# Introduction to photometry

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Optical metrology

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## *Conception & production :*

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# II.Cours

Optical radiometry, or photometry, is the discipline concerning the theoretical and experimental characterization of optical radiation. Its subject is the quantities which define radiation, the laws which govern its emission, propagation and detection. Photometry is concerned with optical radiation which covers the visible, infrared and ultraviolet. It plays a very important role in the design and quality of an optical or optronic instrument. Indeed, it is used to establish the energy budget of the various technical solutions and it makes it possible to theoretically evaluate the performance of the system. Generally, optical sensors allow the translation into electrical signals of information carried by visible light or radiation of neighboring wavelengths: infrared and ultraviolet. However, the spectrum of electromagnetic waves is much richer than the simple spectral window offered by the sensitivity of the eye (figure 1.1).

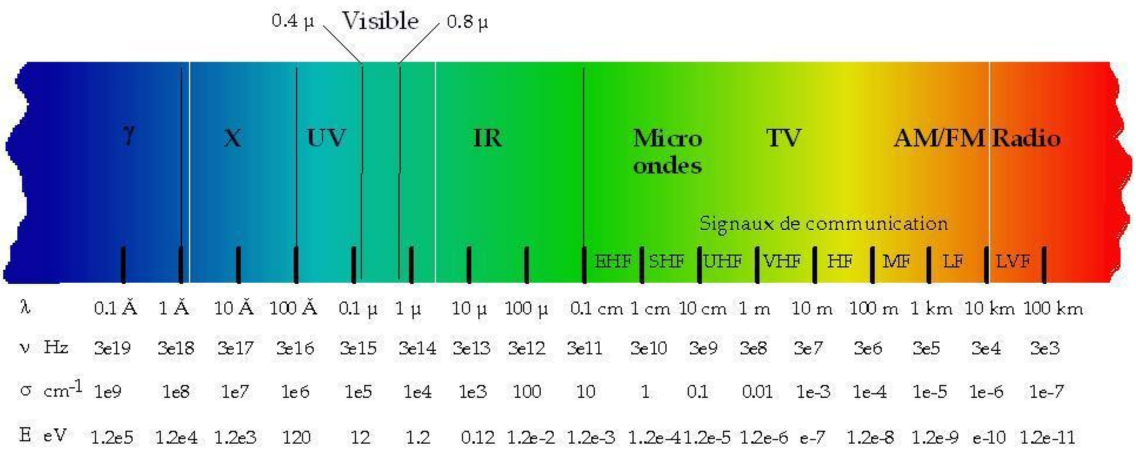


*Figure 1.1 : spectre des ondes électromagnétiques*

Light has both a geometric aspect (see Papers "Geometric and Instrumental Optics"), a wave aspect and a corpuscular aspect. Considered in its wave aspect, light appears to be made up of electromagnetic waves emitted during electronic transitions between levels of energy of the atoms of the source. These waves propagate in a vacuum at the speed $c = 299792458 \text{ m.s}^{-1}$ (i.e. approximately $3 \times 10^8 \text{ m.s}^{-1}$ ) and at a reduced speed c/n in matter The frequency $\nu$ and the wavelength are linked by the relation $\lambda = c/\nu$where $\lambda$ is the wavelength of radiation in a vacuum.

If we consider the corpuscular theory of light, then each photon carries an individual energy E proportional to the frequency of the radiation according to the following relationship

$$E = h\nu = \frac{hc}{\lambda}$$

where $h = 6.62 \times 10^{-34} \text{J.s}$ is Planck's constant.

Radiation is usually emitted by a light source. There are different types of sources.

Tungsten filament lamps consist of a tungsten filament placed in a glass or quartz bulb containing a rare gas or halogen (iodine) intended to limit evaporation of the filament. This is heated by a current whose intensity determines the true temperature. The filament then behaves like a black body and emits light in the visible range. The interest of this type of lamp comes from their very extended spectrum and the radiation throughout the space; on the other hand, their lifespan is limited and they are fragile to shocks, moreover the thermal inertia makes any modulation impossible.

Light-emitting diodes (LED) have narrower spectra than tungsten filament lamps. They work with PN junctions in which the recombination of an electron and a hole leads to the emission of a photon. Their response time is low and these sources are robust and reliable. On the other hand, they are sensitive to temperature and their radiation is low in energy.

In laser sources, there is amplification of light (see "Laser" course, ENSIM 1) which leads to highly energetic radiation with an almost monochromatic spectrum and very high directivity. There are different types of lasers: gas (HeNe, CO2), solid (Neodymium YAG, Titanium-Sapphire, laser diode), dye or chemical (eximers).

This booklet offers an introduction to photometry. It initially focuses on the basic definitions and quantities of photometry. In a second part, the relationships between the quantities and then the photometry of optical systems are discussed.

# 1. Basic definitions

A source is a generator of light from various forms of energy such as electrical (tungsten filament lamp, lasers, diodes, etc.), electronic (cathode ray tube, luminescence lamp, etc.), thermal (radiation depending on temperature, sun, etc.) or optics (natural scenes seen by reflection or diffusion of ambient lighting, etc.). Radiation sources are based on the following characteristics: geometry of the emitter, geometry of its radiation, spectrum, form of emission over time (continuous or pulsed). Elements such as energy efficiency, electrical consumption, mass, dimensions, etc. are generally imposed by the system specifications.

## 1.1. Notion of light flux

All optical radiation carries energy. The resulting flux of energy at each instant, per unit time, is called **energy flux**. We note it $\Phi$ and it is expressed in **Watts (W)**. Electromagnetic theory shows that the energy flow is equal to the flux of its Poynting vector. The time average value of the Poynting vector is given by

$$\langle \|\vec{S}\| \rangle = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\|\vec{E}_0\|^2$$

where $\epsilon$ and $\mu$ are the permittivity and magnetic permeability of the medium, $E_0$ being the electric field vector. The energy flux is also called **power** (in particular we speak of the power of a laser).

The energy flux of radiation made up of photons of the same energy is therefore equal to the product of its photon flux (per unit of time) by the individual energy of each photon. The rate of photons per unit time is called photon flux, denoted $\Phi_p$ . We have :

$$\Phi = E \times \Phi_p = \frac{hc}{\lambda}\Phi_p$$

With : $\lambda$ the wavelength of the radiation, $c \simeq 3 \times 10^8$m/s the velocity of light in vaccuum and $h = 6.62^{-34}$J.s the Planck constant.

We define a third flux called **luminous** flux whose unit is the **lumen (lm)** to quantify the visual simulations of radiation on a standard observer.

### 1.2. Solid angle

The solid angle under which an object is seen from an observation point $O$ is the ratio between the area of the conical projection of the apparent contour of this object on a sphere centered at $O$, by the square of the radius of the sphere ( figure 2.1).

This quantity, which is the ratio between a surface and the square of a distance, is expressed in **steradians (sr)**. It represents the extension in space of the notion of angle which is generally defined in a plane. We have

$$\Omega = \frac{S}{d^2}$$

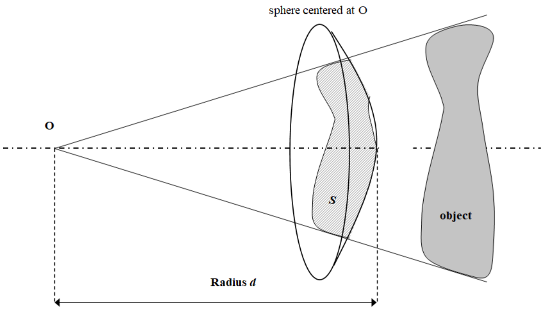


*Figure 2.1 : definition of solid angle*

If the object is planar and if its transverse dimensions are small compared to its distance from point $O$, the elementary solid angle will be expressed

$$d\,\Omega = \frac{dS\cos\theta}{d^2}$$

$dS$ being the real surface of the object and $\theta$ being the angle between the normal of the object and the direction of observation (figure 2.2). The term $dS\cos\theta$ is the apparent surface of the object in the direction of obliquity whose factor is $\cos\theta$ .

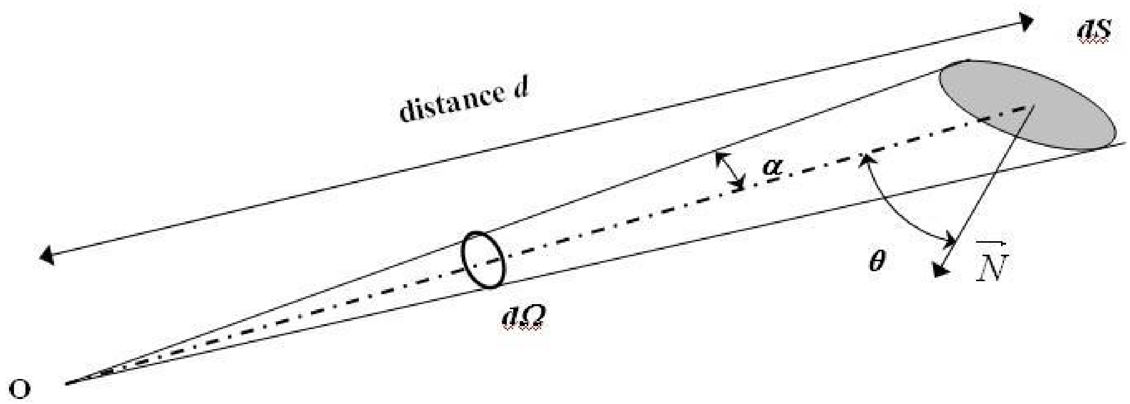


*Figure 2.2 : elementary solid angle*

If the object is perceived in the form of a disk of radius $R (dS = \pi R^2)$ whose angular radius $\alpha$ (half angle at the vertex) is small then the solid angle for this object is given by (figure 2.3)

$$\Omega = \frac{\pi R^2}{d^2} \simeq \pi\alpha^2$$

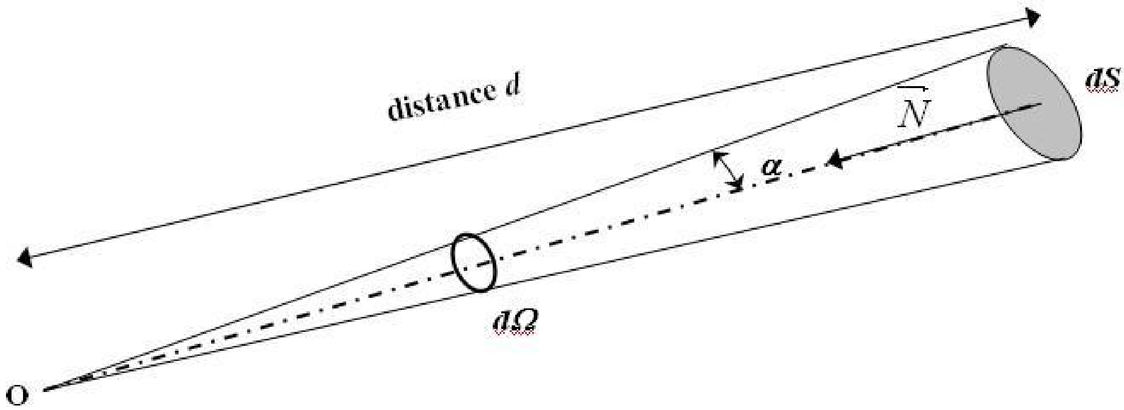


Figure 2.3 : solid angle for a disk

Let us now consider an elementary solid angle defined by a ring of radius $R$ whose average angular radius is $\alpha$ and the angular width $d\alpha$. The disk of radius $R$ has a surface $S = \pi R^2$and for a variation $dR$ of the radius, the surface of the crown $dS = 2\pi R dR$ (figure 2.3). The solid angle is therefore

$$d\,\Omega = \frac{dS\cos\alpha}{d^2} = \frac{2\pi\,RdR\cos(\alpha)}{d^2} = 2\pi\frac{R}{d}\frac{dR}{d}\cos(\alpha)$$

as $\tan(\alpha) = \frac{R}{d}$ and $d\alpha = \frac{dR}{d}$, it comes :

