Diode equivalent

A laser diode in the presence of a target object can be imitated by a single diode equivalent of which the reflection coefficient in amplitude of the electric pathway of the interface on the object's side will be an effective reflection coefficient noted r_{eff}.

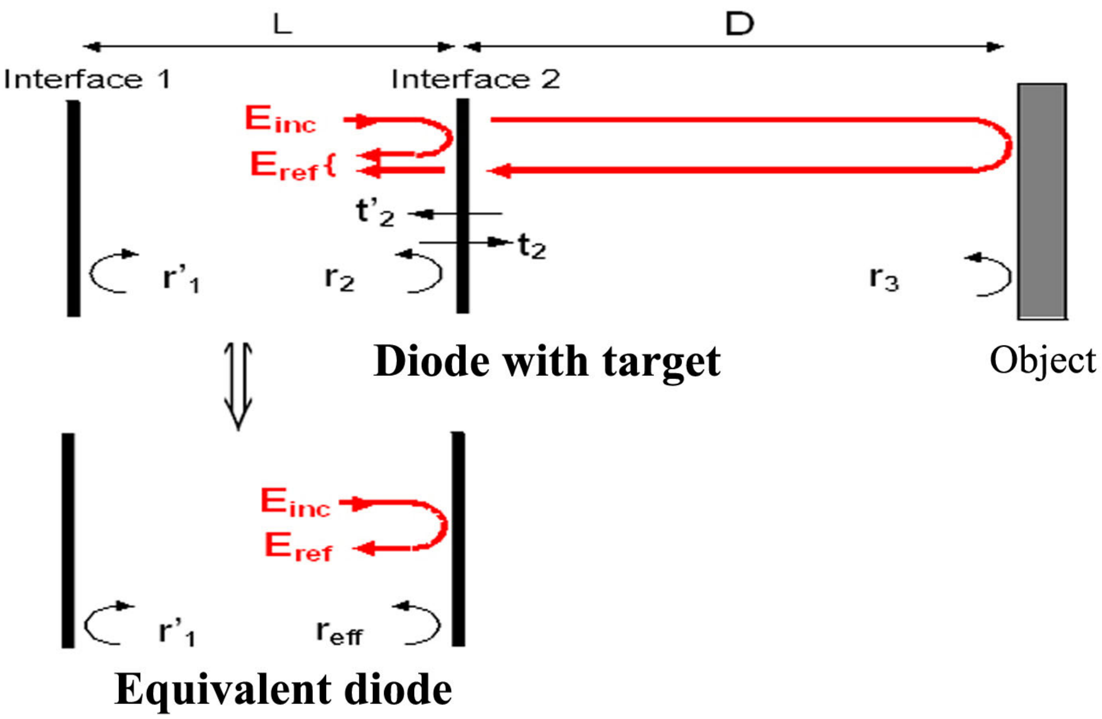

Figure 5 : Model of the equivalent diode

A single laser-object return-trip will be considered, taking into consideration that our study is limited to a weak feedback coupling (a case of non-cooperative objects). This effective coefficient is determined by evaluating in the second interface (in complex notation), the reflected wave E_{ref} according to the incident wave E_{inc} that generated it. The relationship of these pathways gives r_{eff} which has the particularity of depending upon the distance of the object.

$$r_{eff}=r_2(1+\zeta\exp[-j.2\pi\nu_r\tau_D]) \text{ with } \zeta=t_2t_2'\frac{r_3}{r_2} \text{ et } \tau_D=\frac{2D}{c}$$

Where τ_D is the time of a round-trip diode/object flight, ν_r the optical emission frequency with feedback, r_3 the reflection coefficient of the object in amplitude of the electric pathway, and D the distance of the object.

2.2. Threshold Gain

By an analogy with the single laser diode (without target object), the laser emission condition for the equivalent diode is :

$$r'_1 r_{eff} \exp[-j\frac{4\pi\mu_{er}\nu_r}{c}\mathrm{L}+(g_{thr}-\alpha_p).\mathrm{L}]=1$$

Where g_{thr} is the gain of the laser diode with feedback.

Taking into account a weak laser-object coupling (weak r_3), the equalization in module of the laser emission condition with and without object allows one to determine the threshold gain with feedback :

$$g_{thr}=g_{ths}-\frac{\zeta}{L}\cos(2\pi\nu_r\tau_D)$$

The figure below shows the variation of the threshold gain according to the emission frequency.

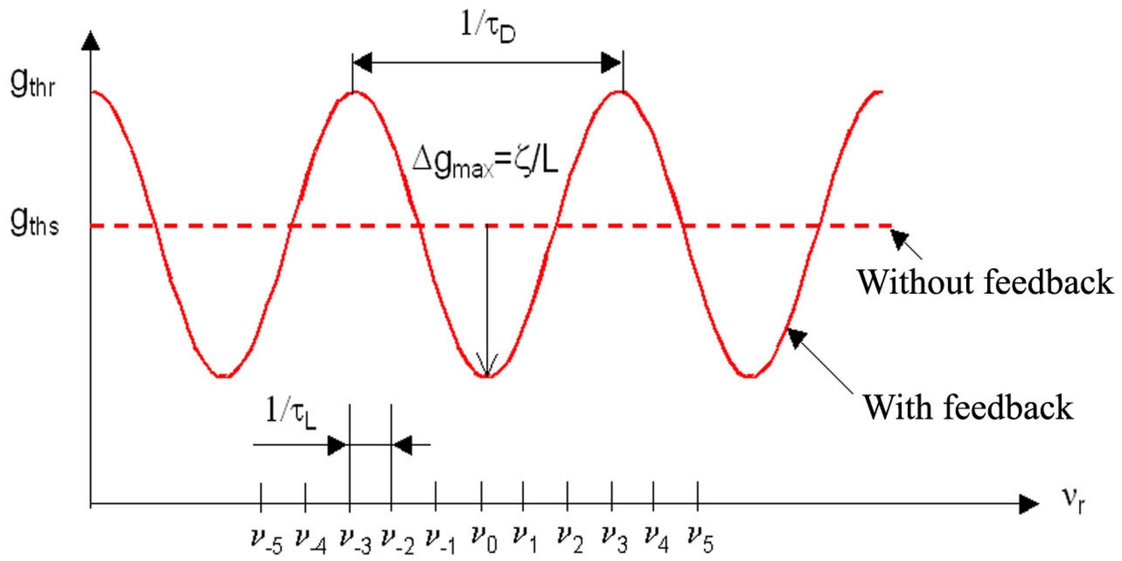

Figure 6 Threshold gain with weak optical retro-injection for weak object distance

Regarding this figure, it seems that the optical feedback can be used to stabilize the emission mode of a monomode laser diode and to eliminate the parasitic lateral modes. For this particular application, it is necessary to have an object close enough to the diode that $\tau_D < \tau_L$. Thus, the threshold gain of the principle mode (ν_0) is reduced by Δg_{max} so that for the lateral modes (ν_{-1}, ν_1), the threshold gains aren't higher. This effect can be increased by raising the reflection coefficient of the object by Δg_{max} or by raising the curve of the threshold gain at the level of the principle mode by choosing a higher τ_D However, if it turns out that τ_D is too high, the values of the threshold gains, being as weak as that of the principle mode, could excite the lateral modes of the latter. The bibliography shows on this matter that $\tau_D \approx 5\tau_L$ is a good compromise.

