

Fractional Fourier analyses to describe light pulses

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I.Présentation

Module :

Fractional Fourier analyses to describe light pulses

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Résumé :

This chapter explains how fractional Fourier analyses can be potentially used to describe the propagation of light pulses in dispersive fields and characterize ultrashort light pulses.

Mots-clés :

Fractional Fourier, light pulses

Pré-requis :

-

Objectif(s) pédagogique(s) :

-

Plan du cours :

- Introduction
- The propagation of light waves in dispersive fields and fractional Fourier transforms
- Characterization of ultrashort light pulses by fractional Fourier analyses
- Conclusion

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II. Lesson

In previous chapters, S. Coëtrnellec developed the properties of fractional Fourier transforms [1 [[01]],2 [[02]],3 [[03]],4 [[04]]], the techniques to implement them digitally, and their applications in digital holography. The hologram of an object is obtained from the diffraction figure given by the object when lit by coherent waves. Holograms can be digitally reconstructed through fractional Fourier transforms since fractional Fourier transforms act as a "refocalisation" on the object, which allows us to find out its shape precisely. The property is based on the fact that fractional Fourier transforms can be considered as Fresnel integrals which accurately describe the propagation of light waves during a diffraction process [5 [[05]]]. The isomorphism between space optics and time optics is well known. Therefore, the same equation describes either the diffraction of waves in free space (space process), or the propagation of light pulses in a linear dispersive field (time process). You only have to replace the diffraction term by a dispersion term and the transverse space coordinate by time. If fractional Fourier transforms are a way to analyze diffraction figures, they can similarly describe chromatic dispersion phenomena [6 [[06]]]. More especially, they should also be a way to describe dispersion-induced phenomena, namely the apparition of frequency drifts within light pulses. Thus, we broach a field very different from digital holography: the characterization of ultrashort pulses [7 [[07]]].

The chapter aims at showing the potential use of fractional Fourier transforms to describe the propagation of light pulses in dispersive fields (*The propagation of light waves in dispersive fields and fractional Fourier transforms*) and characterize ultra-short light pulses (*Characterization of ultrashort light pulses by fractional Fourier analyses*).

1. The propagation of light waves in dispersive fields and fractional Fourier transforms

1.1. Fractional Fourier transforms

If we consider $U(T)$ as a function depending on time T , the fractional Fourier transform $\mathcal{F}_a[U(T)](T_a)$ will be defined as follows [2 [[02]]]:

$$\mathcal{F}[U(T)](T_a) = C(a) \exp\left(i \frac{\pi}{\epsilon^2 \tan(a\pi/2)} T_a^2\right) \int_{-\infty}^{+\infty} U(T) \exp\left(i \frac{\pi}{\epsilon^2 \tan(a\pi/2)} T_a T\right) \exp\left(-i 2\pi \frac{T_a T}{\epsilon^2 \sin(a\pi/2)}\right) dT$$

where:

$$C(a) = \frac{\exp(-i(\pi/4 \operatorname{sign}(\sin(a\pi/2))) - a\pi/4)}{|\epsilon^2 \sin(a\pi/2)|^{1/2}}$$

$C(a)$ the energy conservation in the fractional field, a is the fractional order satisfying $0 < a < 2$, T_a the conjugated variable of T in the fractional field, and ϵ a parameter which has the dimension of time T . Practically speaking, the parameter is chosen at the beginning of the analysis to adapt itself to the rough estimates of the data found during the analysis.

Remarque

Fractional Fourier transforms are an operation with linear frequency drift functions.

1.2. The propagation equation of light pulses in a linear dispersive field

When dispersion prevails (no significant non-linear effects), the propagation of light waves in a linear dispersive field (for example an optical fiber or a glass) is described by the following propagation equation [8 [[08]]]:

$$i \frac{\partial U}{\partial z} = \frac{1}{2} \beta_2 \frac{\partial^2 U}{\partial T^2}$$

In our notation, $U(z, T)$ is the normalized amplitude.

$$U(z, T) = \frac{1}{\sqrt{P_0}} e^{\frac{\alpha z}{2}} A(z, T)$$

where $A(z, T)$ is the slowly variable amplitude of the light pulse envelope, P_0 the peak power of the pulse, α the linear absorption coefficient in the dispersive field, and β_2 the dispersion parameter which is expressed in ps^2m^{-1} . Time T is expressed in the reference table whose propagation follows the group speed of the pulse such that $T = t - z/v_g$, where t is the real time and z the propagation axis.

Remarque

This equation can for instance describe the propagation of light pulses whose peak power remains mild in optical fibers.

1.3. Resolution using fractional Fourier transforms

The standard Fourier transform of $U(z, T)$ is written $\tilde{U}(z, T_a)$. Generally speaking, this is the order 1 fractional Fourier transform of $U(z, T)$. So, we can write:

$$\tilde{U}(z, T_a) = \mathcal{F}_1[U(z, T)](z, T_a)$$

The propagation equation:

$$i \frac{\partial U}{\partial z} = \frac{1}{2} \beta_2 \frac{\partial^2 U}{\partial T^2}$$

becomes in the standard Fourier field:

$$i \frac{\partial \tilde{U}(z, T_a)}{\partial z} = \left(i 2 \pi \frac{T_a}{\epsilon^2} \right)^2 \frac{1}{2} \beta_2 \tilde{U}(z, T_a)$$

whose solution is given by:

where $\tilde{U}(0, T_a)$ is the standard Fourier transform of the field amplitude at the entrance of the dispersive field ($z = 0$), given by: $\tilde{U}(0, T_a) = \mathcal{F}_1[U(0, T)](0, T_a)$.

As $U(z, T)$ is the inverse Fourier transform of $\tilde{U}(z, T_a)$, we obtain the general solution:

$$U(z, T) = \mathcal{F}_{-1}[\tilde{U}(0, T_a) \exp(i 2 \pi^2 \frac{\beta_2 z}{\epsilon^2} T_a^2)]$$

which can also be written as:

$$U(z, T) = C(-\pi/2) \int_{-\infty}^{+\infty} \tilde{U}(0, T_a) \exp\left(i 2 \pi^2 \frac{\beta_2 z}{\varepsilon^4} T_a^2\right) \exp\left(\frac{i 2 \pi T T_a}{\varepsilon^2}\right) dT_a$$

We define:

$$\frac{1}{\tan(a \pi/2)} = 2 \pi \frac{\beta_2 z}{\varepsilon^2}$$

and the new variable s by:

$$\frac{s}{\sin(a \pi/2)} = \frac{T}{\sin(-\pi/2)}$$

The previous equation thus becomes:

$$\hat{U}(z, s) = C(-\pi/2) \int_{-\infty}^{+\infty} \tilde{U}(0, T_a) \exp\left(i \frac{\pi}{\varepsilon^2 \tan(a \pi/2)} T_a^2\right) \exp\left(-i \frac{2 \pi s T_a}{\varepsilon^2 \sin(a \pi/2)}\right) dT_a$$

where

$$\hat{U}(z, s) = U\left(z, s \frac{\sin(-\pi/2)}{\sin(a \pi/2)}\right)$$

Therefore, a fractional Fourier transform appears and, using the composition law of fractional Fourier transforms, we directly obtain:

$$\hat{U}(z, s) = \frac{C(-\pi/2)}{C(a \pi/2)} \exp\left(i \frac{\pi}{\varepsilon^2 \tan(a \pi/2)} s^2\right) \mathcal{F}_{a+1}[U(0, T)](s)$$

Attention

In short, the pulse amplitude can be determined at any time using the fractional Fourier transform of the incident pulse [9 [[09]]]. The required fractional order is directly connected to the distance covered in the dispersive field, i.e. $\beta = a + 1$ with:

$$a = \frac{2}{\pi} \operatorname{atan}\left(\frac{\varepsilon^2}{2 \pi \beta_2 z}\right)$$

1.4. Example

For example, let us consider a time Gaussian pulse. The amplitude of its electric field can be written:

$$U(0, T) = \exp\left(-\frac{T^2}{2 T_0^2}\right) \exp\left(-\frac{i C T^2}{2 T_0^2}\right)$$

where T_0 is the pulse duration in $1/e$. Parameter C controls the initial frequency drift of the pulse. Figure 1 shows the order of the transformation required based on the distance covered in the dispersive field. We can note that the line is a monotonous function: so there is no ambiguity regarding the distance covered as far as the order of the applied transformation is concerned.