$$d\,\Omega = 2\pi\sin(\alpha)\,d\alpha$$

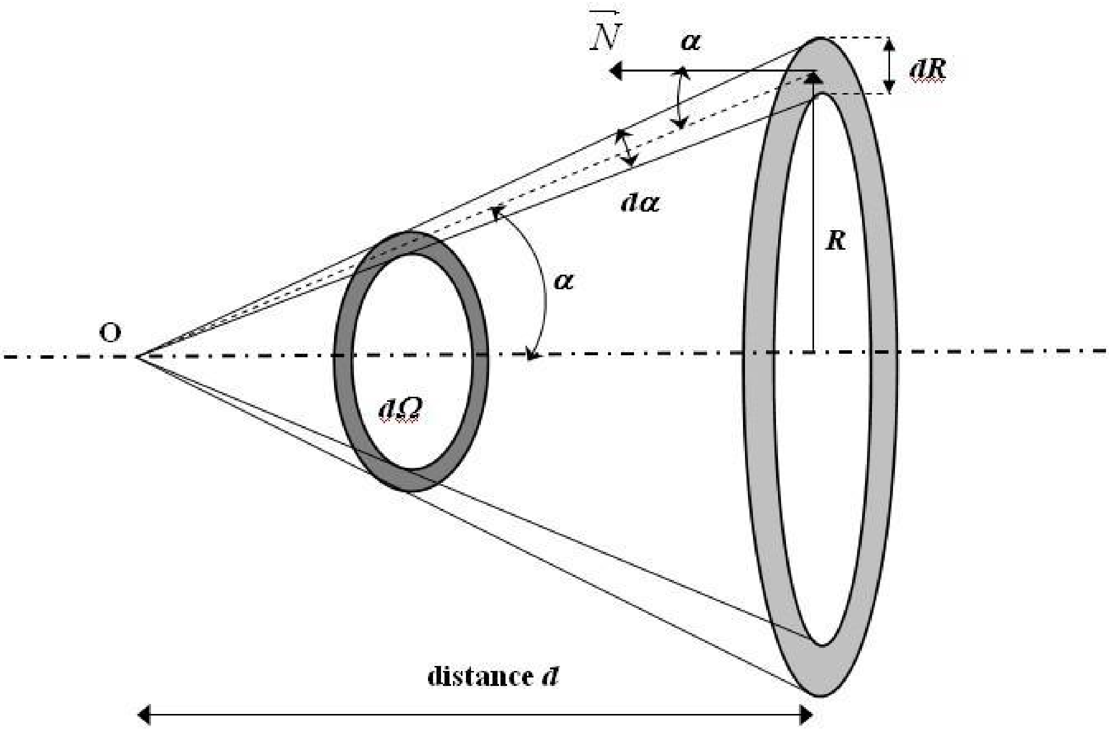


Figure 2.4 : solid angle for a crown

For a cone of revolution with half angle at the vertex $\alpha_M$, the solid angle is given by

$$\Omega=\int_0^{\alpha_M} d\Omega=\int_0^{\alpha_M} 2\pi\sin(\alpha)\,d\alpha=2\pi(1-\cos\alpha_M)$$

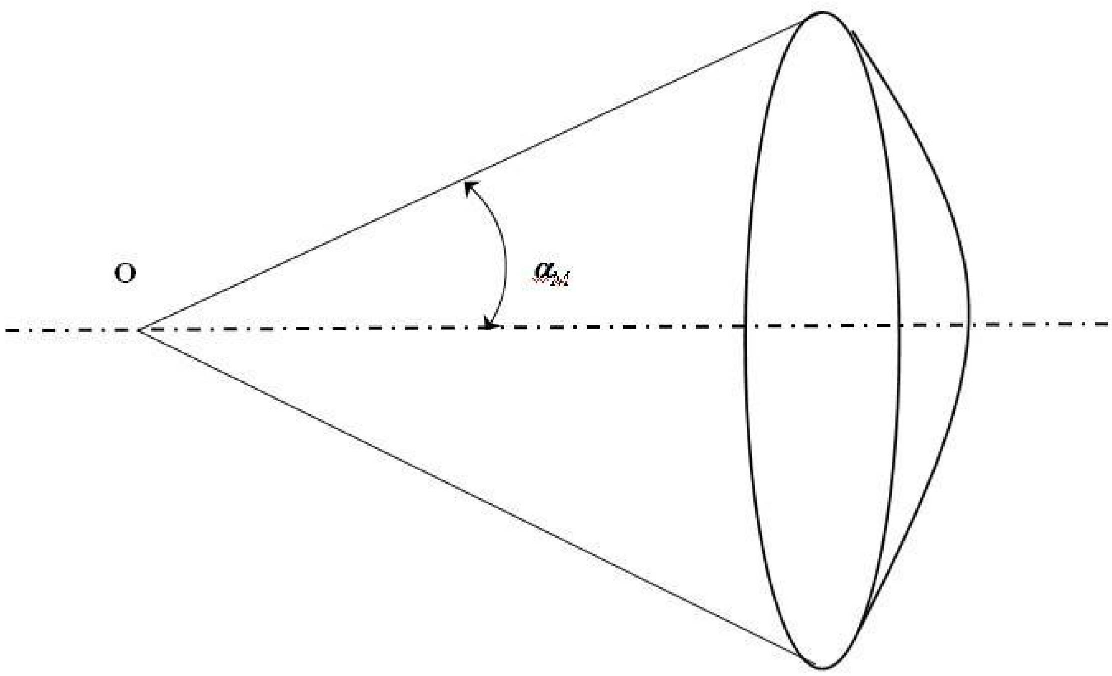


*Figure 2.5 : solid angle for a cone of revolution*

The solid angle corresponding to a half space is given by

$$\Omega_{demi-espace}=\int_0^{\pi/2} d\Omega=2\pi \text{ sr}$$

and for the entire space we have

$$\Omega_{espace}=\int_0^{\pi} d\Omega=4\pi \text{ sr}$$

## 1.3. Intensity

The intensity of an emitter in a given direction is the flux it emits per unit of solid angle in the direction considered. The **intensity** is denoted $I_S$ and is expressed in $W.sr^{-1}$. We have

$$I_S=\frac{d\Phi}{d\Omega_s}$$

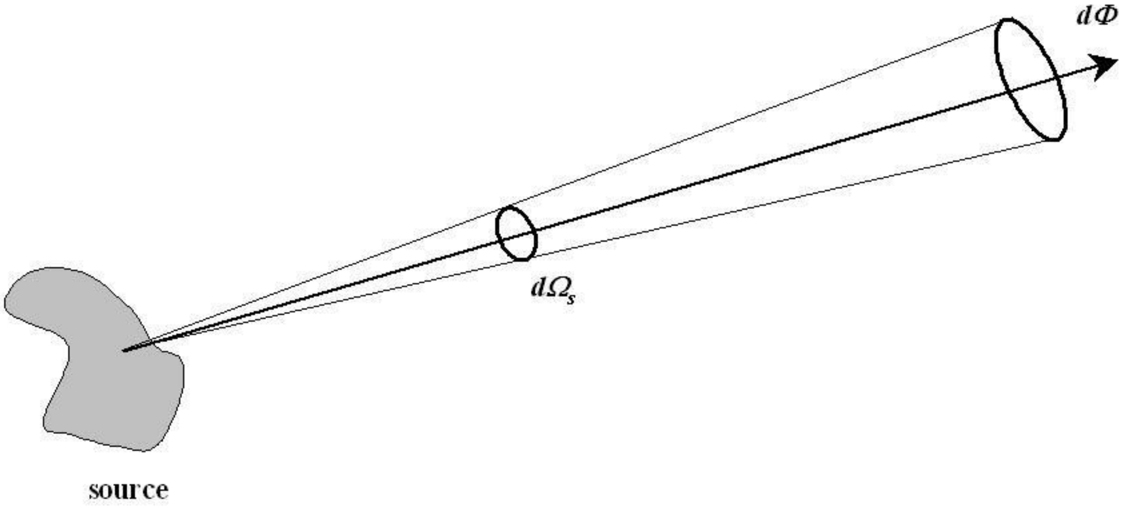


*Figure 2.6 : intensity of a source*

A so-called isotropic source is a source whose intensity is independent of the direction of emission. However, this situation is seldom in practice. The angular behavior of radiation is characterized by the intensity indicator or radiation diagram. The indicator of an isotropic source is a sphere.

## 1.4. Luminance

The notion of intensity does not provide access to the spatial distribution of the source's emitters, their geometry, or their relative importance. To characterize the radiation by its spatial and angular properties, we define the luminance.

Luminance is defined as the intensity per unit of apparent surface area in a given direction, i.e.

$$L_S = \frac{dI_S}{dA_s \cos\theta_s}$$

where $A_S$ is the source surface element and $\theta_S$ is the angle between the source normal and the emission direction. We therefore have

$$L_S = \frac{d^2\Phi}{dA_s \cos\theta_s \, d\,\Omega_s}$$

**Luminance** is expressed in $\mathrm{Wm^{-2}sr^{-1}}$. Luminance is sometimes called "shine" or "brightness". With the previous relationship it comes

$$d^2\Phi = L_S \, dA_s \, d\,\Omega_s \cos\theta_s$$

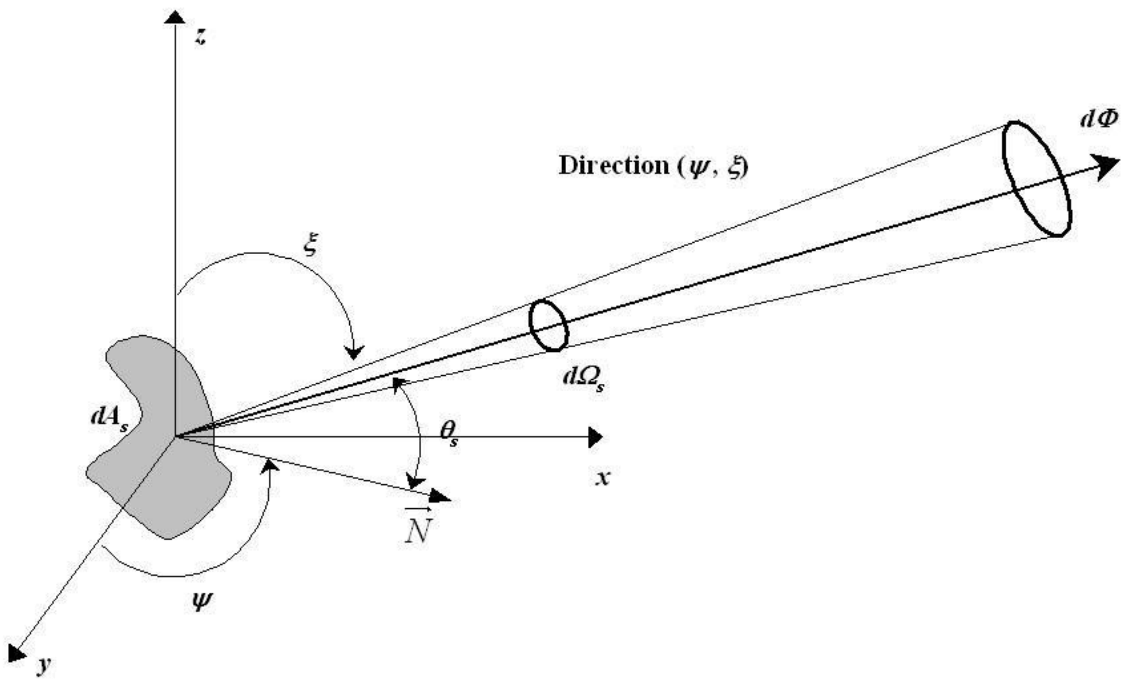


*Figure 2.7 : luminance of a source*

For a given point in space, the luminance generally depends on the direction of emission and the angular properties of the radiation coming from this location are defined by its luminance indicator. This indicator is the locus of the vector $L_S(\psi,\zeta)$ whose origin kept fixed is the point considered.

## 1.5. Illumination

The **illumination** of a plane at a given point is the incident flux per unit surface in this plane, at the point considered. We therefore have

$$E=\frac{\Phi}{A_R}$$

where $A_R$ is the illuminated surface. **Illuminance** is expressed in $\mathrm{Wm}^{-2}$.

## 1.6. Exitance

The emissive surface emitted at a point is the flux emitted in a **half-space** per unit area of the emissive surface centered at this point. We have

$$M=\frac{d\Phi}{dA_s}$$

**Exitance** is expressed in $\mathrm{Wm}^{-2}$ . This name replaces the old terms "emittance" and "radiance".

## 1.7. Energy of light

The energy of light delivered by radiation whose flux is $\Phi$ for a duration $\Delta t = t_2 - t_1$ is given by

$$f = \int_{t_1}^{t_2} \Phi(t)\,dt$$

If the flux is constant over the duration $\Delta t$, it comes $f = \Phi \Delta t$. **The energy of the light** is expressed in joules $(J)$.

## 1.8. Exposure or fluency

The exposure of a surface for a duration $\Delta t$ is given by

$$\zeta = \int_{t_1}^{t_2} E(t)\,dt$$

The **exposure** is expressed in $\mathrm{J.m^{-2}}$.

## 1.9. Geometric etendue

It appears from paragraph 2.4 that

$$d^2\Phi = L_S\, dA_s\, d\Omega_s \cos\theta_s$$

The flux is therefore proportional to the luminance of the radiation and to the quantity

$$d^2 G = dA_s\, d\Omega_s \cos\theta_s$$

which is a characteristic of the emission geometry (surface and solid angle). This quantity is called the **geometric etendue** of the brush of light considered.