2.3. Emission Frequency

The emission frequency with feedback is deduced by conditions of laser emission of the diode with and without an object by the equalization of the phases. An initial evaluation brings us to :

$$\frac{4\pi L}{c}\Delta(\mu_e \nu) = \varphi_{reff} \quad \text{with} \quad \varphi_{reff} = -\zeta \sin(2\pi \nu_r \tau_D)$$

Where $\Delta(\mu_e \nu)$ represents the difference in the state with and without an object, and φ_{reff} represents the phase of the coefficient r_{eff} for a weak laser-object coupling.

The sign of effective refraction being dependent on the density of the major carrier n and on the emission frequency ν .We notate it as:

$$\Delta \mu_e = \left(\frac{\partial \mu_e}{\partial n}\right)\Delta n + \left(\frac{\partial \mu_e}{\partial \nu}\right)\Delta \nu$$

Without forgetting that the sign of group refraction is expressed thus :

$$\bar{\mu}_{es} = \mu_{es} + \nu_s \frac{\partial \mu_e}{\partial \nu}$$

The equalization of the phases becomes :

$$\frac{4\pi L}{c}\left[\nu_s\left(\frac{\partial \mu_e}{\partial n}\right)(n_r - n_s) + \bar{\mu}_{es}(\nu_r - \nu_s)\right] = \varphi_{reff}$$

Where n_r is the density of electrons with feedback.

We have written in the preceding paragraphs that the power flux $I(z)$ spreads itself over the active environment following an exponential law with $(g - \alpha_p)z$. By introducing a sign of complex phase effective refraction μ_{eC} into the expression of the wave associated with this power flux, we thus make explicit another form of the same law, showing a variation in

$$(-\frac{4\pi \nu_s}{c}\mu_e'').z$$

Where μ_e'' is the opposite of the imaginary part of μ_{eC}. Thus for a constant injection current density higher than that of the laser emission threshold, with the respective densities of electrons being that of the threshold, we can therefore express the egality with the following :

$$\left(\frac{\partial \mu_e''}{\partial n}\right) = -\left(\frac{\partial g}{\partial n}\right)\frac{c}{4\pi \nu_s} = -\left(\frac{g_{thr} - g_{ths}}{n_r - n_s}\right)\frac{c}{4\pi \nu_s}$$

By bringing into play the spectral ray widening factor (also known as the “linewidth enhancement factor”) :

$$\alpha=\frac{\partial \mu_e}{\partial \mu_e^{''}}=-\frac{\partial \mathrm{Re}(\mu_{eC})}{\partial \mathrm{Im}(\mu_{eC})}$$

the equalization of the phases brings us to an implicit expression of the mission frequency with feedback :

$$\nu_r-\nu_s+\frac{c}{4\pi L\bar{\mu}_{es}}\zeta\sqrt{1+\alpha^2}\sin(2\pi\nu_r\tau_D+\arctan\alpha)=0$$

By introducing the feedback coefficient

$$C=\frac{\tau_D}{\tau_L}\zeta\sqrt{1+\alpha^2}$$

we obtain a more condensed expression of this frequency :

$$\boldsymbol{\nu_r-\nu_s+\frac{C}{2\pi\tau_D}\sin(2\pi\tau_D\nu_r+\arctan\alpha)=0}$$

Let us notice that the phase $2\pi\tau_D\nu_r$ being equal to $4\pi D/\lambda_r$, a displacement as weak as half a wavelength $\lambda_r/2$ (qqs $1/10\,\mu\mathrm{m}$), produced a period of oscillation of the emission frequency with feedback.

For a coefficient of coupling C inferior to 1 (weak feedback), it would be easy to show, with the help of the preceding formula, that the curve ν_s contingent on ν_r is strictly increasing and, as a result, the diode in presence of the object remains monomode. As the following figure shows, for values of C which are slightly higher and for certain values of τ_D, it is possible that the diode will emit on several longitudinal modes and, as a result, it becomes multimode. The value of C for which the diode allows three authorized longitudinal modes, no matter what the value of τ_D may be, is 4.6.

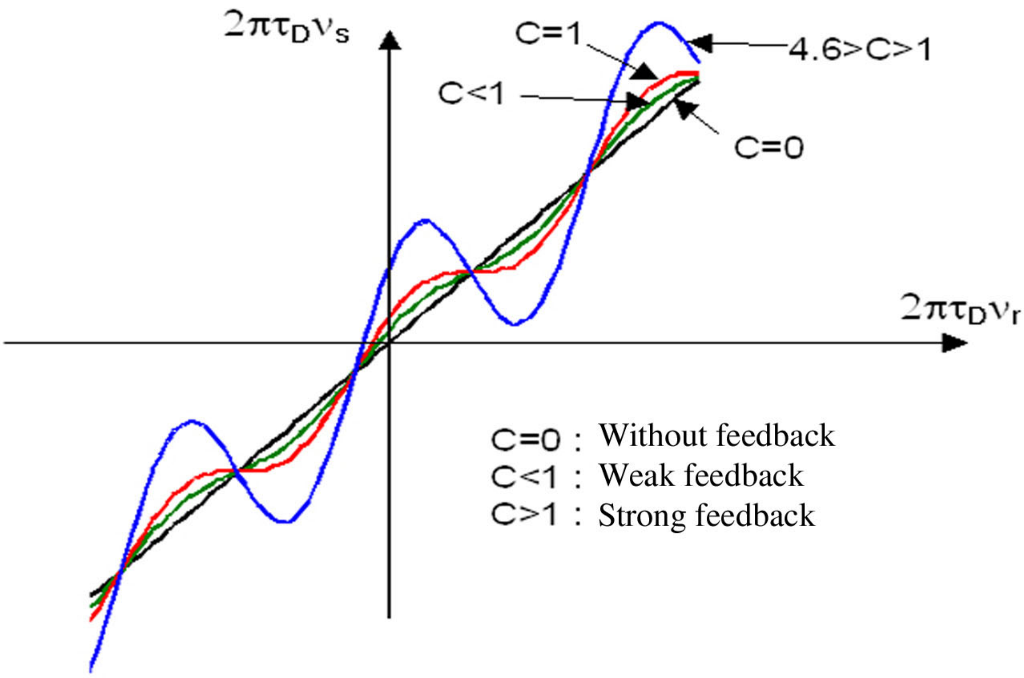

Figure 7 : Longitidunal modes authorized with optical retro-injection

The demonstrated influence of the optical feedback on the spectral behavior of the laser diodes brought us to a phenomenological classification in different systems. The following chart gives this classification for a DFB 1.5 $1.5\ \mu m$ InGaAsP laser diode.

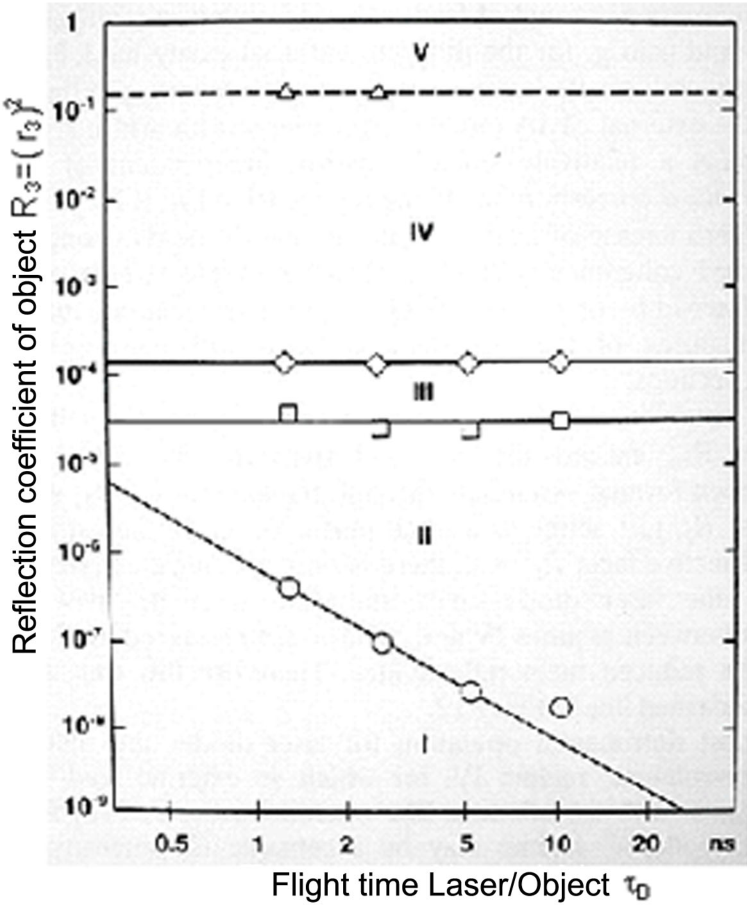

Figure 8 : Classification of optical retro-injection rates (RW Tkach et AR Chraplyvy 1986)