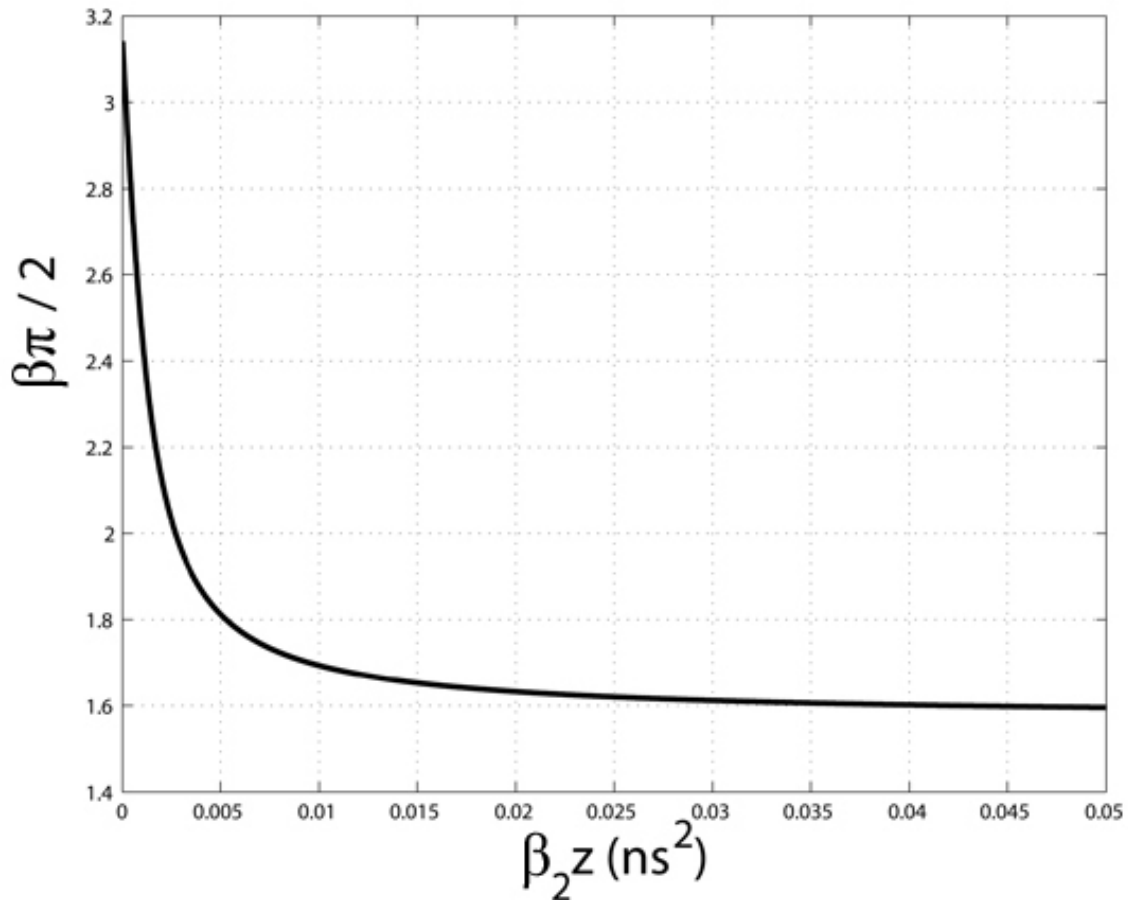


Fig.1 - Fractional order according to the distance covered

Figure 2 then shows the shape of the pulse at the entrance of the dispersive field and then after a distance in the field z which satisfies: $z/L_D = 2$ or 4 (respectively lines ii et iii). Distance L_D is the dispersion length defined by $L_D = T_0^2/|\beta_2|$. These lines were traced using fractional calculation. The fractional orders which correspond to propagation distances $z/L_D = 2$ and $z/L_D = 4$ are respectively $\beta = 1.897$ and $\beta = 1.802$. In this case, the duration of the incident pulse is $T_0 = 10$ ps, and the incident pulse does not have the requisite initial frequency drift: $C = 0$. A gradual widening of the pulse can be noted and the results are in perfect agreement with the standard methods of calculation.

Remarque

We can note that, for a Gaussian pulse, the amplitude is expressed under an analytical form in this particular case, which facilitates the comparison with the results obtained by fractional Fourier transforms and presented here.

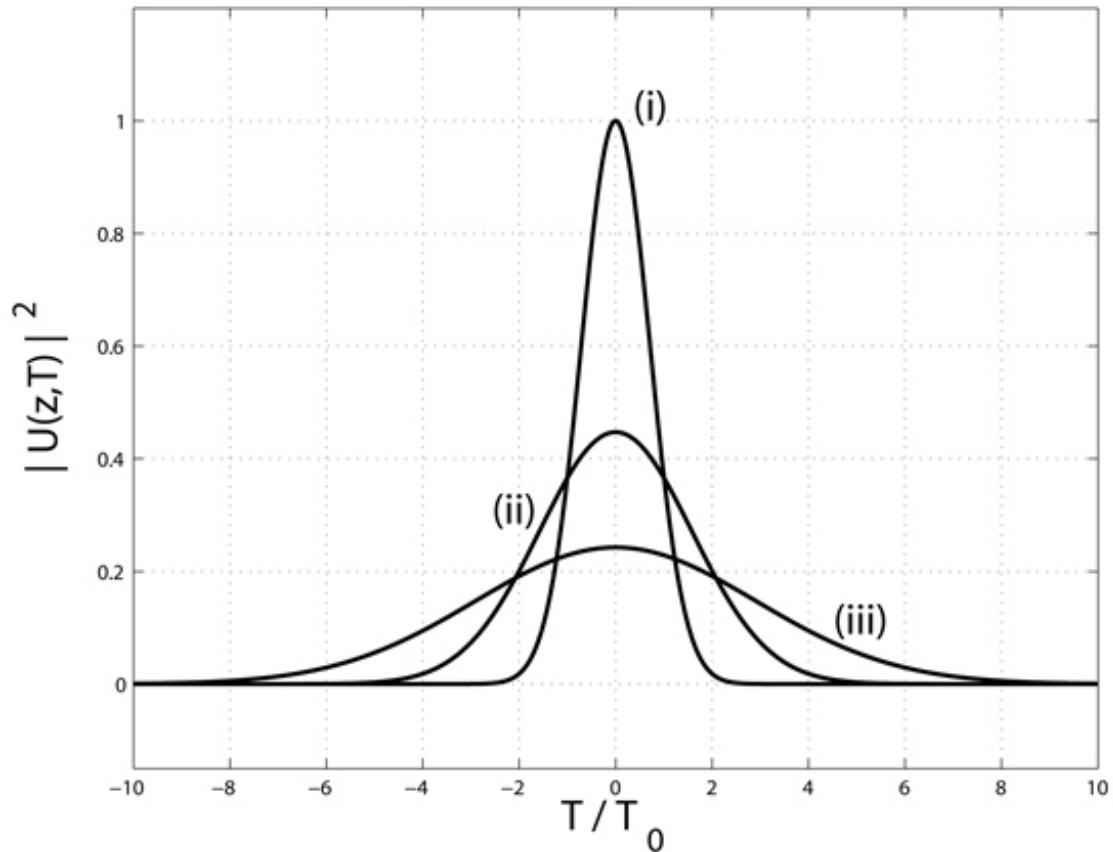


Fig. 2 Widening of a light pulse under the dispersion effect

We will now consider a pulse having an initial frequency drift $C = -1$. It is well known in this case that dispersion leads to a pulse compression in the first part of the dispersive field and to its stretch in the second (for $\beta_2 > 0$). Figure 3 shows the compression rate T_1/T_0 observed during the propagation (T_1 is the duration of the pulse after its propagation in the dispersive field) according to the fractional order applied. Compression is optimal for an order β_{opt} rigorously given by the relation:

$$\beta_{opt} = -\frac{2}{\pi} \operatorname{arccot} \left[\frac{2\pi C T_0^2}{1+C^2 \epsilon^2} \right]$$

where arccot is the inverse function of the cotangent function. In the present case, we obtain $\beta_{opt} = 1.974$ which corresponds to a propagation in the dispersive field (optical fibers) over a distance z such that $\beta_2 z = 5.10^{-5} \text{ ns}^2$. Once again, this relation is in agreement with standard methods of calculation [9 [[09]]].