The solid angle expressed in the relationship above is that at which the source sees the receiver. For a light brush defined by two surface diaphragms $dA_R$ and $dA_S$ located at distance d, the geometric etendue is expressed by one of the following three relationships

$$d^2 G = dA_S\, d\Omega_S \cos\theta_S$$

$$d^2 G = dA_R\, d\Omega_R \cos\theta_R$$

$$d^2 G = \frac{dA_S\, dA_R \cos\theta_S \cos\theta_R}{d^2}$$

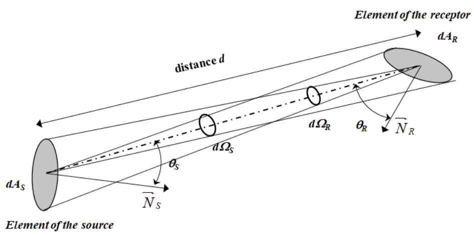


*Figure 2.8 : geometric etendue of a brush of light*

In the general case, if the source and receiver elements are no longer considered "elementary", as in the case of the light brush, the geometric etendue $G$ is the sum of the elementary geometric etendues of all the brushes of which it is constituted. We therefore have :

$$G = \int_{\substack{surface\\source}} \int_{\substack{surface\\récepteur}} d^2 G$$

A frequently encountered case is that where transmitter and receiver are coaxial and where the source sees the circular receiver under an angular radius $\alpha_M$.

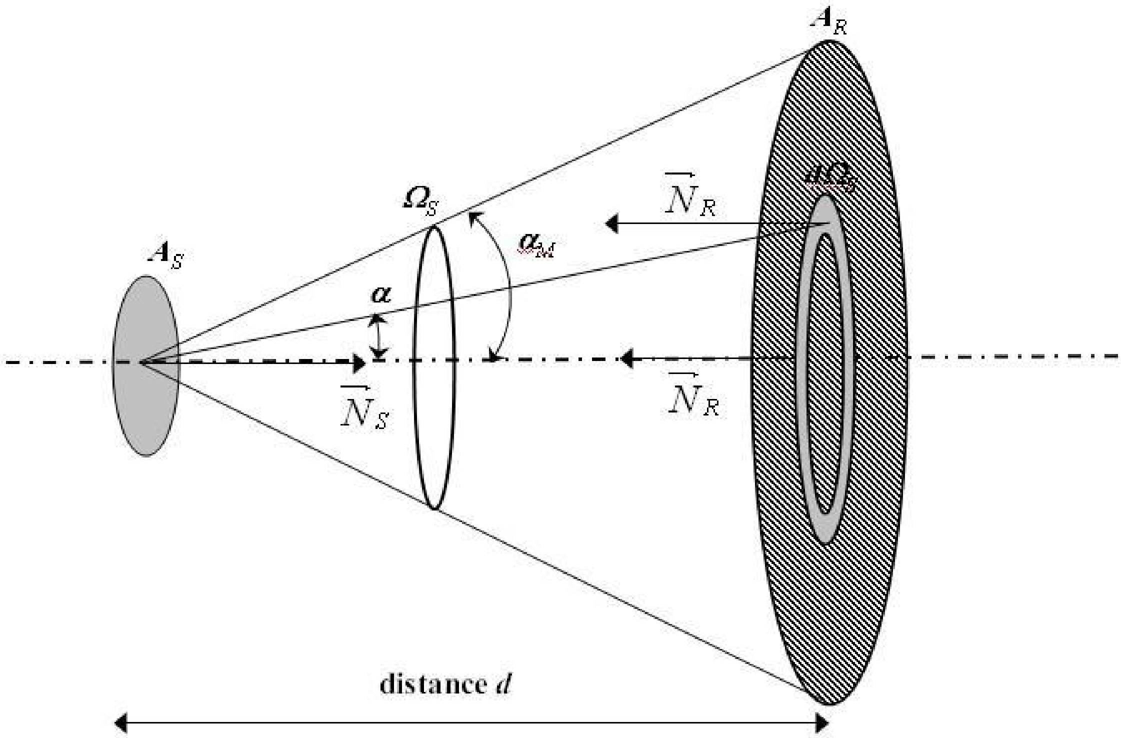


*Figure 2.9 : geometric etendue of a cone beam*

In this case, the elementary geometric etendue is expressed as

$$d^2 G = dA_S \, d\,\Omega_S \cos\theta_S$$

with $\theta_S = \alpha$and , hence $d\Omega_S = 2\pi \sin(\alpha) d\alpha$

$$d^2 G = 2\pi \, dA_S \sin(\alpha) \, d\alpha$$

and we have

$$G = \int_{\substack{surface\\source}} \int_{\substack{angle\\ouverture}} d^2 G = \pi \int_{source} dA_S \int_0^{\alpha_M} 2\sin\alpha \cos\alpha \, d\,\alpha$$

giving

$$G = \pi A_S \sin^2 \alpha_M$$

### a) Luminance conservation

Consider Figure 2.8, if the propagation medium is homogeneous without losses, the flux emitted by the source is also the flux received by the receiver, in the solid angle concerned by the source and the receiver. We therefore have $d^2\Phi_S = d^2\Phi_R$and consequently it appears that the luminance is conserved throughout a light brush, from the source to the receiver .

If the medium introduces losses, the transmission factor of the medium on the path considered is called the ratio

$$T=\frac{d^2\Phi_R}{d^2\Phi_S}$$

The transmission factor of a medium is therefore a flux ratio.

### b) Basic units

The following table summarizes the main radiometric units of the international system that must be used to quantify the energetic, photonic and luminous quantities of radiation.

| Metrics | Energetic units | Photonic units | Visual units |
|---|---|---|---|
| Flux | W | $s^{-1}$ | Lumen (lm) |
| Intensity | $W\ sr^{-1}$ | $s^{-1}\ sr^{-1}$ | Candela (cd) |
| Luminance | $W\ m^{-2}\ sr^{-1}$ | $s^{-1}\ m^{-2}\ sr^{-1}$ | Nits or $cd\ m^{-2}$ |
| Illumination | $W\ m^{-2}$ | $s^{-1}\ m^{-2}$ | Lux |
| Exitance | $W\ m^{-2}$ | $s^{-1}\ m^{-2}$ | $lm\ m^{-2}$ |
| Energy | J | number of photons | lm s |
| Exposure | $J\ m^{-2}$ | number of photons $m^{-2}$ | $lm\ s\ m^{-2}$ |

*Figure 2.11 : table of energy, photonic and visual quantities*

# 2. Relations between quantities

## 2.1. Relations between flux and intensity

According to the definition of intensity, the luminous flux emitted in a solid angle $\Omega_S$ is given by

$$d\Phi=I_S\,d\Omega_S$$

either

$$\Phi=\int_{\Omega_S} I_S\,d\Omega_S$$

If a source is isotropic with intensity $I_0$, then the flux emitted throughout space $(4\pi\text{sr})$ is equal to

$$\Phi_{espace} = 4\pi I_0$$

## 2.2. Relation between flux and luminance

As noted previously, for a luminance source $L_S$, we have

$$d^2\Phi = L_S dA_S d\Omega_S \cos\theta_S = L_S d^2 G$$

and the emitted flux is given by

$$\Phi = \int_{\substack{surface\\source}} \int_{\substack{surface\\récepteur}} L_S d^2 G$$

In the case where the luminance is uniform, the emitted flux is simply

$$\Phi = L_S G$$

## 2.3. Relation between intensity and luminance

Consider the relation $d^2\Phi_S = L_S dA_S d\Omega_S \cos\theta_S$ and evaluate the flux emitted by the entire source in the solid corner element. We have

$$d\Phi_S = \int d^2\Phi_S = d\Omega_S \int_{source} L_S \cos\theta_S dA_S$$

As the intensity is defined by the flux that the source emits per unit of solid angle along the brush of light, we have

$$I_S = \frac{d\Phi_S}{d\Omega_S} = \int_{source} L_S \cos\theta_S dA_S$$

## 2.4. Relation between luminance and exitance

The elementary flux emitted in the solid corner element by the luminance source $LS$ is given by

$$d^2\Phi = L_S dA_S d\Omega_S \cos\theta_S$$

The flux emitted by the source in a half-space is given by the summation of the elementary contributions on all the emission directions, i.e.

$$\Phi_{\substack{S\\demi\\espace}} = dA_s \int_{\substack{demi\\espace}} L_S \cos\theta_S d\Omega_S$$

From the definition of exitance, we deduce that

$$M=\frac{\Phi_{S\ demi-espace}}{dA_S}=\int_{\substack{demi\\espace}} L_S \cos\theta_S \, d\,\Omega_S$$

## 2.5. Relation between illumination and intensity (Bouguer's law)

If a source is small or almost non extended (punctual) with respect to the distance which separates it from the measurement point, the intensity is the parameter which best characterizes it. The elementary flux coming from the source and received at a distance $d$ by an elementary plane surface $dA_R$ illuminated under the angle $\theta_R$ is given by

$$d\Phi_R=I_S\, d\,\Omega_S=I_S\frac{dA_R\cos\theta_R}{d^2}$$

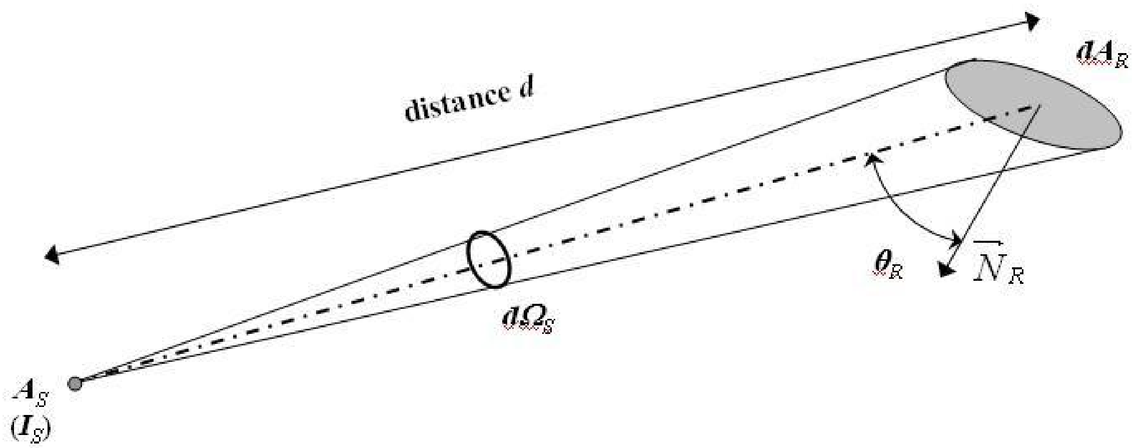


*Figure 3.1 : Bouguer's law*

The illuminance of the surface is therefore

$$E_R=\frac{d\Phi_R}{dA_R}=\frac{I_S\cos\theta_R}{d^2}$$

which constitutes the **Bouguer's law**.

## 2.6. Relation between illumination and luminance

For an extended source, the parameter to consider is luminance. If $L_R$ is the apparent luminance of the illuminated area, $\theta_R$ the angle of incidence and $d\Omega_R$the solid angle of the brush, the elementary flux arriving on the surface $dA_R$ is given by

$$d^2\Phi_R=L_R\, dA_R\, d\,\Omega_R\cos\theta_R$$

and the resulting illumination is equal to

$$dE_R = \frac{d^2\Phi_R}{dA_R} = L_R \cos\theta_R \, d\Omega_R$$

In the case of hemispherical illumination of non-uniform luminance, the illumination contribution $dE_R$ coming from the elementary solid angle must be integrated over the half-space to determine the total illumination. In this case, we will have

$$E_R = \int_{\substack{demi\\espace}} dE_R = \int_{\substack{demi\\espace}} L_R \cos\theta_R \, d\Omega_R$$

## 2.7. Case of light sourcewith uniform luminance

Luminance is the parameter which depends on both a position in space ($x, y$ coordinates on the source) and the direction of emission (angular coordinates $\Psi, \zeta$). For a certain number of sources, the luminance can be considered uniform, i.e. $L_S(x, y, \Psi, \zeta) = L_S$

Consider radiation with uniform luminance passing through a diaphragm consisting of a cone of half angle at the vertex $\Theta_M$(figure 3.2).