Rate 1 is characterized by a weak feedback coefficient $C < 1$ for which the diode remains monomode no matter what the value is of the flight time τ_D. In rate II ($C > 1$), the diode is multimode in principle. As a result of a small displacement of the object (a fraction of the wavelength), the diode "jumps" from one mode to another. Surprisingly, the most stable emission mode is the one associated with the weakest spectral width and not with the weakest threshold gain. In rate III, the variation Δg_{max} of the threshold gain, being still more pronounced, and the spectral width even less, this rate corresponds with a particularly stable monomode emission. Rate IV is called the "coherence collapse rate" because of the large spectral instability of the diode owing to the multiple stimulated longitudinal modes. The spectral width of emission can be several dozen giga hertz, implying a coherence length of less than a centimetre. For an even higher optical coupling, rate V is attained. The external cavity materialized by the interface of the laser diode, opposite the object and the surface of the object, becomes the principal cavity. The new laser diode thus formed can therefore stably emit in monomode rate.

2.4. Emission Power

As part of a weak optical feedback and for an identical supply current, the physical values of the diode with and without object are close. We can therefore differentiate the continuity equation established in the preceding chapters for a laser diode without feedback in order to determine the state of the diode with feedback. In established laser rate resignation, in which the density of electrons is constant and the spontaneous emission of photons is negligible, we can write :

$$\Delta(R_{st}.N)=0 \quad \text{with} \quad R_{st}(n)=v_g.g(n)$$

Where, we remember, R_{st} is the coefficient of the spontaneous emission of photons.

Taking into account the deviation in the obtained gain in the preceding paragraph, the density of photons with optical feedback is :

$$N_r=N_s\left(1+\frac{\zeta}{g_{ths}.L}\cos(2\pi v_r\tau_D)\right)$$

The emission power of the laser diode with optical retro-injection being proportional to the corresponding density of photons, we obtain :

$$\boldsymbol{P_r=P_s\left(1+m.\cos(2\pi v_r\tau_D)\right)} \quad \text{with} \quad m=\frac{\zeta}{g_{ths}.L}$$

Where m is the parameter of power modulation by weak feedback, dependent upon the reflection coefficient of the object.

The dependence of the emission power in the presence of the object with the injection current is introduced by the emission power without the object P_s. By replacing P_s by its expression, we state more clearly this dependence :

$$P_r=\eta.\left(1+m.\cos(2\pi v_r\tau_D)\right).(i-i_{ths})$$

This relationship is only true in the case of a slow modulation frequency of the injection current very far from the relaxation frequencies and from damping of the diode.

The curve below shows the variation of emission power contingent upon a weak displacement of the object of a wavelength. The object is situated $2\,\text{cm}$ away from the diode, and several reflection coefficients of the object have been stimulated to create different parameter couples C and m. We notice the allure in the "sawtooth like" of the emission power characteristic of the self-mixing of the phenomenon. This "sawtooth like" form is marked as much more as the coefficient of retro-injection is raised.

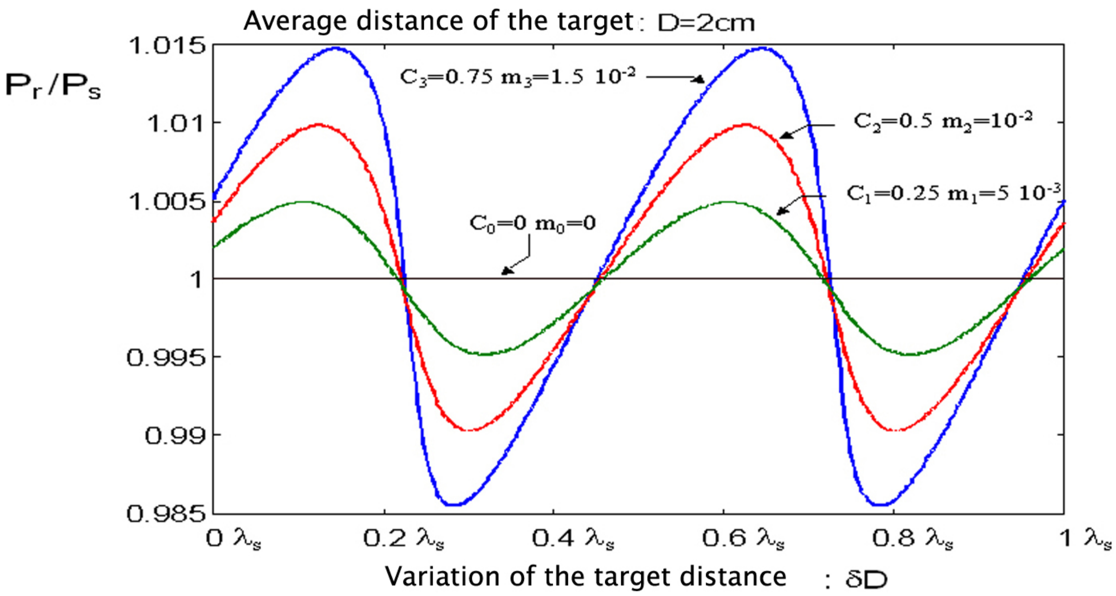

Figure 9 : Classification of the optical retro-injection rates

For example, the figures below show a very realistic simulation of an experimental result obtained for an immobile micro-ball target situated at $D = 2.4\,\text{cm}$ from the diode and for a triangular modulation of the injection current.

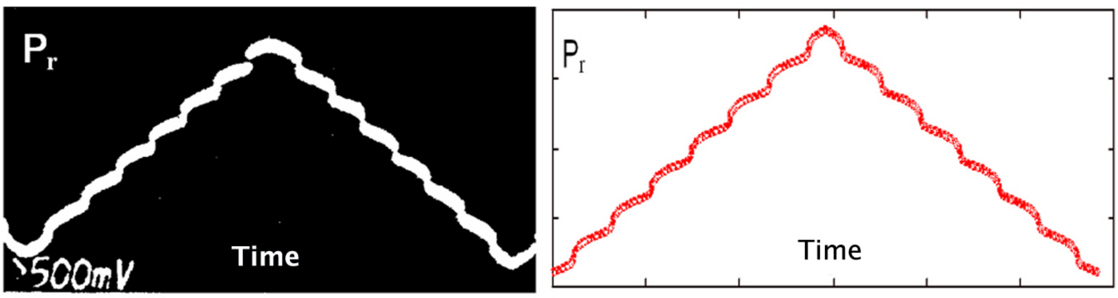

Figure 10 : Experimental emission power simulated with a triangular current modulation

$$\text{D}=2{,}4\,\text{cm}\,;\ \text{C}=0{,}88\,;\ \alpha=3\,;\ \partial\nu_s/\partial i=-6{,}66\,\text{GHz/mA}\,;\ \text{r}_3=0{,}013\,;\ \mu_{es}=3.6\,;\ \bar{\mu}_{es}=4.5$$

* *
*

As it was made clear in the introduction of this course, this study does not allow one to understand the notion of the length of coherence of the diode, and thus, the visibility limit of the disturbances in the emission power contingent upon the characteristics of the feedback. Thus in our study, the modulation parameter m, which is also the visibility of the phenomenon of "self-mixing", appears to be independent of the distance of the object. It should be noted however, that the observation of signals of "self-mixing" is principally limited by the laser-target optical coupling and by the dependence of C with the distance inferring an easily multimodal behavior. For an easily exploitable signal, it is preferable not to go past a distance of a few dozen centimeters.