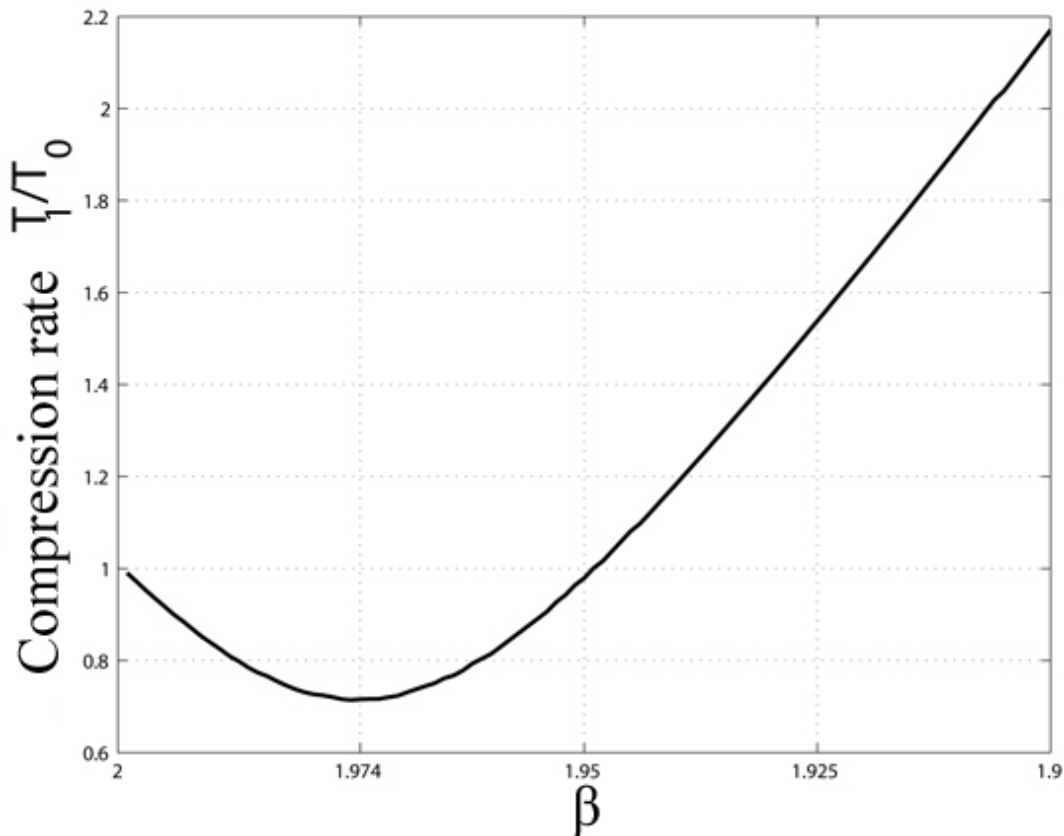


Fig. 3 - Compression of an initially chirp signal

1.5. Propagation of partially coherent beams in a linear dispersive field

We will see in this part that the formalism of fractional Fourier transforms also applies very well to describe the propagation of partially coherent beams in a dispersive field. Such beams are described with statistical tools, in particular by their correlation function of second order $\Gamma(z_1, z_2, T_1, T_2)$ defined by:

$$\Gamma(z_1, z_2, T_1, T_2) = \langle U^*(z_1, T_1) U(z_2, T_2) \rangle$$

where $U(z, T)$ represents the amplitude of the electrical field in z at time T , $*$ the conjugated complex, and $\langle \rangle$ the means over the different representations of the field.

From the propagation equation in a dispersive field:

$$i \frac{\partial U}{\partial z} = \frac{1}{2} \beta_2 \frac{\partial^2 U}{\partial T^2}$$

we show that the propagation of the correlation function follows the two following equations:

$$-i \frac{\partial \Gamma}{\partial z_1}(z_1, z_2, T_1, T_2) = \frac{1}{2} \beta_2 \frac{\partial^2 \Gamma}{\partial T_1^2}(z_1, z_2, T_1, T_2)$$

$$i \frac{\partial \Gamma}{\partial z_2}(z_1, z_2, T_1, T_2) = \frac{1}{2} \beta_2 \frac{\partial^2 \Gamma}{\partial T_2^2}(z_1, z_2, T_1, T_2)$$

We can note:

$$\frac{1}{\tan(a_1 \pi/2)} = -2 \pi \frac{\beta_2 z_1}{\epsilon^2}$$

and

$$\frac{1}{\tan(a_2 \pi/2)} = 2 \pi \frac{\beta_2 z_2}{\epsilon^2}$$

We also define two new variables s_1 and s_2 by the following scale transformations:

$$s_1 = T_1 \frac{\sin(a_1 \pi/2)}{\sin(-\pi/2)}$$

and

$$s_2 = T_2 \frac{\sin(a_2 \pi/2)}{\sin(-\pi/2)}$$

then the standardized function:

$$\hat{\Gamma}(z_1, z_2, s_1, s_2) = \Gamma\left(z_1, z_2, s_1 \frac{\sin(-\pi/2)}{\sin(a_1 \pi/2)}, s_2 \frac{\sin(-\pi/2)}{\sin(a_2 \pi/2)}\right)$$

So, we can write:

$$\hat{\Gamma}(z_1, z_2, s_1, s_2) = \frac{C(-\pi/2)^2}{C(a_1 \pi/2)C(a_2 \pi/2)} \exp(-i \frac{\pi}{\epsilon^2 \tan(a_1 \pi/2)} s_1^2) \exp(-i \frac{\pi}{\epsilon^2 \tan(a_2 \pi/2)} s_2^2) \mathcal{F}_{a_1+1, a_2+1}[\Gamma(0, 0, T_1, T_2)](S_1, S_2)$$

Attention

The correlation function can be determined by a simple two-dimensional fractional Fourier transform whose orders are simply connected to the distance covered in the dispersive field [10 [[10]]].

Exemple

We can consider the partially coherent pulses described by a Gauss-Schell model. The correlation function of second order at the entrance of the dispersive field can be written as:

$$\Gamma(0, 0, T_1, T_2) = G_0 \exp(-(T_1^2 + T_2^2)/2 \tau^2) * \exp(-(T_1 - T_2)^2/4 \tau_c^2)$$

In this model, τ represents the duration of pulses while τ_c represents the "correlation length" within a pulse. We will take here $\tau = 10\text{ps}$ and $\tau_c = 3\text{ps}$. Figure 4 shows $\Gamma(0, 0, T_1, T_2)$ in the plane (T_1, T_2) . Figure 5 represents $\Gamma(z, z, T_1, T_2)$ in the same plane after a propagation z such that $\beta_2 z = 50\text{ps}^2$. This figure shows a widening of both pulse durations and correlation lengths.

In this case, it has been drawn from a fractional formalism. It is in perfect agreement with standard calculation [10 [[10]]].