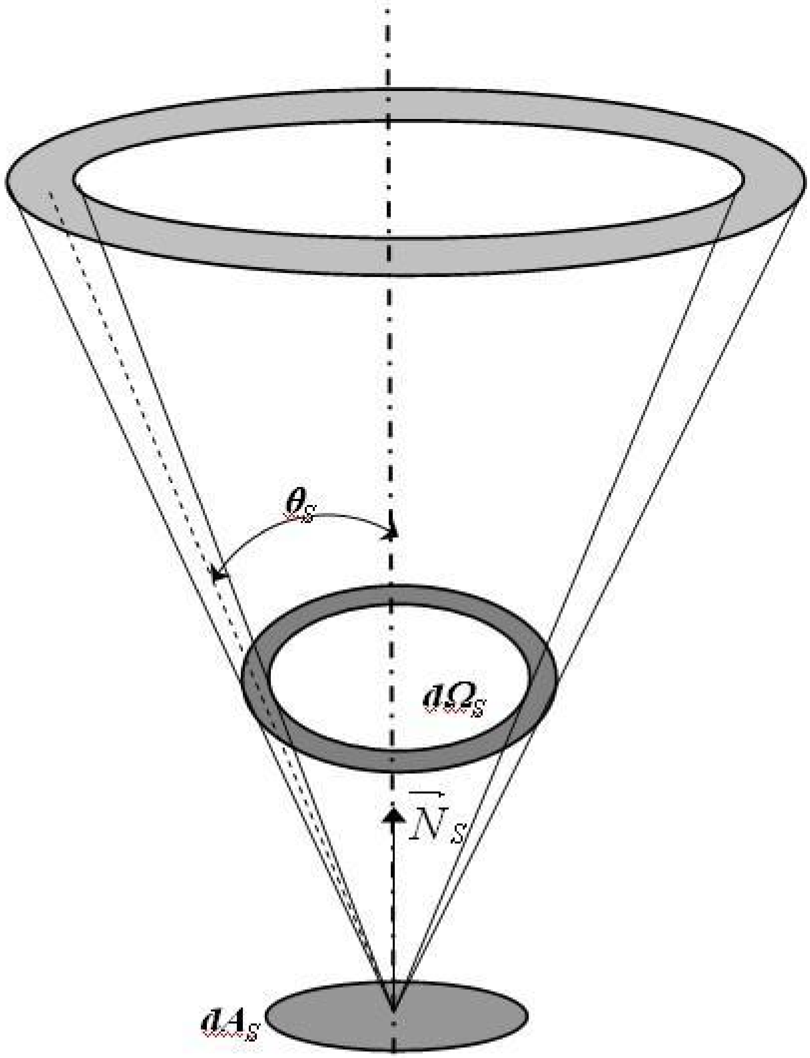


*Figure 3.2 : luminance-exitance relationship*

The solid angle is decomposed into a ring as described in paragraph 2.2. Since the luminance is uniform, the luminance of the source is given by

$$M_S = \frac{\Phi_{S_{cône}}}{dA_S} = L_S \int_{cône} \cos(\theta_S) \, d\Omega_S = L_S \int_0^{\theta_M} 2\pi \sin(\theta_S) \cos(\theta_S) \, d\theta_S$$

given

$$M_S = \pi L_S \sin^2 \theta_M$$

For radiation in half-space, we have $\theta_M = \pi/2$ and it comes

$$M_{S_{demi-espace}} = \pi \, L_S$$

From the previous relation, we deduce the flow emitted by the source. We have

$$\Phi_{S_{source}} = \int_{source} M dA_S = \int_{source} \pi \, L_S \sin^2 \theta_M \, dA_S$$

and for the uniform luminance over the entire surface of the source, we have

$$\Phi_{S_{source}} = \pi \, L_S \, A_S \sin^2 \theta_M$$

For uniform radiation in half-space, we have $\theta_M = \pi/2$ and it comes

$$\Phi_{S_{demi-espace}} = \pi \, L_S \, A_S$$

In the case of globally uniform luminance, it was seen above that the intensity of the source is given by

$$I_S = \int_{source} L_S \cos \theta_S \, dA_S = L_S \int_{source} \cos \theta_S \, dA_S$$

As the term $dA_S \cos \theta_S$is equivalent to the apparent surface of the source, we finally have,

$$I_S = L_S A_{S_{apparente}}$$

Let us now consider the case of the flat surface element $dA_R$ illuminated by uniform luminance radiation inside a finite solid angle $\Omega_R$ assumed to be of revolution around the axis of the surface (figure 3.3).

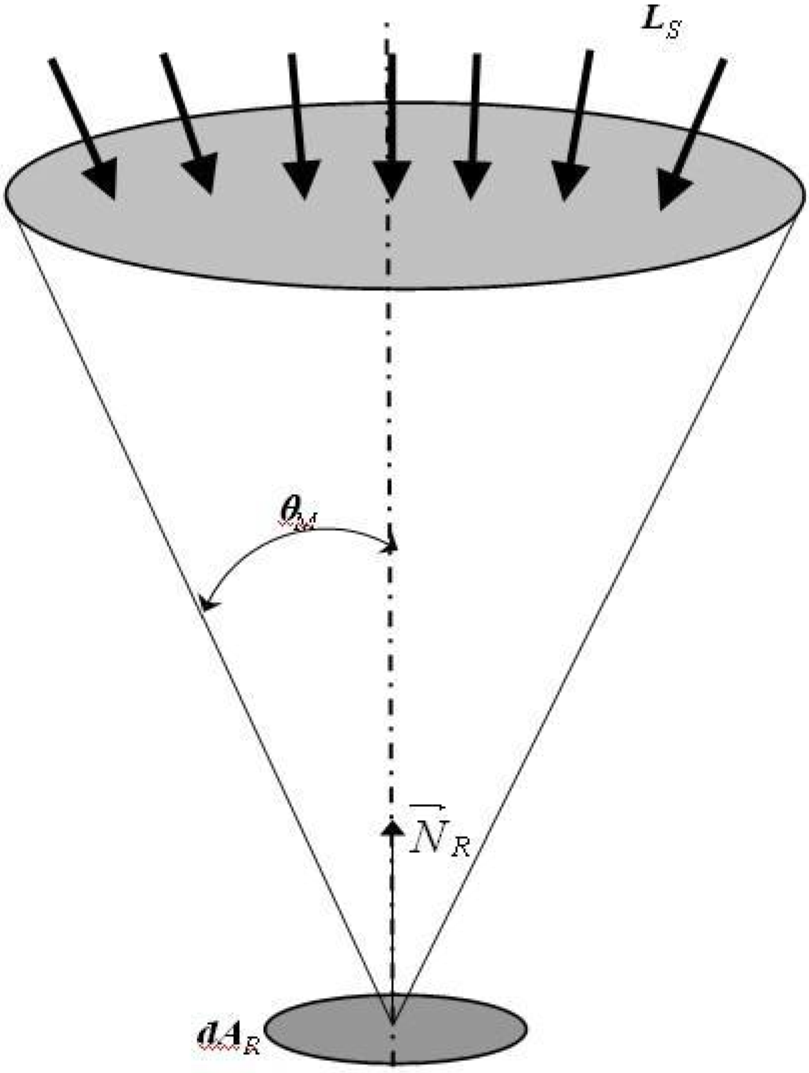


*Figure 3.3 : illuminance-luminance relationship*

The elementary flux received by this surface element is

$$d\Phi_R = \int_{\Omega_R} L_S \, dA_R \, d\,\Omega_R$$

By the symmetry of revolution of the geometry of the problem, we have

$$d\Phi_R = L_S \, dA_R \int_0^{\theta_M} 2\pi \sin(\theta_R)\cos(\theta_R)\, d\theta_R = \pi L_S \, dA_R \sin^2\theta_M$$

and the illumination is therefore written

$$E_R = \frac{d\Phi_R}{dA_R} = \pi L_S \sin^2\theta_M$$

For a small angle $\theta_M$, we can write $E_R = L_S \Omega_M$ where $\Omega_M$ is the solid angle under which the source is seen. In the case where the radiation is uniform in the half-space, we have $\theta_M = \pi/2$ and it comes

$$E_R = \pi L_S$$

## 2.8. Case of perfect diffusers

A **perfect diffuser** is a surface that reflects or transmits the **entire** incident flux with an **uniform luminance** in a half-space regardless of the lighting geometry. This half-space will be upstream for a reflection diffuser and downstream for a transmission diffuser.

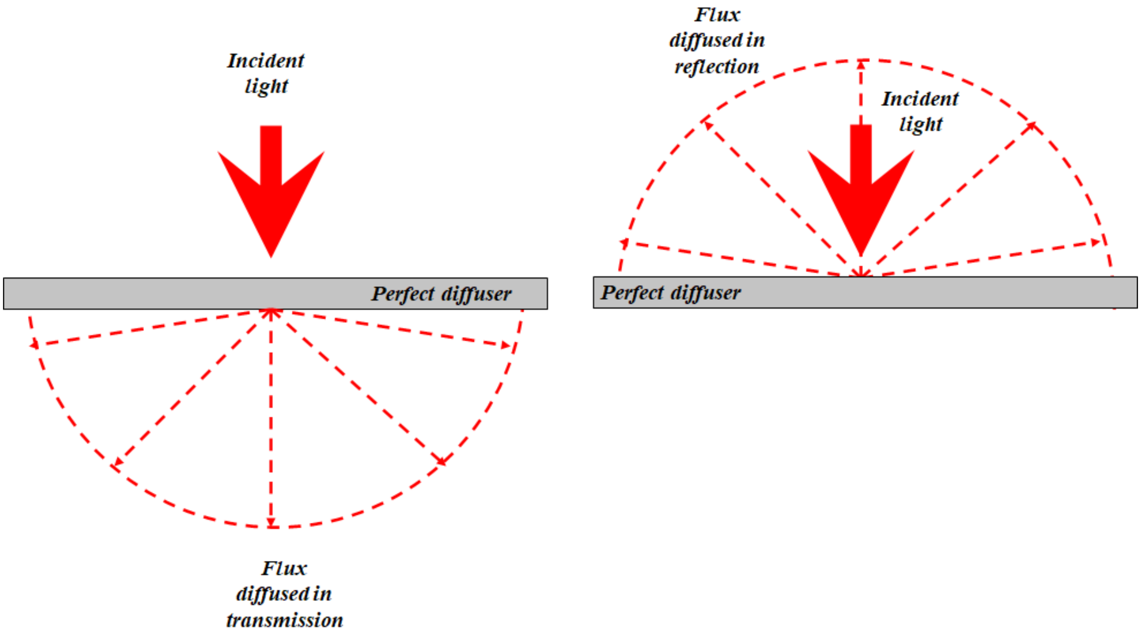


*Figure 3.4 : diffusion of the incident flux in the half-space*

A diffuser will be **orthotropic** or **Lambertian** if it reflects or transmits a **fraction** of the incident flux with **uniform luminance**. This fraction is called surface **albedo** or reflection factor (respectively transmission).

This definition of the perfect diffuser shows that the output of the latter is equal to the illumination received and that its luminance is uniform and given by the relation

$$L_S = \frac{M_S}{\pi} = \frac{E_R}{\pi}$$

In the case of an orthotropic or Lambertian surface, the reflection or transmission factor is no longer equal to 1 and we obtain

$$L_S = \frac{\rho_d E_R}{\pi}$$

where $\rho_d$ is the albedo of the scatterer.

Under the effect of an incident flux $\Phi_R$, the intensity of a Lambertian plane diffuser in a direction making an angle $\theta$ with its normal is given by

$$I_S = \int_{\substack{surface \\ éclairée}} L_S \cos\theta \, dA_S = \int_{\substack{surface \\ éclairée}} \frac{\rho_d E_R \cos\theta}{\pi} dA_S$$

and if the albedo is constant over the entire illuminated surface we have

$$I_S(\theta) = \frac{\rho_d \cos\theta}{\pi} \int_{\substack{surface \\ éclairée}} E_R \, dA_S = \frac{\rho_d \phi_R \cos\theta}{\pi}$$

The reflection or transmission intensity indicator of a Lambertian plane diffuser is a sphere tangent to the diffuser and whose maximum value is obtained along the normal (figure 3.5).

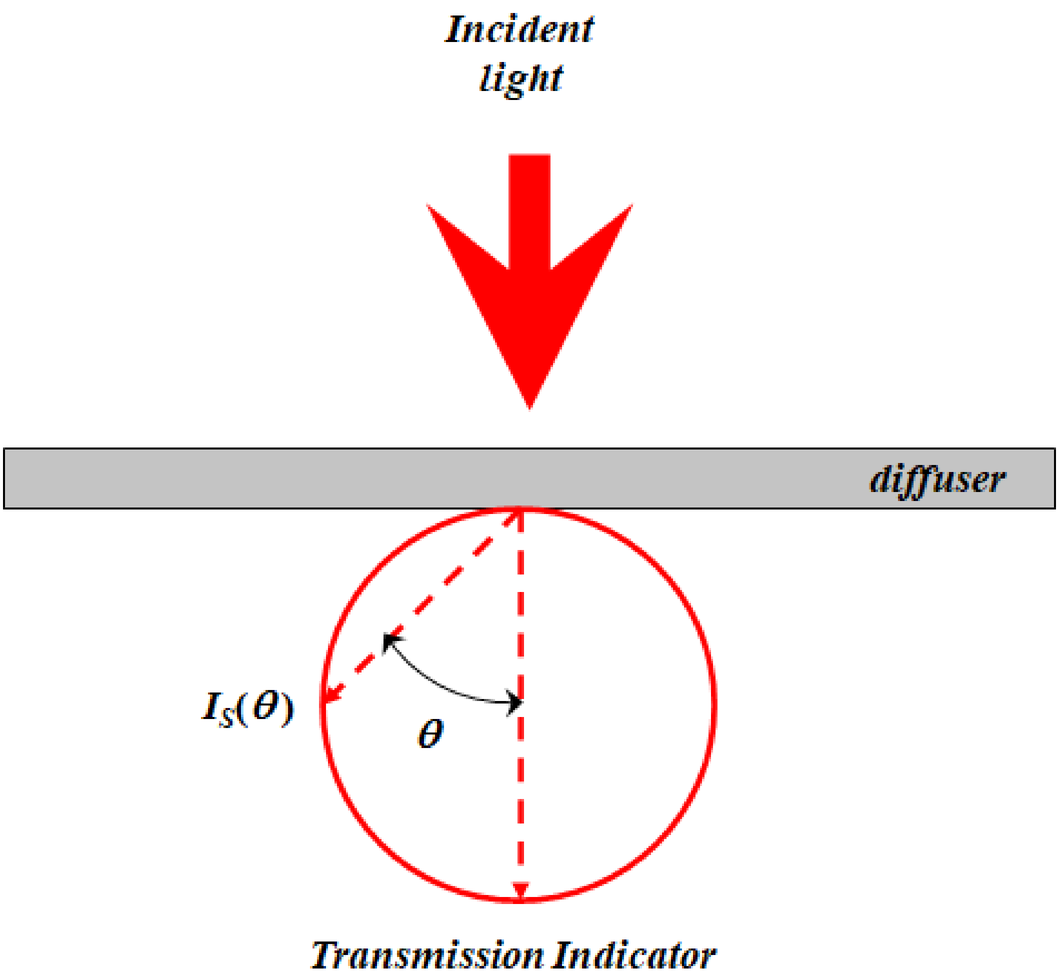


*Figure 3.5 : intensity indicator of the Lambertian diffuser*

## 3. Photometric relationships in optical systems

This paragraph addresses the relationships between the photometric quantities described previously for propagation in different optical media or in components of optical systems.