III.Etude de cas

1. Applications

1.1. Application of Speed Measurement

The experimental device for speed measurement by optical feedback is the one shown in the schema below :

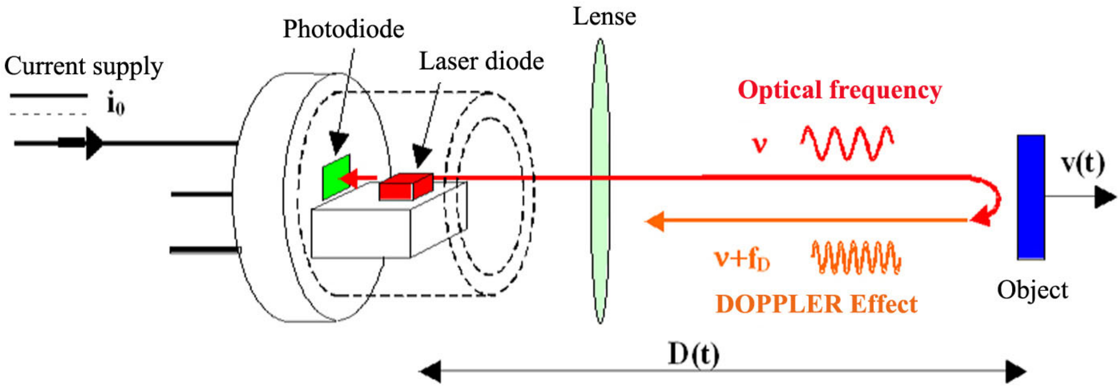

Figure 11 : Experimental device for speed measurement

This device includes a laser-diode fed by a constant current, a mobile object animated by a speed$v$v, and a collimation lens allowing the increase in the optical laser-object coupling. The builders have included in the laser-diode cases a photodiode intended for the control of the laser emission power. In the framework of our application, it would be used to translate the power disturbances generated by optical feedback into electrical signals. Knowing that a laser-diode emits from both sides of its active environment, this photodiode is advantageously situated at the back of the laser diode.

The speed measurement will be inferred from the disturbances of the luminous emission power for which the expression has been established in the preceding chapter. The latter demonstrates a pseudo-periodicity defined by :

$$\delta(\nu_r\tau_D)=\pm 1$$

Where $\delta(\nu_r\tau_D)$ represents the deviation in condition between the beginning and the end of a pseudo-period.

The injection current being constant, the optical frequency is identical to the beginning and to the end of a pseudo-period. By noting f_b the algebraic value of the beat frequency linked to the appearance of the disturbances, we obtain :

$$f_b=\frac{2.\nu_s}{c}\mathrm{v}=\frac{2}{\lambda_s}\mathrm{v}$$

Where λ_s is the wavelength of a single diode.

Let us notice that the beat frequency f_b is equal to the frequency induced by Doppler Effect f_D. This frequency will be positive (resp. negative) once the object moves away (resp. comes closer) from the laser.

One of the characteristics of the velocemitry by optical feedback in the laser diodes is the power to determine simply the direction of movement of an object. In effect, the orientation of the "sawtooth like" form of the power disturbances is a sign of the object's speed as is shown in the following simulations.

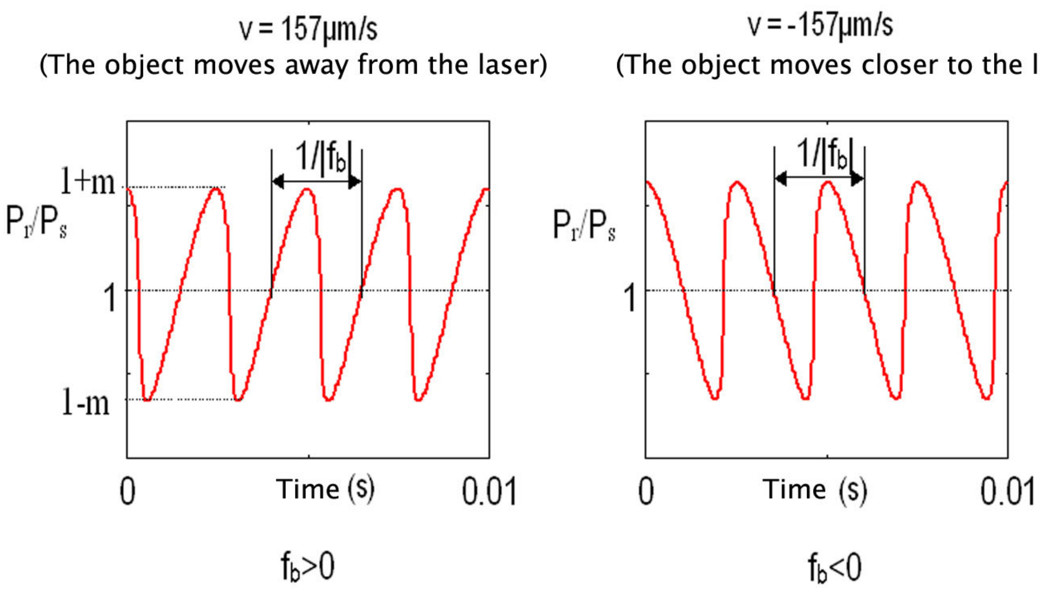

Figure 12 : Emission Power in Velocemitry (C = 0,88)

It should be known that the beat frequency, like the Doppler frequency, depends uniquely upon the training speed of the object marker compared with the laser diode marker. Thus for a diffusing object on which the laser beam slides, this frequency will be independent of the object's shape.