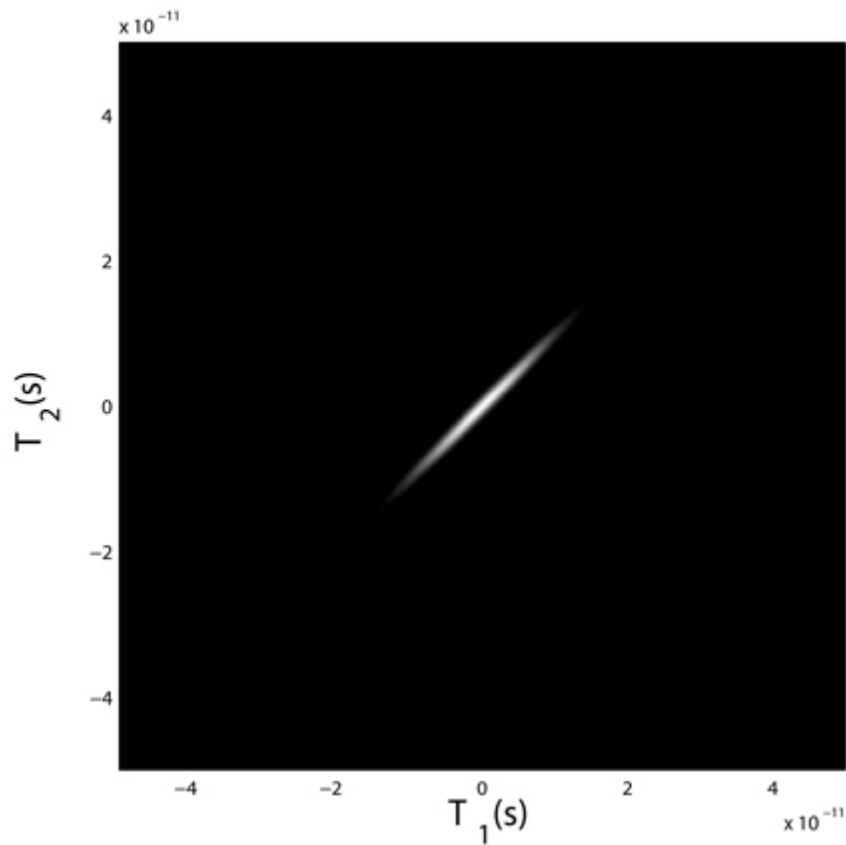


Fig. 4 - Correlation function of order 2

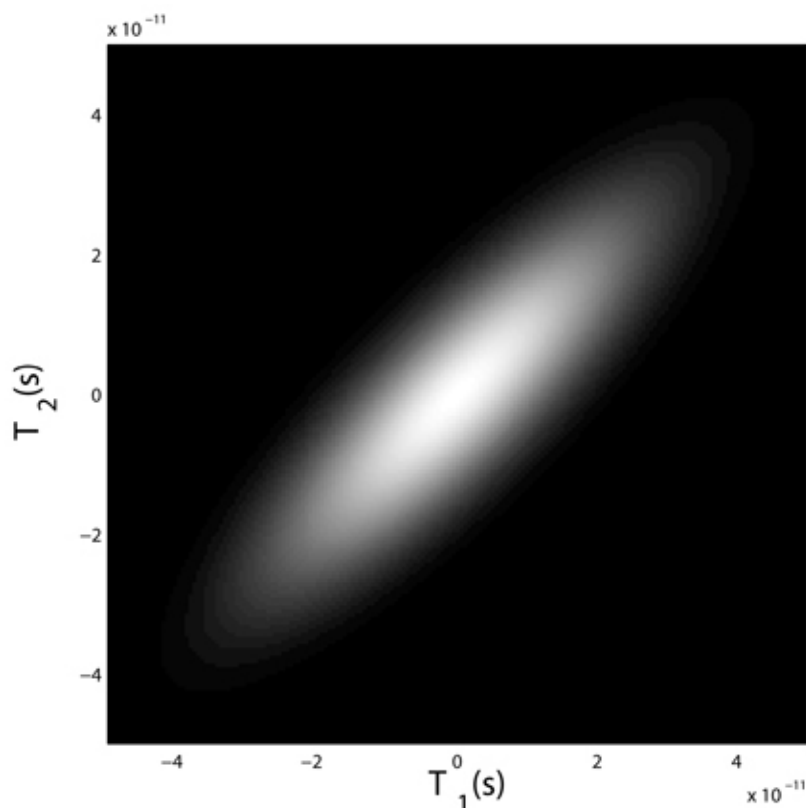


Fig. 5 - Correlation function of order 2 after propagation

2. Characterization of ultrashort light pulses by fractional Fourier analyses

Since the 1980s, the use of femtosecond lasers has become widespread in fields as diverse as physics, chemistry, or biology. Such lasers are indeed excellent tools to conduct dynamic studies with extremely short time scales, but also to generate non-linear effects which are very significant because of the daunting peak power of the emitted pulses. Ultrashort light pulses will inevitably present a broad spectrum: the broader the spectrum the shorter the pulse. As a consequence, the propagation of femtosecond pulses in a field can give rise to complex phenomena, as each spectral component is not affected in the same way by propagation. For example, the properties of prisms are well-known: they disperse the spectral components of polychromatic light. They also disperse the different spectral components of ultrashort pulses, impacting their propagation very significantly. This is only one of the many possible effects, as the interaction with the field can give rise to a multitude of phenomena.

The characterization of light pulses partly consists in finding the frequency drift of these pulses. Since the 1990s, it has become a major field to master the creation of ultrashort pulses and understand their propagation and interaction with the fields they go through [7 [[07]], 11 [[11]], 12 [[12]], 13 [[13]], 14 [[14]]].

We have just observed that the formalism of fractional Fourier transforms is perfectly adapted to describe the propagation of light pulses in dispersive fields. One of the main consequences of dispersion on pulses is the appearance of frequency drifts (linear if the dispersion is in β_2). Therefore, it is obvious that the analysis of fractional Fourier transforms is well-adapted to

characterize the frequency drifts of ultrashort pulses and more generally characterize ultrashort pulses.

In a more mathematical way, the Fourier transform of a monochromatic signal is a Dirac delta. Accordingly, Fourier analyses are a perfect way to understand the spectrum of any signal. Besides, the fractional Fourier transform of signals with a linear frequency drift is a Dirac delta if the fractional order of the transform is good. Therefore, this is the appropriate way to study signals with a linear frequency drift. We will also prove that it can characterize higher order drifts.

First of all, we will show the potential use of these methods on simple signals [15 [[15]], 16 [[16]]]. We will then show how to apply these tools to real measures found with experimental techniques to characterize ultrashort pulses [17 [[17]]].

2.1. Characterization of simple signals

We consider a Gaussian pulse with an amplitude:

$$U(0, T) = \exp\left(-\frac{T^2}{2T_0^2}\right)$$

After propagation over a distance z in a linear dispersive field (characterized by a coefficient β_2), the amplitude of the electrical field of the pulse is written:

$$U(z, T) = \frac{T_0}{(T_0^2 - i\beta_2 z)^{1/2}} \exp\left[\frac{T^2}{2(T_0^2 - i\beta_2 z)}\right]$$

in other words:

$$U(z, T) = \frac{1}{(1 - iz/z_0)^{1/2}} \exp\left[-K^2 \frac{T^2}{2T_0^2}\right] \exp\left[-iL^2 \frac{T^2}{2T_0^2}\right]$$

where $z_0 = T_0^2/\beta_2$, $K^2 = \frac{1}{1 + (z/z_0)^2}$ and $L^2 = \frac{z/z_0}{1 + (z/z_0)^2}$.

We will characterize the amplitude of the latter through fractional Fourier transforms. The a order fractional Fourier transform of this amplitude is written:

$$\mathcal{F} = [U(z, T)](z, T_a) = \frac{C(a)}{1 - iz/z_0^{1/2}} \exp\left(i \frac{\pi T_a^2}{\epsilon^2 \tan(a\pi/2)}\right) \int_{-\infty}^{+\infty} \exp\left[-K^2 \frac{T^2}{2T_0^2}\right] \exp\left[-iL^2 \frac{T^2}{2T_0^2}\right] \exp\left(i \frac{\pi T^2}{\epsilon^2 \tan(a\pi/2)}\right) \exp\left(-i \frac{2\pi T_a T}{\epsilon^2 \sin(a\pi/2)}\right) dT$$

If we have:

$$\frac{L^2}{2T_0^2} - \frac{\pi}{\epsilon^2 \tan(a\pi/2)} = 0 \quad (1)$$

so

$$\mathcal{F}_a[U(z, T)](Z, t_a) = \frac{C(a)}{(1 - iz/z_0)^{1/2}} \exp\left(i \frac{\pi T_a^2}{\epsilon^2 \tan(a\pi/2)}\right) * \delta\left(\frac{T_a}{\epsilon^2 \sin(a\pi/2)}\right)$$

where δ is Dirac distribution. In other words, there is a special fractional order where the frequency drift turned into a Dirac delta. Conversely, once this fractional order is found, the frequency drift can be deducted thanks to relation (1).

Méthode

However, it should be noted that a complex fractional order must be considered. With a real fractional order, a linear frequency drift can also be seen as in figure 6. If an a order fractional Fourier transform is applied to the signal and if we make the order a vary, we can trace $Max(\mathcal{F}_a[U(z, T)])(z, T_a)$ following a . Figure 6 thus represents $Max(\mathcal{F}_a[U(z, T)])(z, T_a)$ following a real fractional order a .

Complément

A peak can be easily identified for an optimal fractional order. Once this fractional order is found, the frequency drift in the pulse can be inferred through relation $\frac{L^2}{2T_0^2} - \frac{\pi}{\epsilon^2 \tan(a\pi/2)} = 0$ [15 [[15]]].

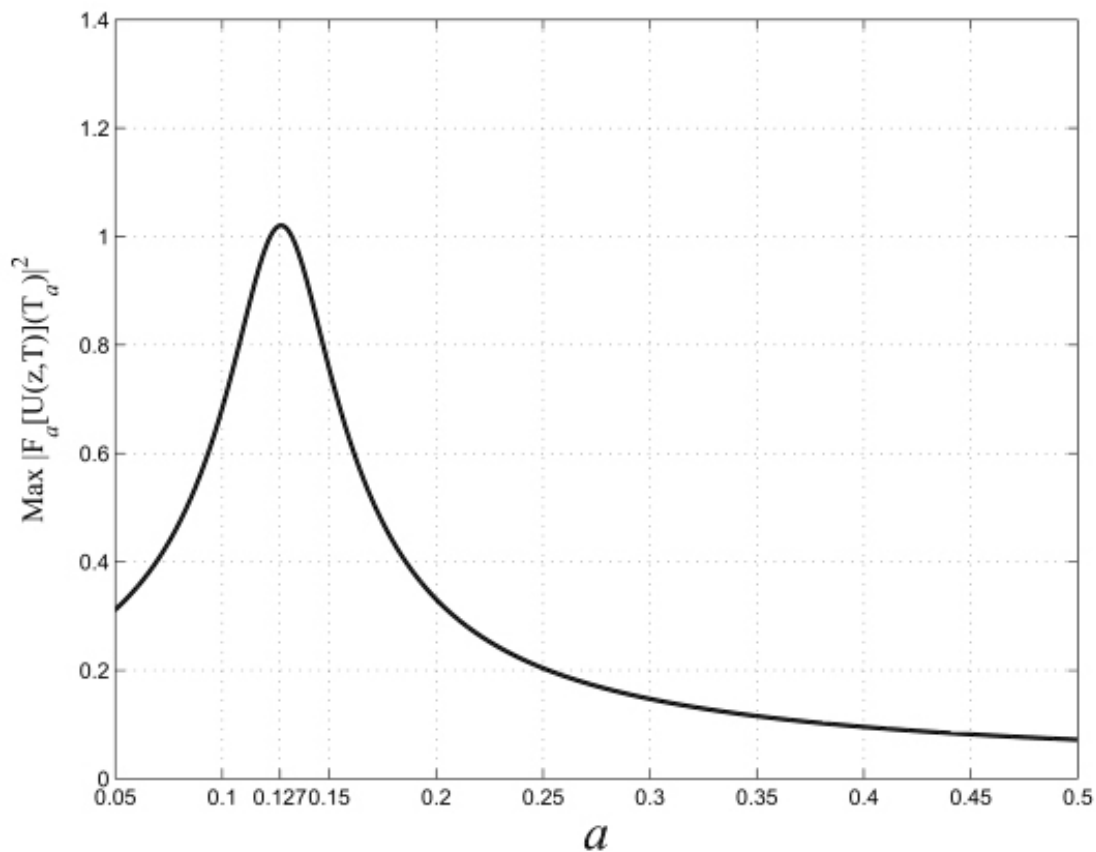


Fig. 6 - Identification of a linear frequency drift

2.2. Characterization of two pulses superimposed in time but having different linear frequency drifts

We now consider a signal which consists of two Gaussian functions superimposed in time but having different linear frequency drifts. If an a order fractional Fourier transform is applied to such a signal and if we make a vary, we can trace $Max(\mathcal{F}_a[U(z, T)])(z, T_a)$ following a . The results can be found in figure 7. Two optimal fractional orders clearly appear which are characteristic of the two frequency drifts in the pulse. Both can be accurately determined through relation (1) [15 [[15]]].