## 3.1. Transfer of geometric etendue and luminance with refraction of light

In the case of propagation in a homogeneous medium with losses, the flow is attenuated along the path. The luminance is therefore also attenuated during propagation, proportionally to the transmission of the medium. However, since the solid angle is invariant, the geometric etendue also remains invariant. If light propagates by changing medium, as is the case with refraction through a diopter, the geometric etendue and luminance along the path must be evaluated. Let us therefore consider a brush of light of etendue $d^2G$, incident on a diopter separating two media of indices $n$ and $n'$. We assume that the geometric etendue is limited by two small diaphragms with sides $(dx, dy)$and $(dx', dy')$ located at the distance $d$ from the interface (figure 4.1). We consider $dx = dy$ and therefore $dx' = dy'$ .

The geometric etendue defined by the incident beam is expressed as

$$d^2G = \frac{dA_S dA_0 \cos\theta_S \cos\theta_R}{d^2} = \frac{dxdy}{d^2} dA_0 \cos^2\theta = d\theta^2 \cos^2\theta \, dA_0$$

The geometric etendue of the brush transmitted by refraction is expressed by

$$d^2G' = \frac{dA_R dA_0 \cos\theta'_S \cos\theta'_R}{d^2} = \frac{dx' dy'}{d^2} dA_0 \cos^2\theta' = d\theta'^2 \cos^2\theta' \, dA_0$$

The law of refraction applied to the problem gives

$$n \sin\theta = n' \sin\theta'$$

and by differentiation we obtain

$$n \cos\theta d\theta = n' \cos\theta' d\theta'$$

given

$$n^2 \cos^2\theta d\theta^2 = n'^2 \cos^2\theta' d\theta'^2$$

and therefore he comes

$$n^2 d^2G = n'^2 d^2G'$$

This result generalizes to bundles of finite dimensions by summing the elementary geometric etendues of the brushes of which it is composed and we obtain

$$n^2 G = n'^2 G'$$

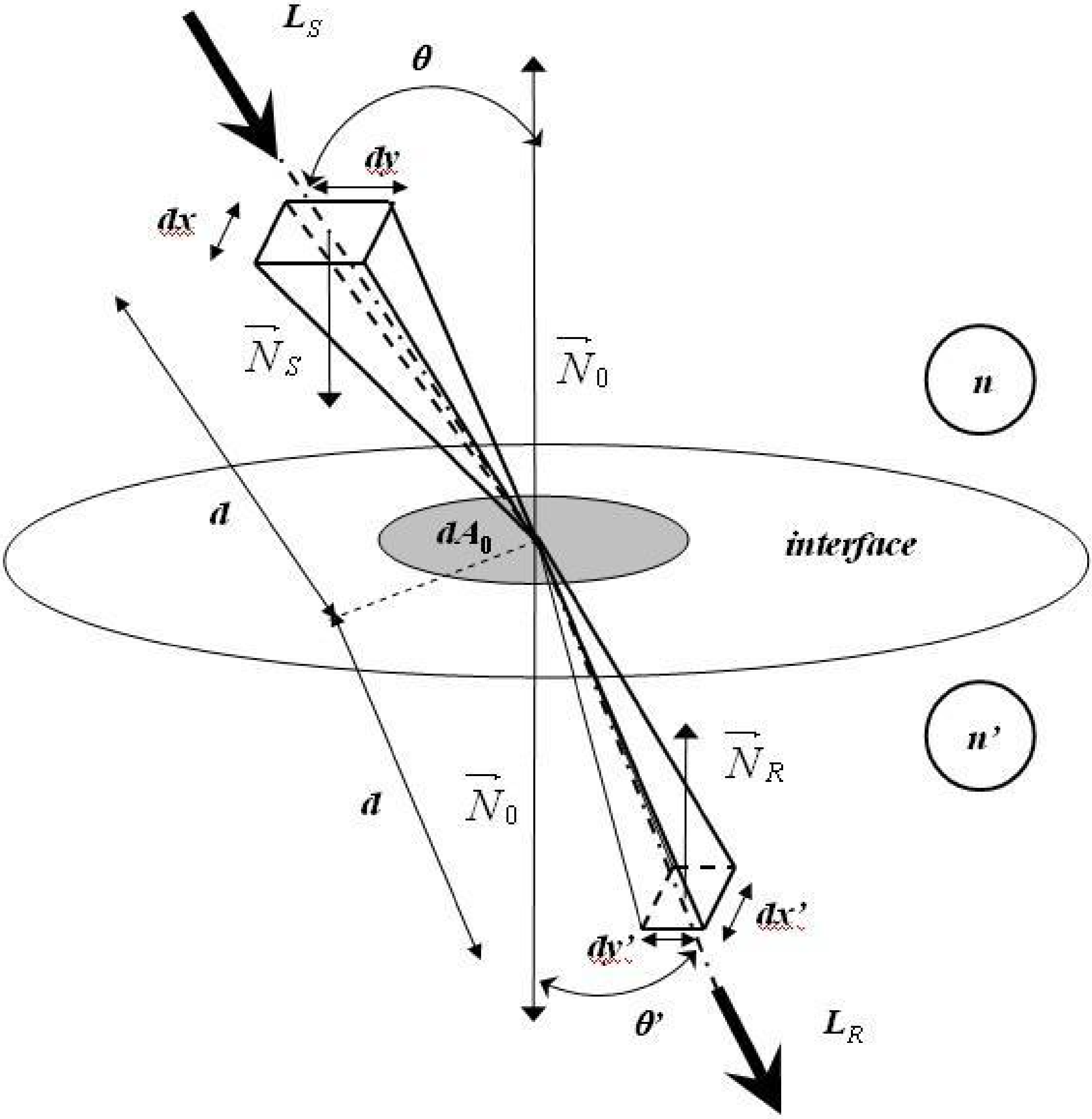


*Figure 4.1 : geometric etendue of refraction*

It follows that for a beam of light which refracts on the surface of a diopter separating two media of different indices, the product of the geometric etendue by the square of the index is constant. The conservation of the geometric etendue of a beam is maximal in a vacuum or in air.

If $T_{opt}$ is the transmission factor of the interface, the elementary flux propagating after refraction is linked to the incident elementary flux by

$$d^2\Phi' = T_{opt}\, d^2\Phi$$

We therefore also have

$$L'_S d^2 G' = T_{opt}\, L_S d^2 G$$

where $L_S$ and $L'_S$ are the respective luminances in the first and second medium. Thus, there comes the relationship between the luminances of the two environments:

$$L'_S \frac{n^2}{n'^2} d^2 G = T_{opt}\ L_S d^2 G$$

given

$$L'_S = T_{opt} \frac{n'^2}{n^2}\ L_S$$

## 3.2. Optical systems for flux collection

A lux collector is an optical system that captures the entire flow emitted by a source. The typical geometry of such a collector is depicted in Figure 4.2.

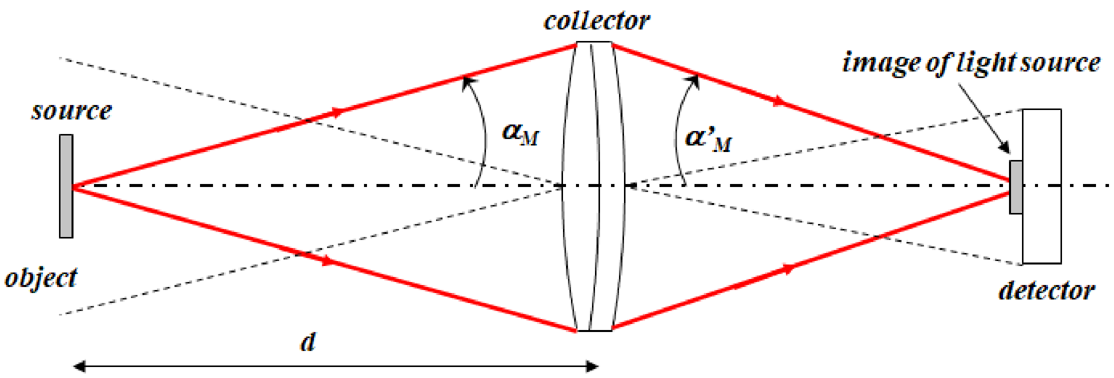


Figure 4.2 : flow collectors

Let $T_{opt}$ and $T$ be the transmission factors of the optics and the surrounding environments, respectively.

The flux incident on the detector is equal to the flux incident on the pupil of the sensor multiplied by the transmission factors.

For a point source of intensity and $I_S$ in the direction of the sensor, we therefore have

$$\Phi_R = T_{opt} T I_S\ \Omega_S$$

where $\Omega_S$ is the solid angle at which the pupil is seen from the source. According to the previous results, we have

$$\Omega_S = 2\pi\left(1 - \cos\alpha_M\right)$$

hence

$$\Phi_R = 2\pi T_{opt} T\ I_S\ \left(1 - \cos\alpha_M\right)$$

In the case where the source is extended with surface $A_S$ and luminance $L_S$, the flux is given by

$$\Phi_R = T_{opt} T L_S G_S$$

where $G_S$ is the object geometric extent from source to pupil. Taking into account the above, we have

$$\Phi_R = \pi T_{opt} T L_S A_S \sin^2 \alpha_M$$

## 3.3. Case of image sensors

An image sensor is more complex than a flow sensor because it must best respect the geometry and photometry of the scene. The object, scene or source is resolved by the sensors which provides a matrix of pixels and the dimension of the object is greater than that of a pixel. The incident flux on each pixel is proportional to the luminance of the image, $L'_S$, and to the geometric extent $G_R$ of reception between optics and pixel. We therefore have

$$\Phi_R = L'_S G_R$$

with, according to the law of conservation of luminance,

$$L'_S = T_{opt} T \frac{n'^2}{n^2} L_S$$

The geometric extent $G_R$can be evaluated in image space (figure 4.3). In the case where the pupil is circular, we have

$$G_R = \pi A_d \sin^2 \alpha'_M$$

In the case where the sensor “looks at infinity”, we also have

$$\sin \alpha'_M = \frac{1}{2N}$$

$N$ being the aperture number of the lens ($N = f'/D, D$diameter of the aperture diaphragm). In this case,

$$G_R = \frac{\pi A_d}{4N^2}$$

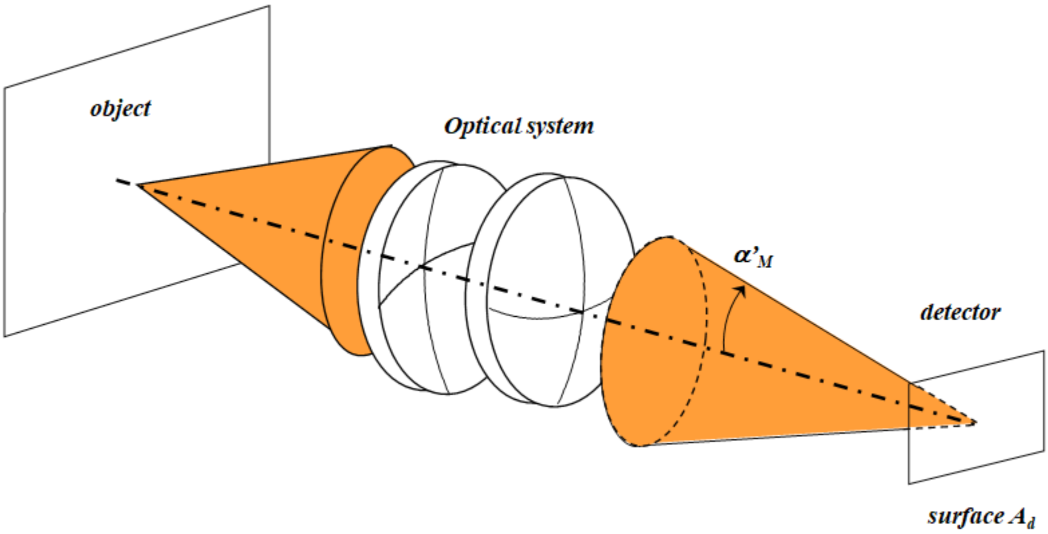


*Figure 4.3 : image sensor*

The flux received in the general case (no observation at infinity) is therefore equal to

$$\Phi_R = \pi T_{opt} T \frac{n'^2}{n^2} L_S A_d \sin^2 \alpha'_M$$

In the case where the image of the object covers the surface $A_d$, the illuminance is given by

$$E_R = \frac{\Phi_R}{A_d} = \pi T_{opt} T \frac{n'^2}{n^2} L_S \sin^2 \alpha'_M$$

If the optical system can be considered thin, that is to say if the pupil magnification between the entrance pupil and the exit pupil of the optical system is close to 1, it is easily shown that

$$\sin \alpha'_M = \frac{1}{2N\left(1-g_y\right)}$$

where $g_y = \frac{n}{n'} \frac{\overline{H'A'}}{\overline{HA}}$ is the transverse magnification between the image plane and the object plane.