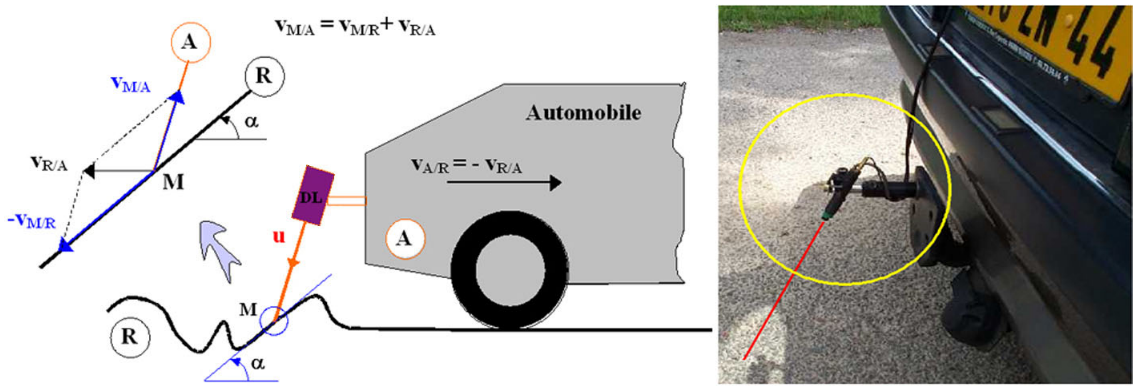

Figure 13 : Application of the speed measurement to an automobile

For example, let us take an automobile (marker A) for which we would like to evaluate the speed $v_{A/R}$ of displacement compared with the road (marker R). The Doppler frequency is induced by the absolute movement of the laser point M in comparison with the diode ($v_{M/A}$) so that by the displacement of this point on the object's surface, it is compared with the object itself ($v_{M/R}$). So, the kinetics shows that the combination of these two speeds gives marker R's training speed in comparison with A. We prove in this way that the Doppler frequency, and therefore the beat frequency, is independent of the shape of the road (presence of potholes, gravel, etc.). This constitutes a particularly interesting result concerning the "robustness" of the measurements by laser Doppler velocemitry.

$$f_D = -f_b = \frac{2}{\lambda_s}\vec{v}_{A/R}.\vec{u}$$

Where $\vec{u}$ is the standard directing vector of the laser axis.

1.2. Application of the Displacement Measurement

The application of the displacement measurement takes over the experimental device and the conditions of using the laser diode for the application of the speed measurement. The pseudo-period of the laser emission power defined by $\delta(\nu_r \tau_D) = \pm 1$ is associated with a quantified displacement worth half a wavelength (qqs $1/10\,\mu\text{m}$).

$$\delta \mathbf{D} = \frac{\lambda_s}{2}$$

In the same way as in the framework of the speed measurement, the displacement direction is obtained by observation of the zig-zag shape of the laser emission power.

It is thus possible to reconstruct the displacement of an object at the resolution of the half wavelength in raising or lowering a counter of the same value at each appearance of a pseudo-period. The following figure presents this reconstruction technique.

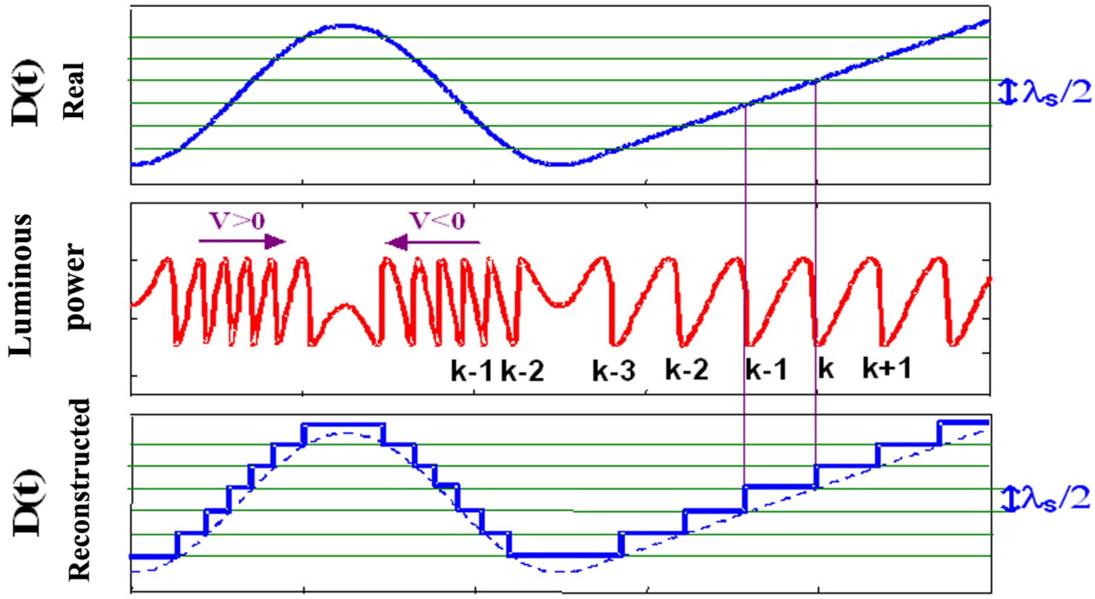

Figure 14 : Reconstruction Method of a Displacement

The figure here below shows an experimental real time measurement by this method applied to the measurement of track vibration for a railway :

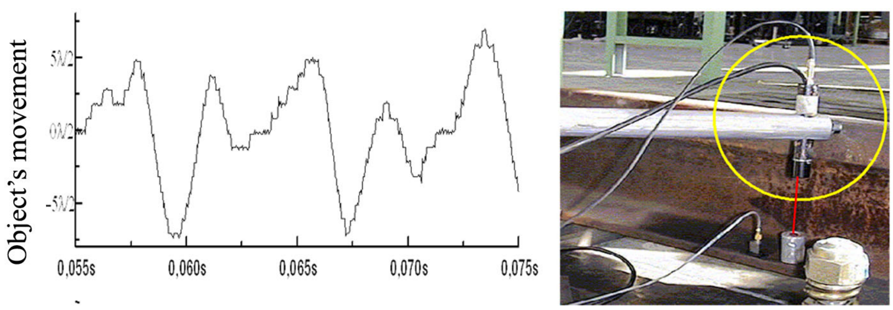

Figure 15 : Application of the vibration measurement of a railway

1.3. Application of the Distance Measurement

The experimental device adapted to the distance measurement differs from the one for speed of displacement measurement by the modulation of the injection current into a symmetrical triangle. As we studied in the preceding chapter, this modulation will have the effect of also imposing a modulation of the emission frequency without retro-injection into the form of a symmetrical triangle. This modulation will be characterized by the slope :

$$\frac{\delta \nu_s}{\delta t}(>0)$$

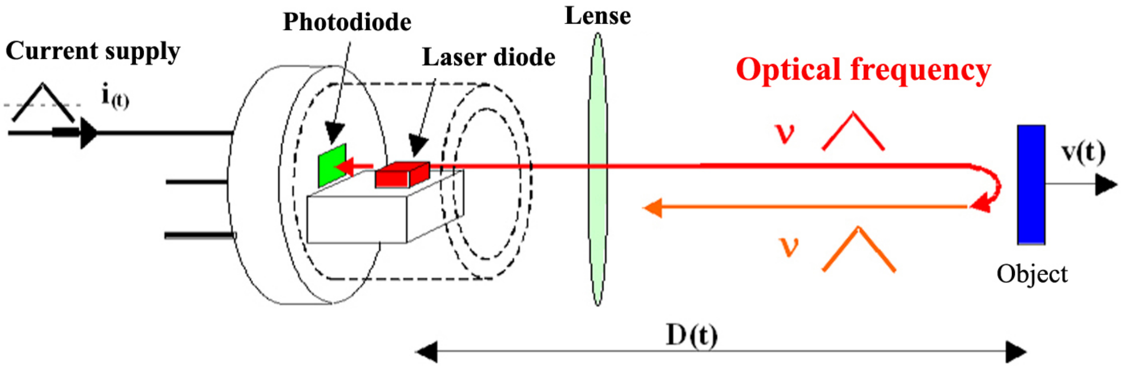

Figure 16 : Experiment Device for Distance Measurement

In these conditions of current supply, the power emitted by the diode with feedback makes a symmetrical triangle shape appear on which a weak disturbance under the form of pseudo-periods applies. The frequency of the appearance of these pseudo-periods will be noted f_{b0} on the positive slope, and f_{b1} on the negative slope, as the following figure displays.