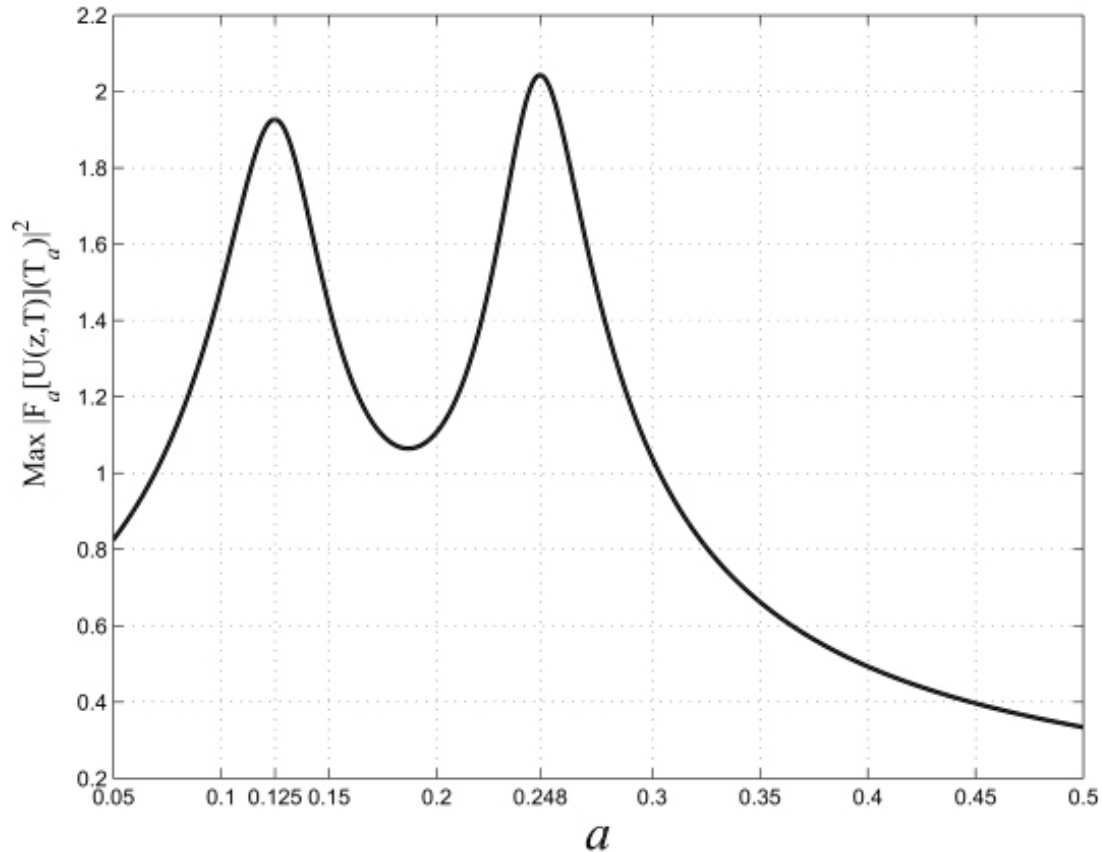


Fig. 7 - Separation and identification of two linear frequency drifts using fractional Fourier transforms

2.3. Decomposition techniques through fractional Fourier series

The decomposition of a signal in Fourier series is a well-known technique. In a rather similar way, decomposition techniques using fractional Fourier series can be developed [18 [[18]]].

Fondamental

For an a order fractional basis, the following basic functions are defined:

$$\Theta_{n,a}(T) = \mathcal{F}_{-a}[\delta(T_a - nk_0)]$$

where δ is the Dirac distribution, κ_0 the central "frequency" and $n = \pm 1, \pm 2, \dots$ the decomposition orders. Thus, these basic functions are given by:

$$\Theta_{n,a}(T) = \frac{C(-a)}{|C(a)|} \frac{\sqrt{\Delta T}}{\Delta T} \exp\left(-i \frac{\pi}{\varepsilon^2 \tan(a\pi/2)} \left(T^2 + \left(\frac{n\varepsilon^2 \sin(a\pi/2)}{\Delta T}\right)^2\right) + i \frac{2\pi n T}{\Delta T}\right)$$

$\Theta_{0,a}(T)$ is a linear frequency drift function called "mother" function; the other functions are expansions of this "mother" function.

If we now consider the aperiodic function $U(T)$ on the interval $[-\Delta T/2, \Delta T/2]$, it can be decomposed using previous functions:

$$U(T) = \sum_{n=-\infty}^{+\infty} C_U(n, a) \Theta_{n, a}(T)$$

where the decomposition coefficients are given by:

$$C_U(n, a) = \int_{-\Delta T/2}^{\Delta T/2} U(T) \bar{\Theta}_{n, a}(T) dT$$

and where $\bar{\Theta}_{n, a}(T)$ represents the complex conjugate of $\Theta_{n, a}(T)$. In practical terms, the reconstruction is shortened at $-N \leq n \leq N$, with an adjustable N to reconstruct it in the most accurate way. If we take the pulse signal:

$$U(T) = \exp\left(-\frac{T^2}{2T_0^2}\right) \exp\left(-iL_0 \frac{T^2}{2T_0^2}\right)$$

where $T_0 = 100\text{fs}$ is the duration halfway of the pulse and $L_0 = 1$ a parameter of linear frequency drift. Figure 8 represents the module of the different coefficients $C_U(n, a)$ in the plane (n, a) . The favorite fractional order $a_{opt} = 0.804$ is particularly significant. This is the optimal order which verifies:

$$\tan(a_{opt} \pi/2) = \frac{\pi}{L_0} \frac{T_0^2}{\varepsilon^2}$$

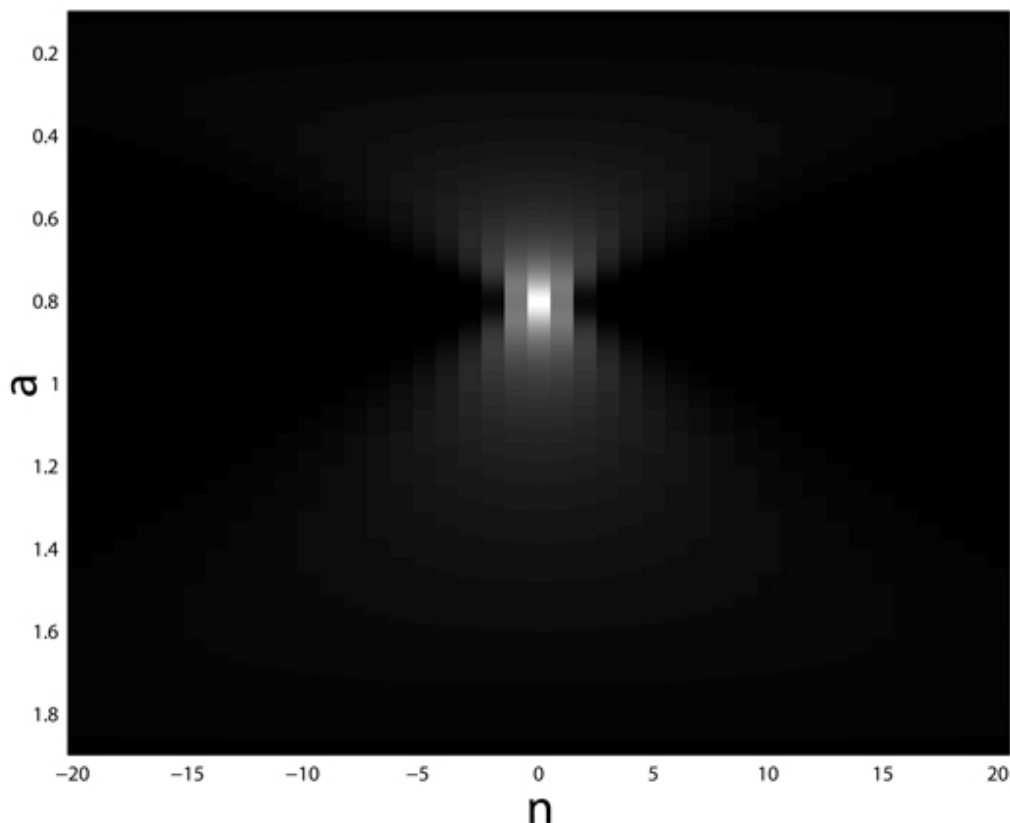


Fig. 8 : Decomposition coefficients in fractional Fourier series in the plane (n, a)

Finding this optimal order enables us to discover the linear frequency drift of the pulse. Here we chose $\epsilon = T_0$. So, we obtain $L_0 = 1$ which is true to the value of L_0 chosen during the simulation. Figure 9 represents $|C_U(n, 0.804)|^2$ in more details following the decomposition coefficient n .