The illumination therefore becomes

$$E_R = \frac{\pi \; n'^2 T_{opt} T L_S}{4 n^2 N^2 \left(1-g_y\right)^2}$$

Generally, the lens diaphragm ring is graduated for values of $N$ in geometric progression of ratio $\sqrt{2}$, for example, $N = \{2; 2.8; 4; 5.6; 8; 11,3; 16; 22,6; 32\}$. Thus each increase in the value of $N$ induces a reduction in illumination by a factor of 2.

## 3.4. Case of wide field systems

The previous paragraph dealt with the flux received by a detector and the corresponding illumination in the case where the source objects are located around the axis of the system.

However, in the case of strong fields, greater than $30$, the illumination in the image plane can vary greatly between the center of the field and its edges. Consider the image sensor in Figure 4.4. The image geometric etendue for an object in the vicinity of the axis is given by

$$G_R = \frac{A_d A_{pup}}{p'^2}$$

where $A_{pup}$ is the area of the exit pupil and $p'$is the distance between the exit pupil and the image plane.

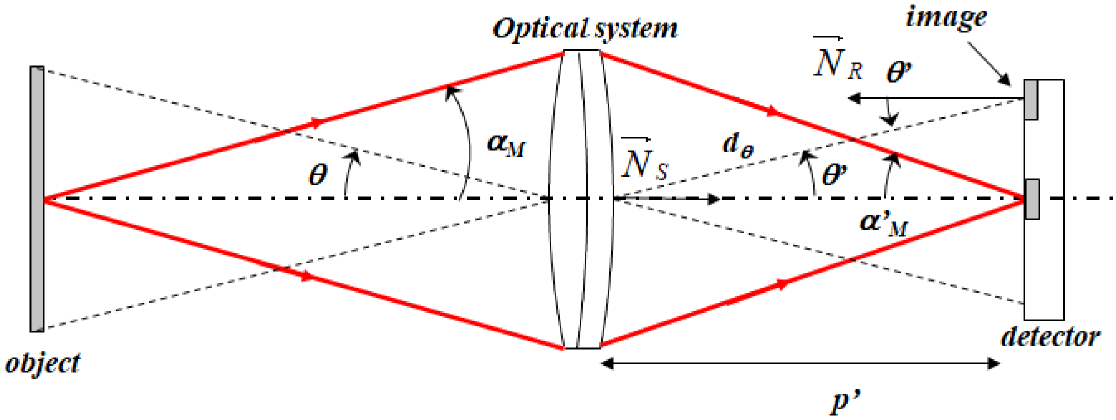


*Figure 4.4 : high field sensor*

In the case where the object is very extended, for a field angle of $\theta$, the geometric etendue of the image now becomes

$$G_R^\theta = \frac{(A_d \cos\theta')(A_{pup}\cos\theta')}{d_\theta^2} = \frac{(A_d \cos\theta')(A_{pup}\cos\theta')}{(p'/\cos\theta')^2} = G_R \cos^4\theta'$$

and the illumination at the edge of the field is now written

$$E_R^\theta = E_R \cos^4\theta' = \frac{\pi\, n'^2 T_{opt} TL_S \cos^4\theta'}{4 n^2 N^2 (1-g_y)^2}$$

It therefore results that the illumination decreases in the field following a law known as "cosine 4 theta".

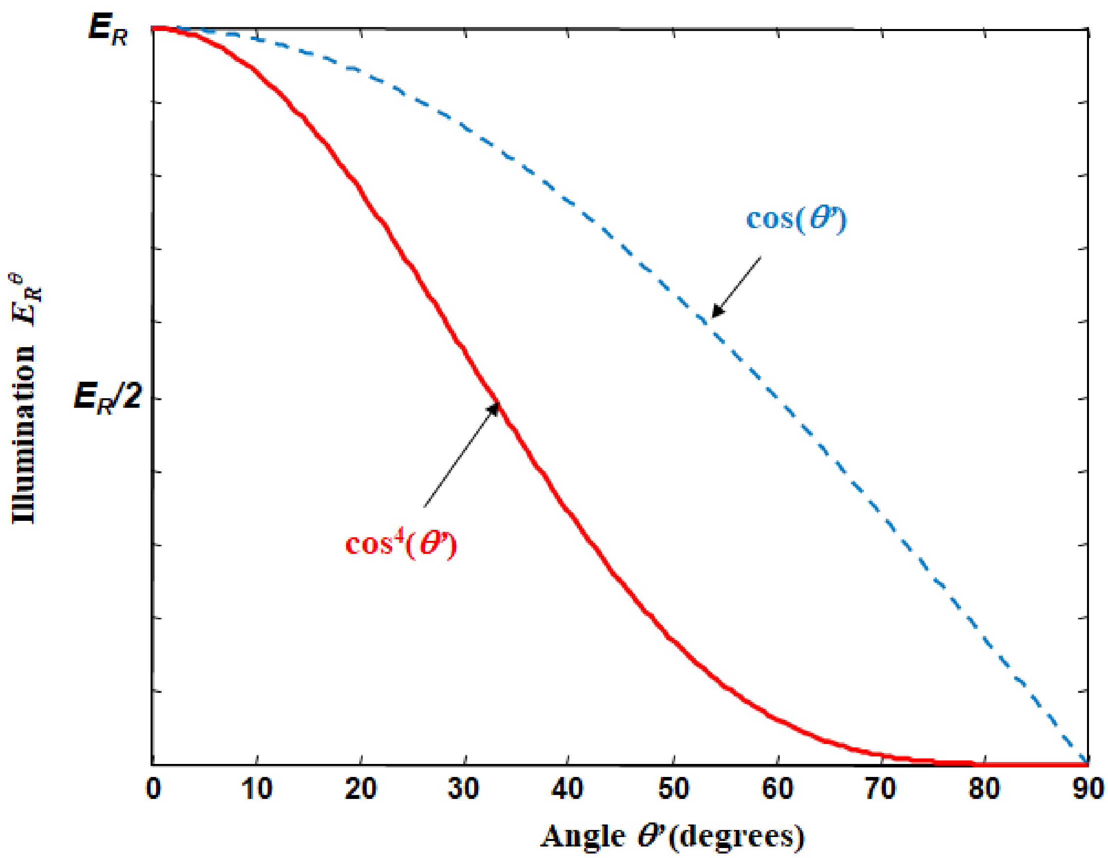


*Figure 4.5 : variation of illuminance in the field*

# 4. Spectral quantities

## 4.1. Spectrum of radiation

Any source has an emission spectrum which can be quasi-monochromatic or more or less extended in wavelengths. In the case where the spectrum is monochromatic, the radiation is qualified by parameters such as flux, luminance, intensity or illuminance. In the case where the radiation is broad spectrum, it is characterized by "spectral quantities" which are the densities of the photometric quantities flux, luminance, intensity or illuminance, per unit of spectral parameter (wavelength, wave number or photon energy). For example if $g$ is the photometric quantity of interest (flux, luminance, intensity or illuminance), and if the radiation emits an elementary contribution $dg$ which comes from the spectral domain between $\lambda$ and $\lambda + d\lambda$, we will say that this radiation has, at wavelength $\lambda$, a spectral quantity $\delta g / \delta \lambda$ defined by

$$dg = \frac{\partial g}{\partial \lambda} d\lambda$$

The following table presents the different spectral quantities.

| Metrics | Name | Notation | Units |
|---|---|---|---|
| Flux | Spectric Flux | $\frac{\partial \Phi}{\partial \lambda}$ | W mm$^{-1}$<br>s$^{-1}$ mm$^{-1}$<br>lm mm$^{-1}$ |
| Intensity | Spectric Intensity | $\frac{\partial I}{\partial \lambda}$ | W sr$^{-1}$ mm$^{-1}$<br>s$^{-1}$ sr$^{-1}$ mm$^{-1}$<br>cd mm$^{-1}$ |
| Luminance | Spectric Luminance | $\frac{\partial L}{\partial \lambda}$ | W m$^{-2}$ sr$^{-1}$ mm$^{-1}$<br>s$^{-1}$ m$^{-2}$ sr$^{-1}$ mm$^{-1}$<br>cd m$^{-2}$ mm$^{-1}$ |
| Illumination | Spectric Illumination | $\frac{\partial E}{\partial \lambda}$ | W m$^{-2}$ mm$^{-1}$<br>s$^{-1}$ m$^{-2}$ mm$^{-1}$<br>lux mm$^{-1}$ |

## 4.2. Properties of surfaces and media

Optical components, surfaces and propagation media, like radiation, have a behavior which depends on the wavelength, depending on their chemical composition, their concentration, their surface state, etc. Thus we define different parameters to characterize this spectral dependence.

The regular reflection spectral factor is defined as the ratio of the values of the luminance of the radiation after $(r)$ and before $(i)$ reflection

$$L_r(\lambda)=\rho_r(\lambda)L_i(\lambda)$$

Similarly, we define the diffuse reflection spectral factor for a Lambertian surface by

$$L_r(\lambda)=\frac{\rho_d(\lambda)}{\pi}E_i(\lambda)$$

and the spectral reflection factor by the ratio between the spectral fluxes after and before reflection

$$\Phi_r(\lambda)=R(\lambda)\Phi_i(\lambda)$$

## 4.3. Conversion between energy units and photonic units

The conversion of a spectral energy flux $(\Phi_e)$ into a spectral photon flux $(\Phi_p)$ is obtained simply from the energy of the photon at the wavelength $\lambda$. Either

$$\frac{\partial \Phi_e}{\partial \lambda} = \frac{hc}{\lambda} \frac{\partial \Phi_p}{\partial \lambda}$$

If a radiation is characterized by its energy spectral distribution between $\lambda_1$ and $\lambda_2$, its photon flux between $\lambda_1$ and $\lambda_2$ is given by

$$\Phi_p = \int_{\lambda_1}^{\lambda_2} \frac{\partial \Phi_p}{\partial \lambda} d\lambda = \frac{1}{hc} \int_{\lambda_1}^{\lambda_2} \lambda \frac{\partial \Phi_e}{\partial \lambda} d\lambda$$

## 4.4. Conversion between energy units and visual units

The observation of radiation by a human being results in a set of visual stimulations interpreted by the brain in terms of colors and levels. The aim of visual photometry is to quantify the levels of visual stimulation of an observer in the face of light radiation. The essential results are based on the one hand on the measurement of the relative spectral sensitivity of the eye and on the other hand on the absolute connection of the light units with the energy units. The human eye is sensitive to radiation with wavelengths between 0.4 and 0.7 $\mu\mathrm{m}$, which corresponds to the so-called "visible" range. On the other hand, sensitivity in this area is not uniform and for the same person it varies depending on environmental conditions, age and health. The sensitivity curves are therefore average values obtained with a large number of people. Thus, we can define 2 standard curves of relative sensitivities of the eye depending on the ambient lighting conditions.

The first curve, V($\lambda$) corresponds to the daytime vision, called photopic, for illuminance levels greater than $10\ \mathrm{cd.m^{-2}}$. This curve involves vision through the cones of the retina. The maximum sensitivity of the eye in day vision is at $\lambda = 0.555 \mu\mathrm{m}$.

The second curve, V'($\lambda$) corresponds to night vision, called scotopic, for illumination levels lower than $0.001\ \mathrm{cd.m^{-2}}$. This curve involves vision through the rods of the retina. The maximum sensitivity of the eye in day vision is at $\lambda = 0.510 \mu\mathrm{m}$.

For intermediate illuminance levels between $0.001\ \mathrm{cd.m^{-2}}$ and $10\ \mathrm{cd.m^{-2}}$ vision is called mesopic and the corresponding curve depends on the level of lighting.