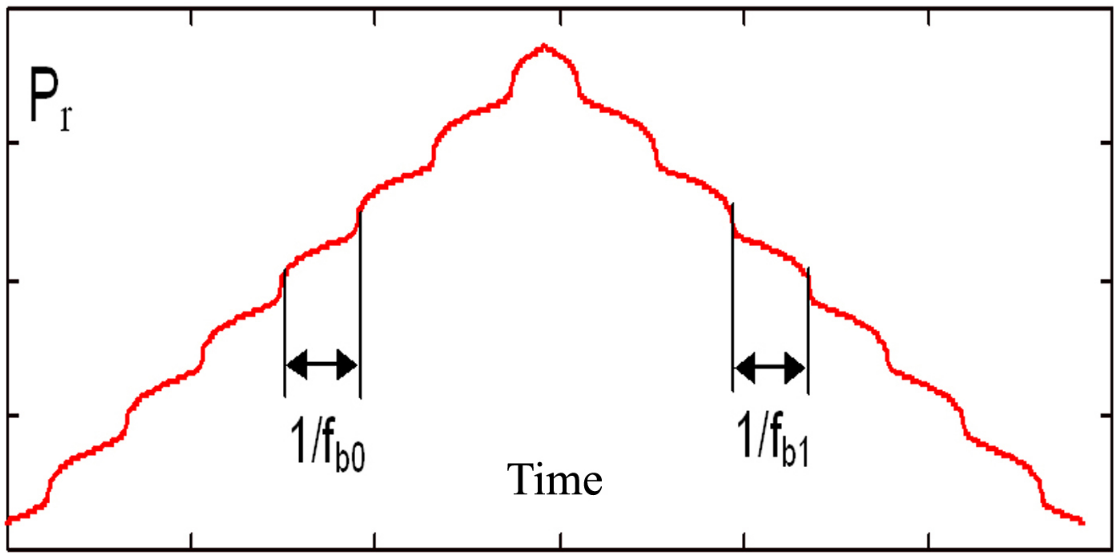

Figure 17 : Emission power disturbance for distance measurement

Knowing that a pseudo-period is defined by $\delta(\nu_r \tau_D) = \pm 1$ and taking into account a weak object speed, we obtain for the positive and negative slope this equation system :

$$\begin{cases} \frac{D}{c}\frac{\delta \nu_s}{\delta t} - \frac{2}{\lambda_s} v = \frac{f_{b0}}{2} \\ \frac{D}{c}\frac{\delta \nu_s}{\delta t} + \frac{2}{\lambda_s} v = \frac{f_{b1}}{2} \end{cases}$$

The evaluation of the object's distance so that its speed can thus be determined from the beat frequency is given by :

$$D = \frac{c}{4\frac{\delta \nu_s}{\delta t}} (f_{b0} + f_{b1}) \quad \text{and} \quad v = \frac{\lambda_s}{4}(f_{b1} - f_{b0})$$

The simulation below shows the influence of the distance and the speed on the luminous power signal. The principle parameters of this simulation are

$$\frac{\delta \nu_s}{\delta t} = 8000 \text{ GHz/s}$$

$\lambda_s = 780$ nm and a retro-injection coefficient $C = 0.88$. In agreement with the previously established relationships, we notice that the beat frequencies are equal when the speed is worthless. In addition, they are proportional to the object's distance. We notice also that a displacement induces a difference between the frequencies that change signs once the object's direction of movement is inversed.

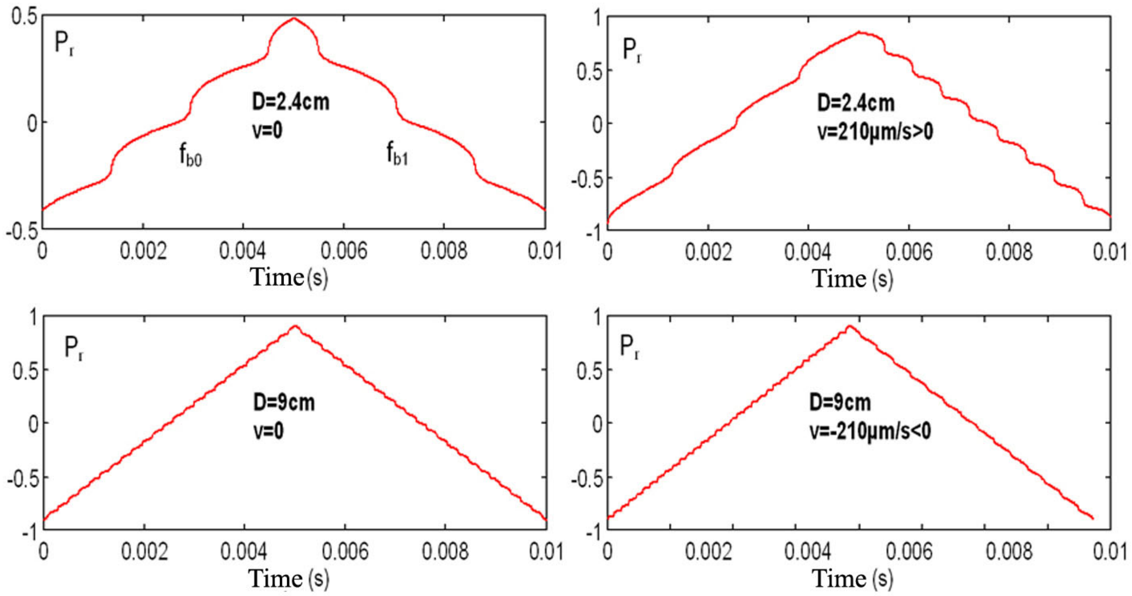

Figure 18 : Influence of the object's distance and speed on the beat frequencies of the signal of luminous power

The image below in distorted colors gives an example of the application of the phenomenon of “self-mixing” in the shape of a helix. The object is placed on a map sitated 148 cm from the laser diode. A scanning system with two motorized mirrors allows one to carry out this measurement point by point (1000, 000 points).

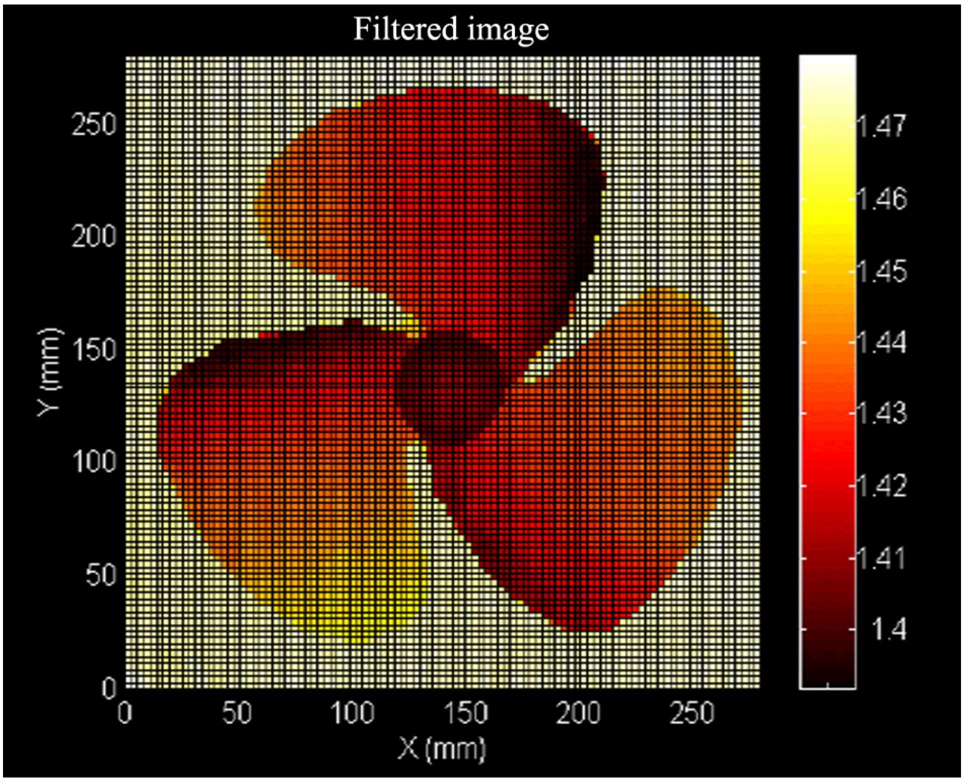

Figure 19 : Experimental 3D measurement of a helix

IV. Exercices

1. Exercice n°1

Answer the questions

Question 1

[Solution n°1 p 25]