It should be noted that, if the pulse had no Gaussian envelope, only one coefficient not equal to 0 would be found in $n = 0$. Besides, if the signal had a higher order frequency drift, the number of coefficients not equal to 0 would increase. More precisely, a dissymmetry would appear between the n positive or negative coefficients for another quadratic frequency drift [16 [[16]]].

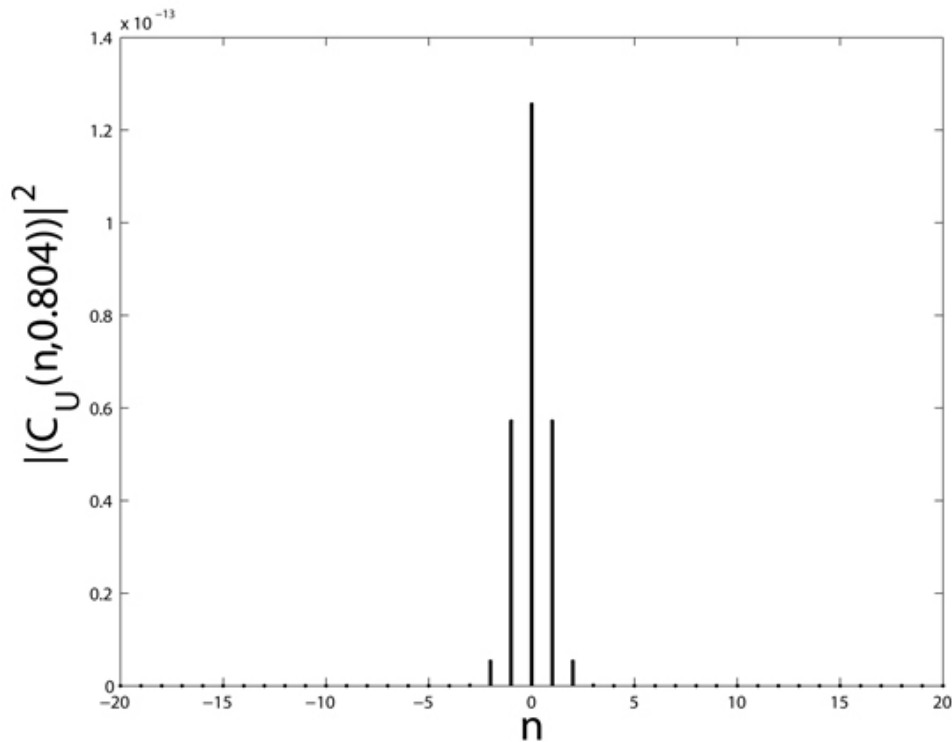


Fig. 9 : Decomposition coefficients in fractional Fourier series according to n for $a = a_{opt}$

Finally, when the coefficients $C_U(n, a)$ are set, it is possible to rebuild the initial function, as shown in figure 10. It should be noted that, in this case, the reconstruction is very accurate. The real and imaginary parts of the original and reconstructed functions are merged [16 [[16]]].

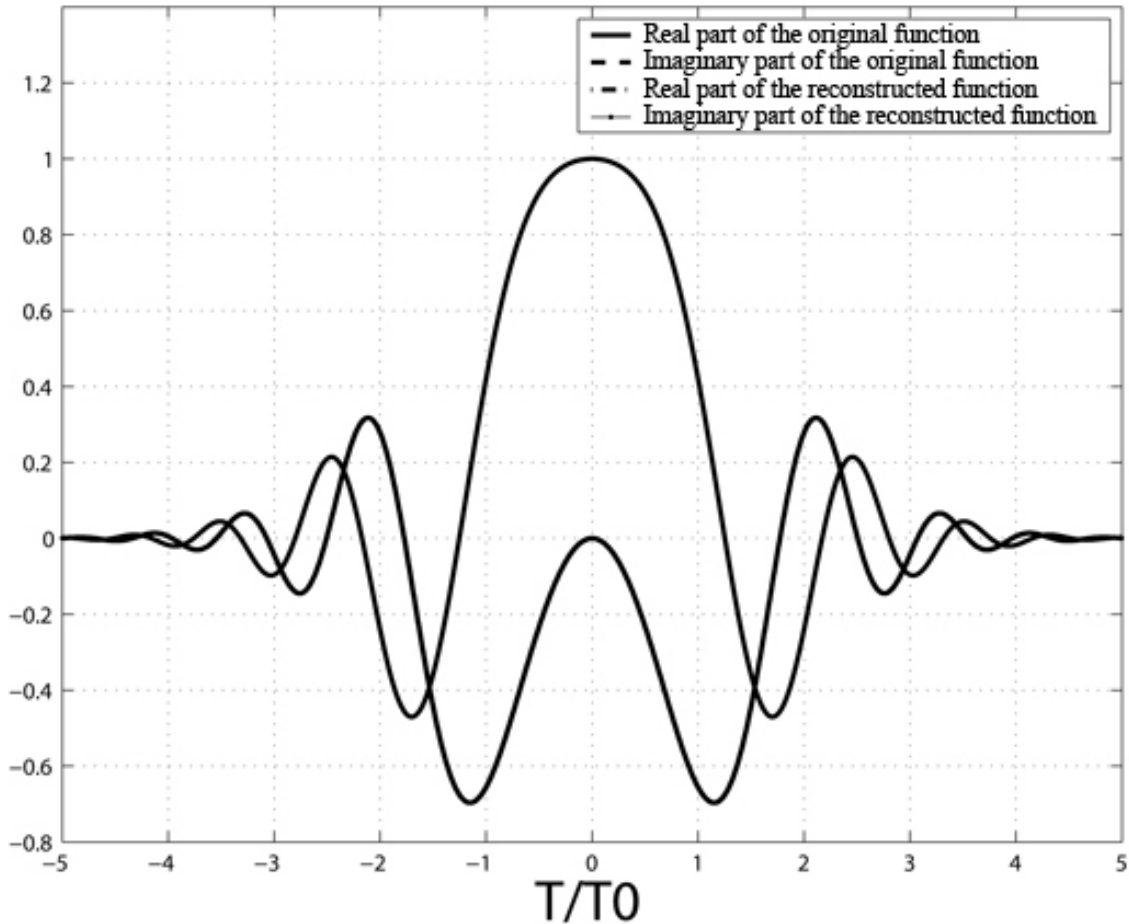


Fig. 10 : Original and reconstructed signals

2.4. An experimental example: analysis of SPIRIT signals

In this last part, we will apply these different methods to an experimental signal to characterize ultrashort pulses.

The SPIRIT (Spectral Interferometry Resolved In Time) method developed in XLIM consists in creating 2 time-delayed replicas of the pulse to be measured [19 [[19]], 20 [[20]]]. These replicas are then sent towards a spectroscopy, both slightly space-delayed with each other, which is similar to a spectral shift. Such an experiment produces an unsteady interferogram given by [19 [[19]], 20 [[20]]] :

$$S(\omega, t) = |\tilde{E}(\omega)|^2 + |\tilde{E}(\omega + \Omega)|^2 + 2|\tilde{E}(\omega)\tilde{E}(\omega + \Omega)|\cos[\omega\tau + \Omega t + \phi(\omega) - \phi(\omega + \Omega)]$$

where $\tilde{E}(\omega)$ is the Fourier transform of the amplitude of the incident field, $\phi(\omega)$ its spectral phase, Ω the spectral shift, and τ the time delay between the 2 replicas. t is the time. A simplified notation can be:

$$\phi(\omega) - \phi(\omega + \Omega) = -\phi'(\omega)\Omega$$

We should now consider the case of pulses presenting a spectral phase which has quadratic and cubic terms: $\phi(\omega) = \frac{1}{2}\phi^{(2)}(\omega - \omega_0)^2 + \frac{1}{3}\phi^{(3)}(\omega - \omega_0)^3$. Therefore, the SPIRIT interferogram contains the terms of the form $\cos[\omega\tau + \Omega t - \Omega\phi^{(2)}(\omega - \omega_0) - \Omega\phi^{(3)}(\omega - \omega_0)^2]$ and linear frequency drift terms: the term $\Omega\phi^{(3)}(\omega - \omega_0)^2$. So, there is a specific fractional order which enables us to find the linear drift. The correspondence between the optimal fractional order a_{opt} and the corresponding phase term is given by:

$$\frac{\pi}{\varepsilon^2 \tan(a_{opt} \pi / 2)} = \Omega \phi^{(3)}$$

Conversely, the term of phase is directly obtained from the optimal fractional order by the relation:

$$\phi^{(3)} = \frac{\pi}{\Omega \varepsilon^2 \tan(a_{opt} \pi / 2)}$$

Figure 11 shows an experimental example of this type of SPIRIT signal.