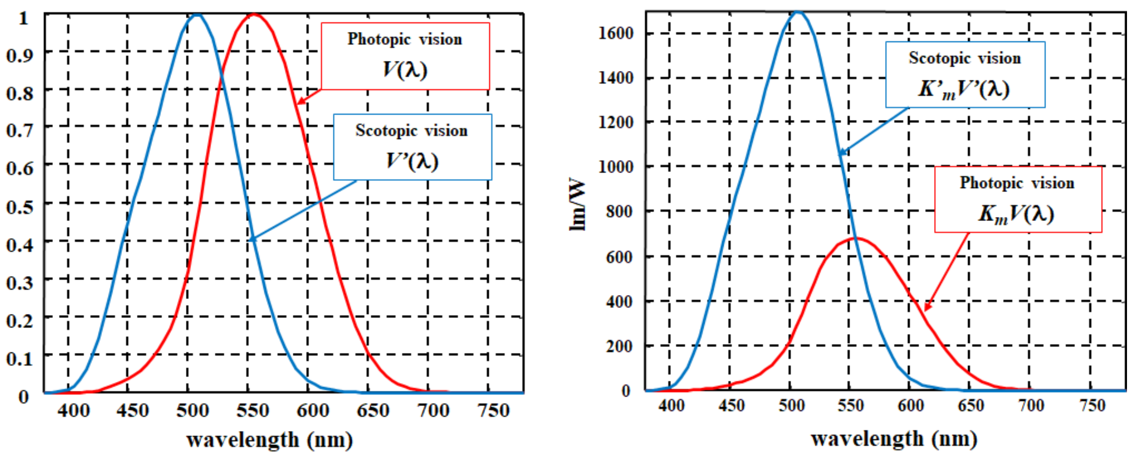


*Figure 5.2 : relative (left) and absolute (right) spectral sensitivity curves of the eye*

The connection between the energy system and the visual system is based on a correspondence in the case of monochromatic sources.

In **photopic** vision, the ratio between the visual flow $\Phi_v$ and the energy flow $\Phi_e$ is given by

$$\Phi_v(\lambda)=K_m V(\lambda)\Phi_e(\lambda)$$

with $K_m = 683 \text{lm.W}^{-1}$ .
In scotopic vision, the ratio between the visual flow $\Phi_v$ and the energy flow $\Phi_e$ is given by

$$\Phi_v(\lambda)=K'_m V'(\lambda)\Phi_e(\lambda)$$

with $K'_m = 1703 \text{ lm.W}^{-1}$.

## 4.5. Black body radiation

Any body receiving radiation absorbs, reflects and transmits this radiation. If we note $\alpha(\lambda)$ the absorption coefficient, $\rho d(\lambda)$ the albedo and $\tau(\lambda)$ the transmission coefficient of this body then, by conservation of energy, we have

$$\alpha(\lambda)+\tau(\lambda)+\rho_d(\lambda)=1$$

For an opaque body, we have $\tau(\lambda) = 0$ and $\alpha(\lambda) + \rho(\lambda) = 0$.
Kirchhoff's law postulates that the emissive power of a body is equal to its absorption coefficient, that is to say that the body re-emits all the radiation it absorbs.
A black body is a body for which the emissive power is constant and independent of the wavelength, i.e.

$$\varepsilon(\lambda)=\rho_d(\lambda)=1$$

Such a body behaves like a Lambertian source, its luminance is independent of the direction of emission.

$$\varepsilon(\lambda)=\rho_d(\lambda)=1$$

Max Planck showed that the spectral luminance of a black body is given by the following relation

$$\frac{\partial L_{CN}}{\partial \lambda}=\frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda kT}\right)-1} \quad \left(W.m^{-2}.\text{sr}^{-1}.m^{-1}\right)$$

where $T$ is the body temperature and $k = 1.380662 \times 10^{-23} \text{ J.K}^{-1}$ is the Boltzmann constant. Figure 5.3 shows the spectral luminance of a black body as a function of its temperature. The warmer the body, the more its maximum luminance tends towards the visible region.

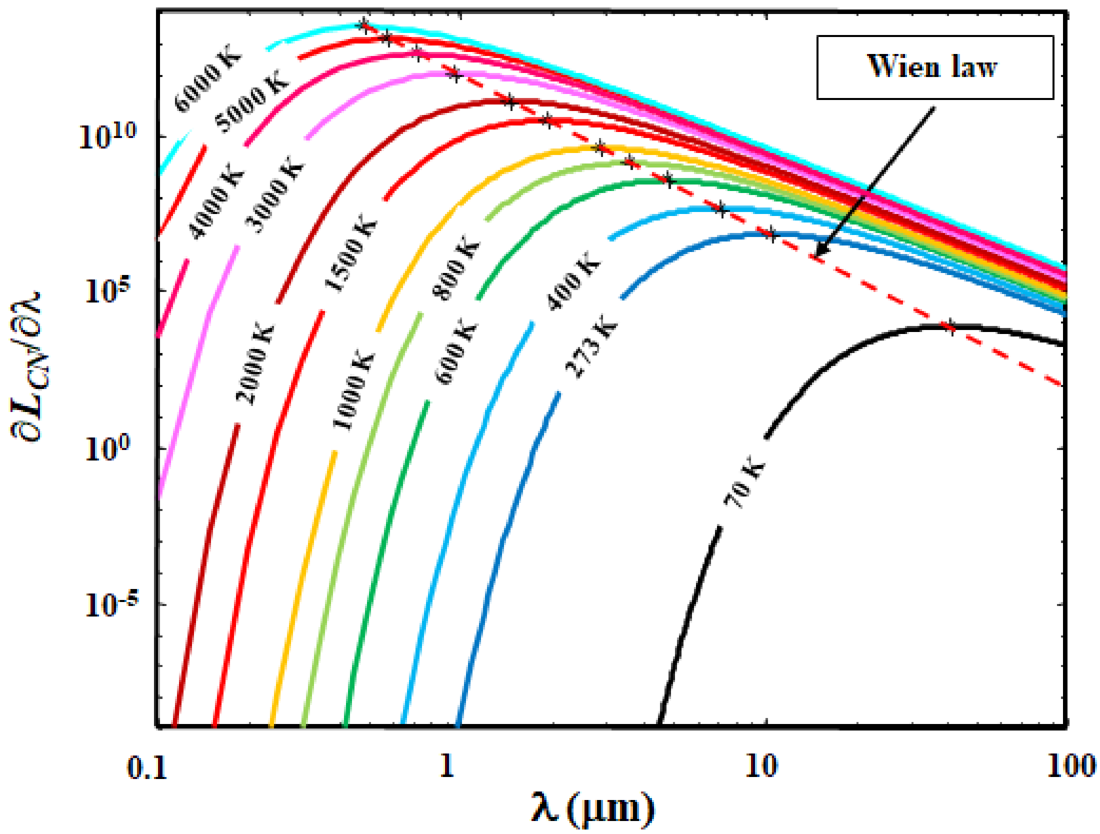


*Figure 5.3 : black body spectral luminance curves*

For example, barbecue embers behave like a black body. It is black to the naked eye, when you blow on it, it heats up and it then appears orange. This is due to the increase in its temperature which induces a shift in its radiation towards the visible.

The maximum spectral luminance is given by Wien's law,

$$\frac{\partial L_{CN}}{\partial \lambda}\bigg|_{\max} = \frac{2\times 5^5 k^5}{h^4 c^3 (e^5-1)} T^5$$

where

$$\frac{2\times 5^5 k^5}{h^4 c^3 (e^5-1)} = 4,095\times 10^{-6}\ W.m^{-2}.\mathrm{sr}^{-1}.m^{-1}.K^{-1}$$

The wavelength of the emission maximum is given by

$$\lambda_{\max} T = \frac{hc}{5k} = 2897,79\ \mu m.K$$

At temperature $T$, a black body has a total luminance which is given by Stephan's law,

$$L_{CN} = \int_0^{+\infty} \frac{\partial L_{CN}}{\partial \lambda} d\lambda = \frac{\pi^5}{15} \frac{2k^4}{h^3 c^2} T^4 \quad (W.m^{-2}.\mathrm{sr}^{-1})$$

with $\sigma = \dfrac{2\pi^2 k^4}{15h^3c^2} = 1.806 \times 10^{-8}\ \mathrm{W{\cdot}sr^{-1} \cdot m^{-2} \cdot K^{-4}}$

The black body emits most of its light in the infrared range. The eye is therefore only sensitive to a very small part of its wavelength spectrum. The luminous efficiency of a blackbody at temperature $T$, is defined as the ratio of its visual luminance to its energetic luminance. In photopic vision, we have :

$$\eta_{CN}(T) = K_m \frac{\int_0^{+\infty} \frac{\partial L_{CN}}{\partial \lambda} V(\lambda)\, d\lambda}{\int_0^{+\infty} \frac{\partial L_{CN}}{\partial \lambda}\, d\lambda} = \frac{K_m}{L_{CN}} \int_0^{+\infty} \frac{\partial L_{CN}}{\partial \lambda} V(\lambda)\, d\lambda$$

The following figure shows the efficiency curve as a function of temperature.

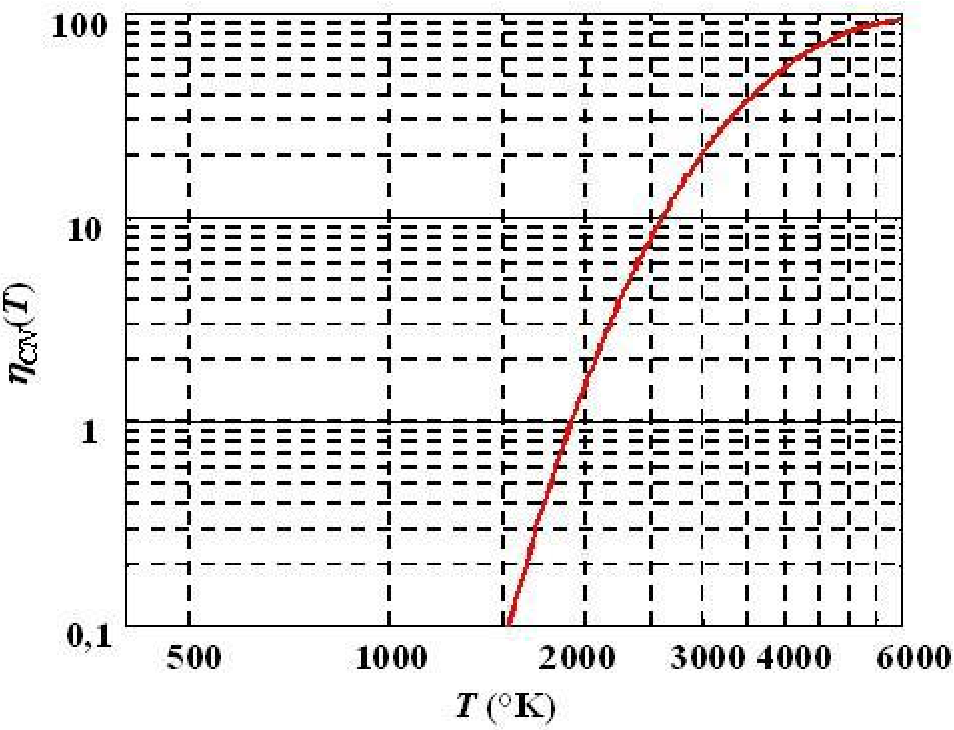


*Figure 5.4 : black body luminous efficiency curves in photopic vision*

The efficiency values as a function of temperature are given in the appendix.