With the help of the indications given in the course, find the implicit expression of the emission frequency with optical feedback :

$$\nu_r - \nu_s + \frac{c}{4\pi L \bar{\mu}_{es}} \zeta \sqrt{1+\alpha^2} \sin(2\pi \nu_r \tau_D + \arctan` \alpha) = 0$$

Question 2

[Solution n°2 p 26]

Using the differential $\delta(\nu_r \tau_D) = \pm 1$ characterising a pseudo-period in the luminous emission power, prove that :

a) For the speed measurement :

$$f_b = \frac{2.\nu_s}{c} v = \frac{2}{\lambda_s} v$$

b) For the displacement measurement :

$$\delta D = \frac{\delta_s}{2}$$

c) For the distance measurement :

$$D = \frac{c}{4 \frac{\delta \nu_s}{\delta t}} (f_{b0} + f_{b1}) \text{ and } v = \frac{\lambda_s}{4} (f_{b1} - f_{b0})$$

2. Exercice n°2

Answer the questions

Question 1

[Solution n°3 p 27]

Find the speed of an automobile for a beat frequency $f_b = 13,154 \text{ MHz}$ and a laser beam inclination angle $\theta = 20$ (the wavelength is :$\lambda_s = 1300 \text{ nm}$).

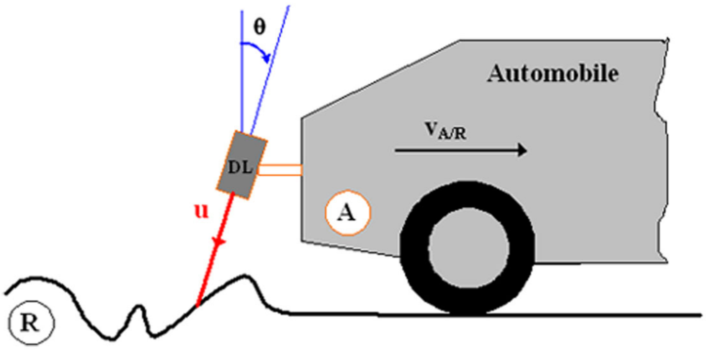

Figure 20

What would be the beat frequency for $\theta = 0$?

Question 2

[*Solution n°4 p 28*]

In displacement measurement, how many laser emission pseudo-periods must we count for a displacement of $\Delta D = 1\ \text{mm}$? What is the percentage (%) for this measurement? (The wavelength is : $\lambda_s = 780\ \text{nm}$).

Question 3

[*Solution n°5 p 28*]

Measure the speed distance of an object with the help of a « self-mixing" signal, given by the following figure (we give you :$\delta \nu_s = 5\ \text{GHz}$ et $\lambda_s = 780\ \text{nm}$) :

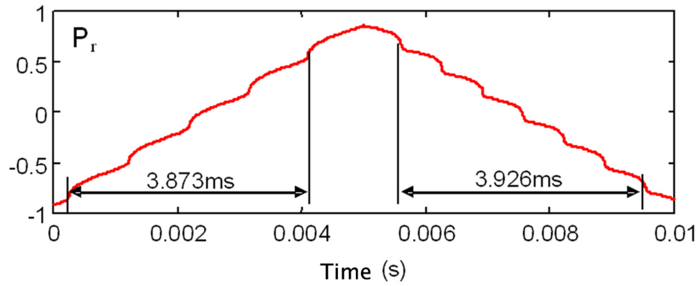

Figure 21

Solution des exercices

>Solution n°1 *(exercice p. 23)*

The equalization of the phases of the laser emission conditions with and without an object gives:

$$\frac{4\pi L}{c}\Delta(\mu_e \nu)=\varphi_{reff} \quad \Leftrightarrow \quad \frac{4\pi L}{c}(\nu_s \Delta\mu_e+\mu_{es}\Delta\nu)=\varphi_{reff}$$

With :

$$\Delta\mu_e=\left(\frac{\partial\mu_e}{\partial n}\right)\Delta n+\left(\frac{\partial\mu_e}{\partial\nu}\right)\Delta\nu$$

$$\frac{4\pi L}{c}\left(\nu_s\frac{\partial\mu_e}{\partial n}\Delta n+\left(\mu_{es}+\left(\frac{\partial\mu_e}{\partial\nu}\right)\nu_s\right)\Delta\nu\right)=\varphi_{reff}$$

So :

$$\bar{\mu}_{es}=\mu_{es}+\nu_s\frac{\partial\mu_e}{\partial\nu}$$

$$\frac{4\pi L}{c}\left(\nu_s\frac{\partial\mu_e}{\partial n}\Delta n+\bar{\mu}_{es}\Delta\nu\right)=\varphi_{reff} \quad \Leftrightarrow \quad \frac{4\pi L}{c}\left(\nu_s\frac{\partial\mu_e}{\partial\mu_e''}\frac{\partial\mu_e''}{\partial n}\Delta n+\bar{\mu}_{es}\Delta\nu\right)=\varphi_{reff}$$

We know that :

$$\left(\frac{\partial\mu_e''}{\partial n}\right)=-\frac{\Delta g_{th}}{\Delta n}\frac{c}{4\pi\nu_s} \text{ and } \alpha=\frac{\partial\mu_e}{\partial\mu_e''}$$

$$\frac{4\pi L}{c}\left(-\alpha\frac{c}{4\pi}\Delta g_{th}+\bar{\mu}_{es}\Delta\nu\right)=\varphi_{reff}$$

On the other hand :

$$\Delta g_{th}=g_{thr}-g_{ths}=-\frac{\zeta}{L}\cos(2\pi\nu_r\tau_D) \text{ and } \varphi_{reff}=-\zeta\sin(2\pi\nu_r\tau_D)$$

$$\frac{4\pi L}{c}\left(\alpha\frac{c}{4\pi}\frac{\zeta}{L}\cos(2\pi\nu_r\tau_D)+\bar{\mu}_{es}\Delta\nu\right)+\zeta\sin(2\pi\nu_r\tau_D)=0$$

$$\Leftrightarrow \; \alpha\zeta\cos(2\pi\nu_r\tau_D)+\frac{4\pi L}{c}\bar{\mu}_{es}\Delta\nu+\zeta\sin(2\pi\nu_r\tau_D)=0$$

$$\Leftrightarrow \; \frac{4\pi L}{c}\bar{\mu}_{es}\Delta\nu+\zeta(\alpha\cos(2\pi\nu_r\tau_D)+\sin(2\pi\nu_r\tau_D))=0$$

$$\Leftrightarrow \; \frac{4\pi L}{c}\bar{\mu}_{es}\Delta\nu+\zeta\sqrt{1+\alpha^2}\left(\frac{\alpha}{\sqrt{1+\alpha^2}}\cos(2\pi\nu_r\tau_D)+\frac{1}{\sqrt{1+\alpha^2}}\sin(2\pi\nu_r\tau_D)\right)=0$$