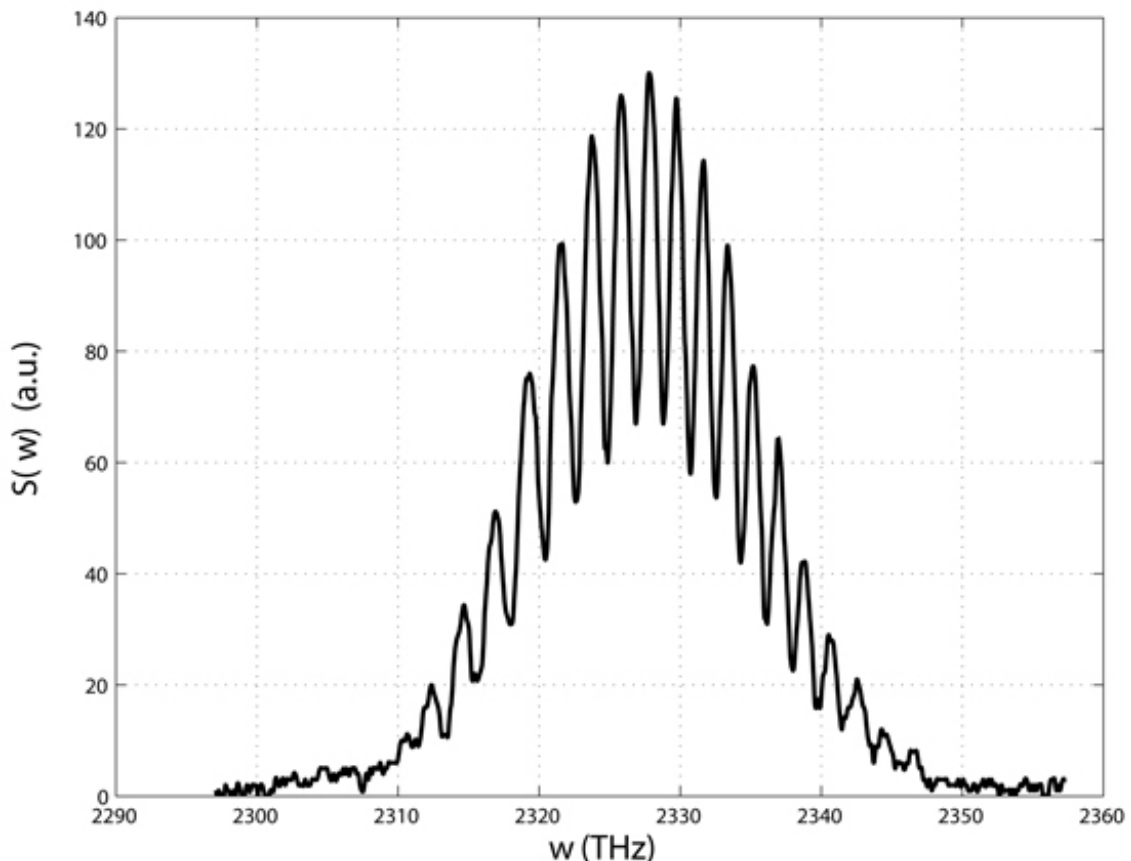


Fig. 11 SPIRIT experimental interferogram

- First of all, we made fractional Fourier transforms of different orders of this interferogram. Figure 12 shows the results. There is a specific order ($a_{opt} = 1.8$) which allows us to highlight the linear frequency drift in the signal: we notice a very thin peak. As a way of comparison, this peak does not appear for another order $a = 1.5$.

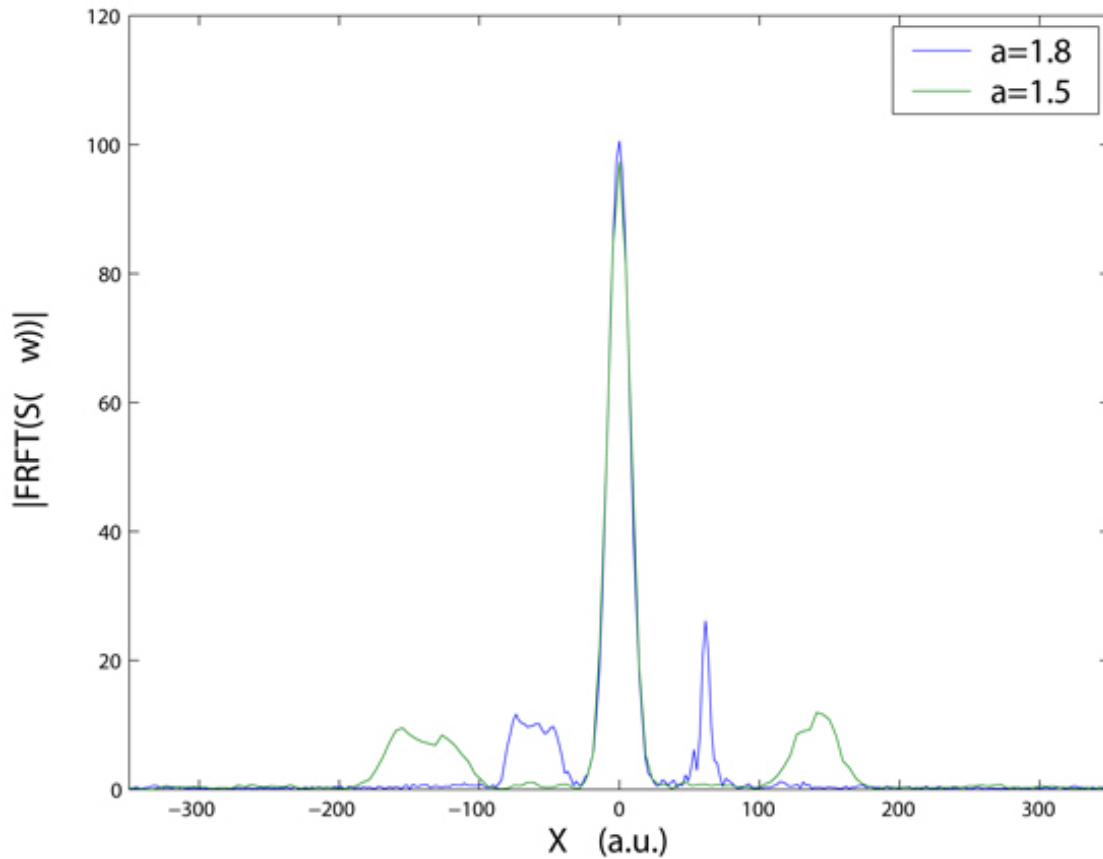


Fig. 12 - Two fractional Fourier transforms of different orders of the experimental SPIRIT interferogram.

- Secondly, we applied the method of decomposition in fractional Fourier series to the interferogram. Figure 13 shows the values of the different orders of decomposition $|C_s(n, a)|$ in the plane (n, a) .

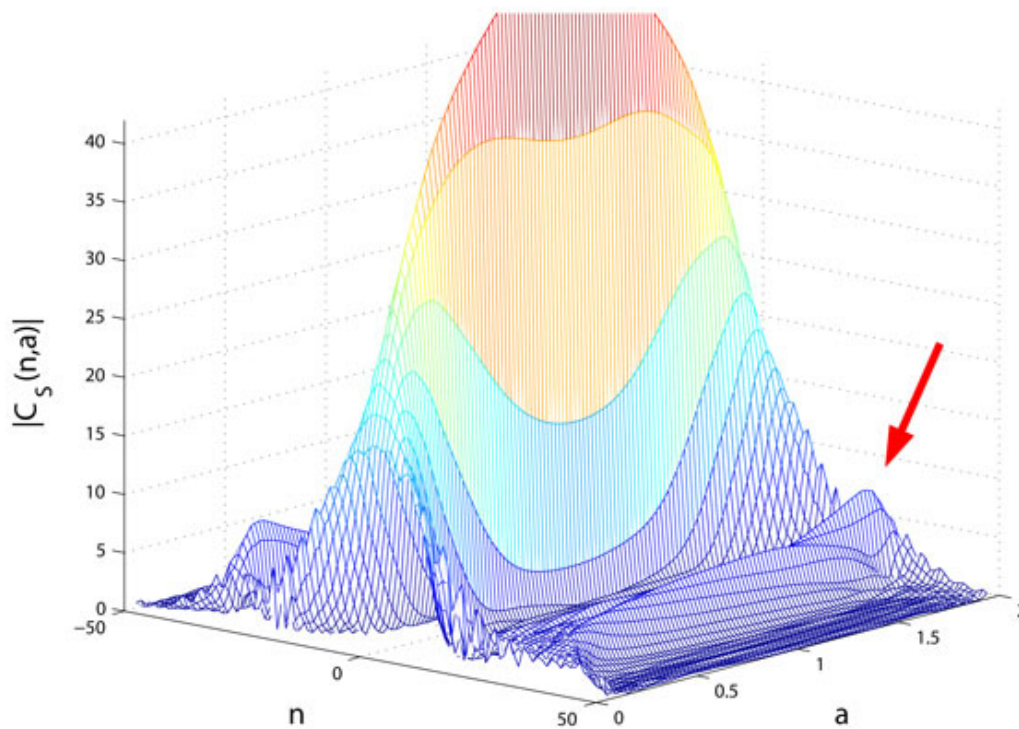


Fig. 13

* Fig. 13 - Cartography of the decomposition coefficients $|C_s(n,a)|$

We can differentiate a particular domain (see the arrow). It is due to the term of linear frequency drift of the SPIRIT signal. This peak is in position $a = a_{opt} = 1.8$ (as seen before). We then have:

$$\frac{\pi}{\epsilon^2 \tan(a_{opt} \pi / 2)} - \Omega \phi^{(3)} = 0$$

We took $\epsilon = 20\text{THz}$ in these simulations, which allowed us to infer $\phi^{(3)} \simeq -0.009\text{ps}^3$. Besides, this peak is in position $n = n_{opt} = 30$ and we can write:

$$\tau - \Omega \phi^{(2)} - \frac{2\pi n_{opt}}{\Delta\omega} = 0$$

where $\Delta\omega$ is the basis of the interferogram. In our case $\Delta\omega = 60.04\text{THz}$, $\Omega = 2.87\text{THz}$ and $\tau = 2.97\text{ps}$. We can then deduct:

$$\phi^{(2)} = \frac{1}{\Omega} \left(\tau - \frac{2\pi n_{opt}}{\Delta\omega} \right) \simeq -0.06\text{ps}^2$$

Remarque

These values have been experimentally confirmed [17 [[17]]].