## 4.6. Some orders of magnitude

The following tables give the orders of magnitude of natural light sources such as the Sun and the Moon.

| Source | Luminance | Illumination |
|---|---|---|
| Sun on a frontal plane at Earth level. outside the atmosphere | $2\times10^9$ cd.m$^{-2}$<br>$2\times10^7$ W.sr$^{-1}$.m$^{-2}$ | $1.27\times10^5$ lux<br>$1.35\times10^3$ W.m$^{-2}$ |
| Sun on a frontal plane at sea level under blue sky | $1\times10^4$ cd.m$^{-2}$<br>$1.3\times10^7$ W.sr$^{-1}$.m$^{-2}$ | $1\times10^5$ lux<br>$1\times10^3$ W.m$^{-2}$ |
| Sun on a ground-level front plane. orthotropic albedo diffuser $\rho_d = 0.6$ | $1.9\times10^4$ cd.m$^{-2}$<br>$1.9\times10^2$ W.sr$^{-1}$.m$^{-2}$ | |
| Day with very bright overcast sky | $1\times10^4$ cd.m$^{-2}$ | |
| Day under blue sky at 75° from the Sun | $1.5\times10^3$ cd.m$^{-2}$ | |
| Overcast day | $< 1\times10^3$ cd.m$^{-2}$ | |
| Night with full Moon at zenith | $2\times10^3$ cd.m$^{-2}$ | |
| Night with full Moon at zenith on a ground front plane | | $8\times10^{-2}$ lux |
| Night with full Moon at zenith on an albedo ground plane | $7.6\times10^{-3}$ cd.m$^{-2}$ | |
| Earth's average emission | $6.1\times10^1$ W.sr$^{-1}$.m$^{-2}$ | |
| Tungsten filament lamp with Argon-Azote atmosphere | $7\times10^6$ cd.m$^{-2}$ | |
| Tungsten filament lamp with iodine | $1\times10^7$ cd.m$^{-2}$ | |
| Black body at 2850 K | $1.8\times10^7$ cd.m$^{-2}$ | |

*Figure 5.26 : orders of magnitude of light sources*

The table below gives the orders of magnitude of the laser illuminance limits for the eye and the skin with the use of continuous lasers. The exposure time limit is $\Delta t$.

| Laser | $\lambda$ (nm) | Eye | Skin |
|---|---|---|---|
| Helium-Cadmium<br>Helium-Neon<br>Argon<br>Krypton<br>YAG frequency doubled | 441.5<br>632.8<br>488 - 514.5<br>647.1<br>532 | 2.5 mW.cm$^{-2}$ for $\Delta t = 0.25$s<br>10 mJ.cm$^{-2}$ for $\Delta t = 10$ to 10000s<br>10 µW.cm$^{-2}$ for $\Delta t > 10000$s | 0.2 W.cm$^{-2}$ for $\Delta t > 10$s |
| $YAG{:}Nd^{3+}$<br>AsGa | 1064<br>905 | 2.84 mW.cm$^{-2}$ for $\Delta t > 100$s<br>0.57 mW.cm$^{-2}$ for $\Delta t > 100$s | 1 W.cm$^{-2}$ for $\Delta t > 10$s<br>0.5 W.cm$^{-2}$ for $\Delta t > 10$s |
| Helium-Cadmium<br>Azote | 325<br>337.1 | 1 J.cm$^{-2}$ for $\Delta t > 10$s | 1 J.cm$^{-2}$ for $\Delta t = 10$ to 1000s<br>1 mW.cm$^{-2}$ for $\Delta t > 1000$s |
| $CO_2$<br>and other lasers<br>1.4 to 1000µm | 10.6 µm | 0.1 W.cm$^{-2}$ for $\Delta t > 10$s | 0.1 W.cm$^{-2}$ for $\Delta t > 10$s |

*Figure 5.27 : orders of magnitude of limiting illumination for the use of continuous lasers*

The table below gives the orders of magnitude of the laser illuminance limits for the eye and the skin with the use of pulsed lasers. The exposure time limit is $\Delta t$.

| Laser | λ (nm) | Pulse duration | Eye | Skin |
|---|---|---|---|---|
| Rubis (relaxed) | 694.3 | ≅ 1 ms | $10^{-5}$ J.cm$^{-2}$ | 0.2 J.cm$^{-2}$ |
| Rubis (Q-switch) | 694.3 | 5 to 100 ns | $5 \times 10^{-7}$ J.cm$^{-2}$ | 0.02 J.cm$^{-2}$ |
| Colorant (R6G) | 500 – 700 | 10 to 20 µs | $5 \times 10^{-7}$ J.cm$^{-2}$ | 0.07 J.cm$^{-2}$ |
| YAG :Nd (relaxed) | 1064 | ≅ 1 ms | $5 \times 10^{-5}$ J.cm$^{-2}$ | 1.0 J.cm$^{-2}$ |
| YAG :Nd (Q-switch) | 1064 | 5 to 100 ns | $5 \times 10^{-6}$ J.cm$^{-2}$ | 0.1 J.cm$^{-2}$ |

*Figure 5.28 : orders of magnitude of limiting illumination for the use of pulsed lasers*

## 4.7. Appendix: Visual efficiency of the black body

| T (°K) | Efficiency | T (°K) | Efficiency | T (°K) | Efficiency | T (°K) | Efficiency |
|---|---|---|---|---|---|---|---|
| 1500.00 | 0.09 | 2630.04 | 10.73 | 3760.09 | 46.63 | 4890.13 | 79.26 |
| 1520.18 | 0.10 | 2650.22 | 11.19 | 3780.27 | 47.33 | 4910.31 | 79.68 |
| 1540.36 | 0.12 | 2670.40 | 11.67 | 3800.45 | 48.03 | 4930.49 | 80.09 |
| 1560.54 | 0.14 | 2690.58 | 12.15 | 3820.63 | 48.72 | 4950.67 | 80.50 |
| 1580.72 | 0.16 | 2710.76 | 12.65 | 3840.81 | 49.42 | 4970.85 | 80.90 |
| 1600.90 | 0.18 | 2730.94 | 13.15 | 3860.99 | 50.11 | 4991.03 | 81.30 |
| 1621.08 | 0.21 | 2751.12 | 13.66 | 3881.17 | 50.80 | 5011.21 | 81.69 |
| 1641.26 | 0.24 | 2771.30 | 14.19 | 3901.35 | 51.48 | 5031.39 | 82.08 |
| 1661.44 | 0.27 | 2791.48 | 14.72 | 3921.52 | 52.17 | 5051.57 | 82.45 |
| 1681.61 | 0.31 | 2811.66 | 15.26 | 3941.70 | 52.85 | 5071.75 | 82.83 |
| 1701.79 | 0.35 | 2831.84 | 15.81 | 3961.88 | 53.52 | 5091.93 | 83.19 |
| 1721.97 | 0.40 | 2852.02 | 16.37 | 3982.06 | 54.19 | 5112.11 | 83.55 |
| 1742.15 | 0.45 | 2872.20 | 16.93 | 4002.24 | 54.86 | 5132.29 | 83.91 |
| 1762.33 | 0.51 | 2892.38 | 17.51 | 4022.42 | 55.53 | 5152.47 | 84.25 |
| 1782.51 | 0.57 | 2912.56 | 18.09 | 4042.60 | 56.19 | 5172.65 | 84.60 |
| 1802.69 | 0.63 | 2932.74 | 18.68 | 4062.78 | 56.84 | 5192.83 | 84.93 |
| 1822.87 | 0.70 | 2952.91 | 19.28 | 4082.96 | 57.50 | 5213.00 | 85.26 |
| 1843.05 | 0.78 | 2973.09 | 19.89 | 4103.14 | 58.15 | 5233.18 | 85.58 |
| 1863.23 | 0.86 | 2993.27 | 20.50 | 4123.32 | 58.79 | 5253.36 | 85.90 |
| 1883.41 | 0.95 | 3013.45 | 21.12 | 4143.50 | 59.43 | 5273.54 | 86.21 |
| 1903.59 | 1.05 | 3033.63 | 21.75 | 4163.68 | 60.06 | 5293.72 | 86.52 |
| 1923.77 | 1.15 | 3053.81 | 22.38 | 4183.86 | 60.69 | 5313.90 | 86.82 |
| 1943.95 | 1.26 | 3073.99 | 23.02 | 4204.04 | 61.32 | 5334.08 | 87.11 |
| 1964.13 | 1.38 | 3094.17 | 23.67 | 4224.22 | 61.94 | 5354.26 | 87.40 |
| 1984.30 | 1.51 | 3114.35 | 24.32 | 4244.39 | 62.56 | 5374.44 | 87.68 |
| 2004.48 | 1.64 | 3134.53 | 24.97 | 4264.57 | 63.17 | 5394.62 | 87.96 |
| 2024.66 | 1.78 | 3154.71 | 25.63 | 4284.75 | 63.77 | 5414.80 | 88.23 |
| 2044.84 | 1.93 | 3174.89 | 26.30 | 4304.93 | 64.37 | 5434.98 | 88.50 |
| 2065.02 | 2.09 | 3195.07 | 26.97 | 4325.11 | 64.97 | 5455.16 | 88.75 |
| 2085.20 | 2.26 | 3215.25 | 27.64 | 4345.29 | 65.56 | 5475.34 | 89.01 |
| 2105.38 | 2.44 | 3235.43 | 28.32 | 4365.47 | 66.14 | 5495.52 | 89.26 |
| 2125.56 | 2.63 | 3255.61 | 29.01 | 4385.65 | 66.72 | 5515.70 | 89.50 |
| 2145.74 | 2.83 | 3275.78 | 29.69 | 4405.83 | 67.29 | 5535.87 | 89.74 |
| 2165.92 | 3.03 | 3295.96 | 30.38 | 4426.01 | 67.86 | 5556.05 | 89.97 |
| 2186.10 | 3.25 | 3316.14 | 31.08 | 4446.19 | 68.42 | 5576.23 | 90.19 |
| 2206.28 | 3.48 | 3336.32 | 31.77 | 4466.37 | 68.98 | 5596.41 | 90.42 |
| 2226.46 | 3.71 | 3356.50 | 32.47 | 4486.55 | 69.53 | 5616.59 | 90.63 |
| 2246.64 | 3.96 | 3376.68 | 33.17 | 4506.73 | 70.07 | 5636.77 | 90.84 |
| 2266.82 | 4.22 | 3396.86 | 33.87 | 4526.91 | 70.61 | 5656.95 | 91.05 |
| 2287.00 | 4.49 | 3417.04 | 34.58 | 4547.09 | 71.14 | 5677.13 | 91.25 |
| 2307.17 | 4.77 | 3437.22 | 35.29 | 4567.26 | 71.67 | 5697.31 | 91.44 |
| 2327.35 | 5.06 | 3457.40 | 35.99 | 4587.44 | 72.19 | 5717.49 | 91.63 |
| 2347.53 | 5.36 | 3477.58 | 36.70 | 4607.62 | 72.70 | 5737.67 | 91.81 |
| 2367.71 | 5.67 | 3497.76 | 37.41 | 4627.80 | 73.21 | 5757.85 | 91.99 |
| 2387.89 | 6.00 | 3517.94 | 38.12 | 4647.98 | 73.71 | 5778.03 | 92.17 |
| 2408.07 | 6.33 | 3538.12 | 38.83 | 4668.16 | 74.21 | 5798.21 | 92.34 |
| 2428.25 | 6.68 | 3558.30 | 39.55 | 4688.34 | 74.70 | 5818.39 | 92.50 |
| 2448.43 | 7.03 | 3578.48 | 40.26 | 4708.52 | 75.18 | 5838.57 | 92.66 |
| 2468.61 | 7.40 | 3598.65 | 40.97 | 4728.70 | 75.66 | 5858.74 | 92.81 |
| 2488.79 | 7.78 | 3618.83 | 41.68 | 4748.88 | 76.13 | 5878.92 | 92.96 |
| 2508.97 | 8.17 | 3639.01 | 42.39 | 4769.06 | 76.59 | 5899.10 | 93.11 |
| 2529.15 | 8.57 | 3659.19 | 43.10 | 4789.24 | 77.05 | 5919.28 | 93.25 |
| 2549.33 | 8.98 | 3679.37 | 43.81 | 4809.42 | 77.51 | 5939.46 | 93.38 |
| 2569.51 | 9.40 | 3699.55 | 44.52 | 4829.60 | 77.95 | 5959.64 | 93.51 |
| 2589.69 | 9.83 | 3719.73 | 45.22 | 4849.78 | 78.39 | 5979.82 | 93.64 |
| 2609.87 | 10.28 | 3739.91 | 45.93 | 4869.96 | 78.83 | 6000.00 | 93.76 |

*Figure 6.1 : visual efficiency of the black body in photopic vision*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2347,53 | 5,36 | 3477,58 | 36,7 | 4607,62 | 72,7 | 5737,67 | 91,81 |
| 2367,71 | 5,67 | 3497,76 | 37,41 | 4627,8 | 73,21 | 5757,85 | 91,99 |
| 2387,89 | 6 | 3517,94 | 38,12 | 4647,98 | 73,71 | 5778,03 | 92,17 |
| 2408,07 | 6,33 | 3538,12 | 38,83 | 4668,16 | 74,21 | 5798,21 | 92,34 |
| 2428,25 | 6,68 | 3558,3 | 39,55 | 4688,34 | 74,7 | 5818,39 | 92,5 |
| 2448,43 | 7,03 | 3578,48 | 40,26 | 4708,52 | 75,18 | 5838,57 | 92,66 |
| 2468,61 | 7,4 | 3598,65 | 40,97 | 4728,7 | 75,66 | 5858,74 | 92,81 |
| 2488,79 | 7,78 | 3618,83 | 41,68 | 4748,88 | 76,13 | 5878,92 | 92,96 |
| 2508,97 | 8,17 | 3639,01 | 42,39 | 4769,06 | 76,59 | 5899,1 | 93,11 |
| 2529,15 | 8,57 | 3659,19 | 43,1 | 4789,24 | 77,05 | 5919,28 | 93,25 |
| 2549,33 | 8,98 | 3679,37 | 43,81 | 4809,42 | 77,51 | 5939,46 | 93,38 |
| 2569,51 | 9,4 | 3699,55 | 44,52 | 4829,6 | 77,95 | 5959,64 | 93,51 |
| 2589,69 | 9,83 | 3719,73 | 45,22 | 4849,78 | 78,39 | 5979,82 | 93,64 |
| 2609,87 | 10,28 | 3739,91 | 45,93 | 4869,96 | 78,83 | 6000 | 93,76 |

*Figure 6.2 : visual efficiency of the black body in photopic vision (continued)*