We recognize the trigonometric formula : $\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b)$ with $a=\arctan\alpha$ and $b=2\pi\nu_r\tau_D$.

$$\frac{4\pi L}{c}\bar{\mu}_{es}\Delta\nu+\zeta\sqrt{1+\alpha^2}\sin(2\pi\nu_r\tau_D+\arctan\alpha)=0$$

$$\boldsymbol{\nu_r-\nu_s+\frac{c}{4\pi L\bar{\mu}_{es}}\zeta\sqrt{1+\alpha^2}\sin(2\pi\nu_r\tau_D+\arctan\alpha)=0}$$

>Solution n°2 *(exercice p. 23)*

a) The laser emission frequency without retro-injection is constant. So once the object's distance is raised $\delta(\nu_r\tau_D)>0$ therefore $\delta(\nu_r\tau_D)=1$.

$$\delta(\nu_r\tau_D)=1 \;\Leftrightarrow\; \nu_s\delta(\tau_D)=1$$

$$\Leftrightarrow\; \nu_s\delta\left(\frac{2D}{c}\right)=1$$

$$\Leftrightarrow\; \frac{\delta D}{\delta t}=\frac{c}{2\nu_s}\frac{1}{\delta t}$$

$$\Leftrightarrow\; \mathrm{v}=\frac{c}{2\nu_s}f_b \text{ and so } \boldsymbol{f_b=\frac{2}{\lambda_s}\mathrm{v}}$$

b) The laser emission frequency without retro-injection is constant. So once the object's distance is raised $\delta(\nu_r\tau_D)>0$ therefore $\delta(\nu_r\tau_D)=1$.

$$\delta(\nu_r\tau_D)=1 \;\Leftrightarrow\; \nu_s\delta\left(\frac{2D}{c}\right)=1$$

$$\Leftrightarrow\; \frac{2\nu_s}{c}\delta D=1$$

$$\Leftrightarrow\; \boldsymbol{\delta D=\frac{\lambda_s}{2}}$$

On the self-mixing signal portion associated with the beat frequency f_{b0}, the optical frequency descends : $\delta(\nu_r\tau_D)=-1$(weak speed)

$$\delta(v_r \tau_D) = -1 \Leftrightarrow \tau_D \delta v_r + v_r \delta \tau_D = -1$$
$$\Leftrightarrow v_s \frac{\delta D}{\delta t_0} + D \frac{\delta v_s}{\delta t_0} = -\frac{c}{2} \frac{1}{\delta t_0}$$
$$\Leftrightarrow v_s \mathrm{v} - D \frac{\delta v_s}{\delta t} = -\frac{c}{2} f_{b0}$$

On the self-mixing signal portion associated with the beat frequency f_{b1},the optical frequency ascends :$\delta(v_r \tau_D) = 1$(weak speed). By a similar rational, we obtain :

$$v_s \mathrm{v} + D \frac{\delta v_s}{\delta t} = \frac{c}{2} f_{b1}$$

We must solve the set of equations following :

$$\begin{cases} v_s \mathrm{v} - D \dfrac{\delta v_s}{\delta t} = -\dfrac{c}{2} f_{b0} \\ v_s \mathrm{v} + D \dfrac{\delta v_s}{\delta t} = \dfrac{c}{2} f_{b1} \end{cases} \Leftrightarrow \begin{cases} 2D \dfrac{\delta v_s}{\delta t} = \dfrac{c}{2} f_{b0} + \dfrac{c}{2} f_{b1} \\ 2 v_s \mathrm{v} = \dfrac{c}{2} f_{b1} - \dfrac{c}{2} f_{b0} \end{cases}$$
$$\Leftrightarrow \begin{cases} \boldsymbol{D = \dfrac{c}{4 \dfrac{\delta v_s}{\delta t}} (f_{b0} + f_{b1})} \\ \boldsymbol{\mathrm{v} = \dfrac{\lambda_s}{4} (f_{b1} - f_{b0})} \end{cases}$$

>Solution n°3 *(exercice p. 23)*

The algebraic beat frequency is worth : $f_b = \frac{2}{\lambda_s} \mathrm{v}$ with v being the speed protection $V_{R/A} = -V_{A/R}$ on the axis of the laser beam.

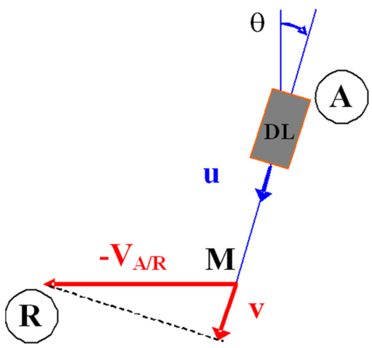

Figure 22

Taking into account the incidence angle, we obtain :

$$f_b = -\frac{2}{\lambda_s}\vec{V}_{A/R}.\vec{u} = \frac{2}{\lambda_s} V_{A/R} \sin(\theta) \;\Leftrightarrow\; \boldsymbol{V_{A/R} = \frac{\lambda_s}{2\sin(\theta)} f_b}$$

Remarque

$$V_{A/R} = \frac{1300.10^{-9}}{2\sin(20°)}(13,154.10^6) \;\Leftrightarrow\; V_{A/R} = 25 \text{ m/s} = 90 \text{ km/h}$$

For an angle $\theta = 0$, the beat frequency is nothing, whatever may be the automobile's speed and the bumps of the roadway.

>Solution n°4 *(exercice p. 24)*

A pseudo-period corresponds with

$$\delta D = \frac{\lambda_s}{2}$$

so that the number of pseudo-periods for a displacement of déplacement $\Delta D = 1$ mm is :

$$\boldsymbol{N = \frac{2\,\Delta D}{\lambda_s}}$$

Remarque

$$\text{N} = \frac{2.10^{-3}}{780.10^{-9}} \;\Leftrightarrow\; \mathbf{N = 2564}$$

When the accounting error comes from a pseudo-period, the precision is thus : $p = 1/2564$ so that $p = 0.039\%$

>Solution n°5 *(exercice p. 24)*

On the ascending part (resp. descending) of the luminous emission power, we can count 4 pseudo-periods for $t_0 = 3.873$ ms (resp. 6 pseudo-periods pour $t_1 = 3.926$ ms). The associated beat frequencies are thus : $fb_0 = 1.033$ kHz et $fb_1 = 1.528$ kHz.

The optical frequency excursion is $\delta\%\nu_s = 5$ GHz. This excursion is effectuated in time-frame corresponding to the half-period of the triangular feed signal. Regarding this curve, the time is : $\delta t = 0,01/2 = 5$ ms .

$$\frac{\delta\nu_s}{\delta t} = 1000 \text{ GHz/s}$$

We can now use the relationships giving the object's distance and speed :

$$D=\frac{c}{4\frac{\delta v_s}{\delta t}}(f_{b0}+f_{b1}) \quad \text{and} \quad v=\frac{\lambda_s}{4}(f_{b1}-f_{b0})$$

Remarque

$$D=\frac{3.10^8}{4.1000.10^9}(1,033.10^3+1,528.10^3) \Leftrightarrow \mathbf{D=19{,}2\ cm}$$

$$v=\frac{780.10^{-9}}{4}(1,528.10^3-1,033.10^3) \Leftrightarrow \mathbf{v \simeq 96{,}5\,\mu\,m/s > 0}$$

(the object moves away from the laser)