Attention

The fractional methods allow us to determine the coefficients of quadratic and cubic spectral phase.

Remarque

When we perform a decomposition in fractional Fourier series, we can then rebuild the original signal based on this decomposition. A screening is then possible if we adjust the number of functions considered in the reconstruction process.

Figure 14 illustrates such a reconstruction from an interferogram with interferences (this is a case of simulation). The reconstruction was made by taking into account all the functions between $n = -80$ and $n = 80$. Noise is efficiently eliminated via reconstruction.

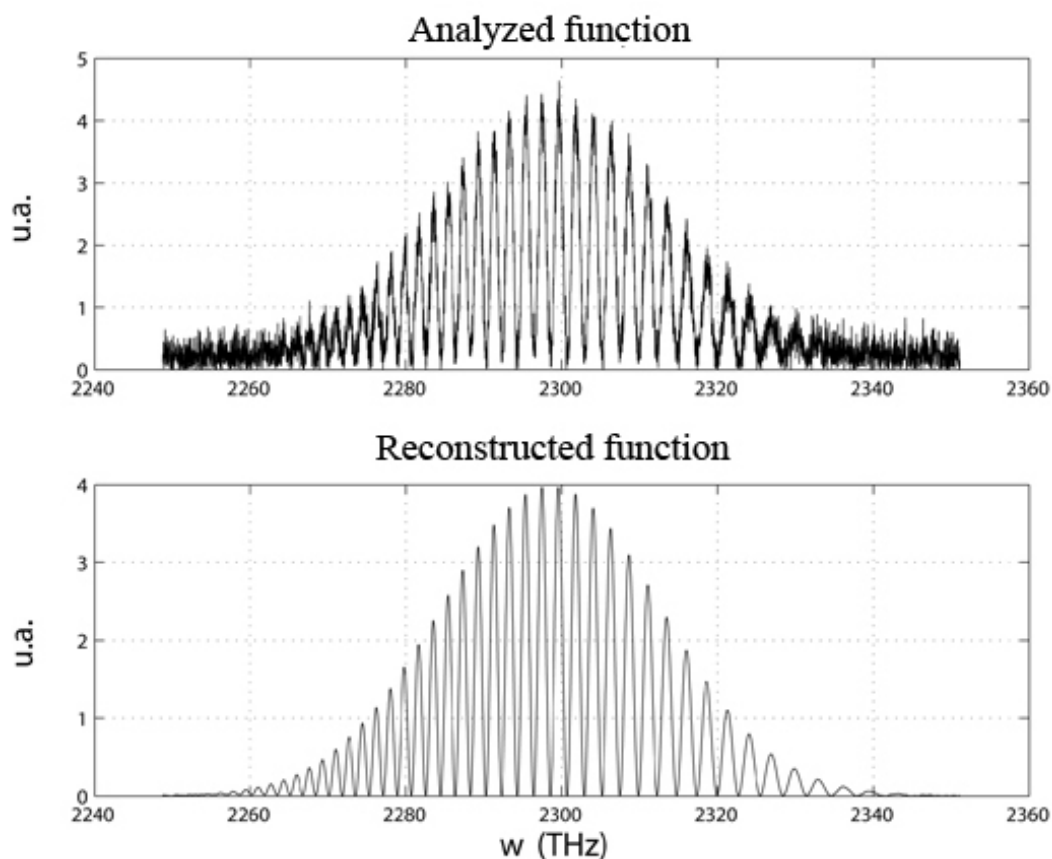


Fig. 14 - The noisy signal and its reconstruct

* *
*

The aim of this chapter was to present the potential use of fractional Fourier analyses to describe light pulses. We have more particularly shown that this formalism very well describes the propagation of light pulses in dispersive fields. It is also a suitable way to characterize frequency drift light pulses.

It should be noted that we do not want to change standard existing methods, but rather present other little-known methods which have true potential of analysis.

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Bibliographie

- [[01]] V. NAMIAS, *The Fractional Order Fourier Transform and its Application to Quantum Mechanics*, J. Inst. Math. Appl. 25, 241-265, 1980.
- [[02]] A. W. LOHMANN, *Image rotation, Wigner rotation, and the fractional Fourier transform*, J. Opt. Soc. Am. A 10, 2181-2186, 1993.
- [[03]] D. MENDLOVIC, H. OZAKTAS, *Fractional Fourier transforms and their optical implementation : I*, Opt. Soc. Am. A 10, 1875-1881 1993.
- [[04]] H. OZAKTAS, D. MENDLOVIC, *Fractional Fourier optics*, J. Opt. Soc. Am. A 12, 742-751, 1995.
- [[05]] P. PELLAT-FINET, *Fresnel diffraction and fractional-order Fourier transform*, Opt. Lett. 19, 1388-1390, 1994.
- [[06]] A. W. LOHMANN, D. MENDLOVIC, *Fractional fourier transform : photonic implementation*, Appl. Opt. 33, 7661-7664, 1994.
- [[07]] R. TREBINO, K. W. DELONG, D. N. FITTINGHOFF, J. N. SWEETSER, M. A. KRUMBÜGEL, B.A. RICHMAN, D.J. KANE, *Measuring ultrashort laser pulses in the time-frequency domain using frequency-resolved optical gating*, Rev. Sci. Instrum. 68, 3277-3295, 1997.
- [[08]] GP AGRAWA, *Nonlinear Fiber Optics*, CA :Academic, San Diego, 1995.
- [[09]] S. COËTMELLECC, M. BRUNEL, D. LEBRUN, J.B. LECOURT, *Group Velocity Dispersion with fractional-order Fourier formalism*, J. Opt. A : Pure Appl. Opt. 5, 26-30, 2003.
- [[10]] M. BRUNEL, S. COËTMELLECC, *Fractional-order Fourier formulation of the propagation of partially-coherent light pulses*, Opt. Commun. 230, 1-5, 2004.
- [[11]] R. TREBINO, D.J. KANE, *Using phase retrieval to measure the intensity and phase of ultrashort pulses : frequency-resolved optical gating*, J. Opt. Soc. Am. A 10, 1101-1110, 1993.
- [[12]] J.A. CHILLA, O.E. MARTINEZ, *Direct determination of the amplitude and the phase of femtosecond light pulses*, Opt. Lett. 16, 39-41, 1991.
- [[13]] C. IACONIS, I.A. WALMSLEY, *Spectral phase interferometry for direct electric-field reconstruction of ultrashort optical pulses*, Opt. Lett. 23, 792-794, 1998.
- [[14]] C. IACONIS, I.A. WALMSLEY, *Self-referencing spectral interferometry for measuring ultrashort optical pulses*, IEEE J. Quantum Electron. 35, 501-509, 1999.
- [[15]] S. COËTMELLECC, M. BRUNEL, D. LEBRUN, C. ÖZKUL, *Characterization of chirped pulses with the fractional-order Fourier transformation*, J. Opt. Soc. Am. A 18, (11), 2754-2759, 2001.
- [[16]] S. COËTMELLECC, M. BRUNEL, D. LEBRUN AND J.B. LECOURT, *Fractional-order fourier series expansion for the analysis of chirped pulses*, Opt. Commun. 249, 145-152, 2005.
- [[17]] M. BRUNEL, S. COËTMELLECC, M. LELEK, F. LOURADOUR, *Fractional-order Fourier analysis for ultrashort pulse characterization*, J. Opt. Soc. Am. A. 24, No. 6, 1641-1646, 2007.

[[18]] SOO-CHANG PEI, MIN-HUNG YEH, TZYI-LIANG LUO, *Fractional Fourier series expansion for finite signals and dual extension to discrete-time fractional Fourier transform*, IEEE Trans. On Signal Processing 47, 2883-2888, 1999.

[[19]] V. MESSEGER, F.LOURADOUR, C.FROEHLI, A.BARTHELEMY, *Coherent measurement of short laser pulses based on spectral interferometry resolved in time*, Opt. Lett. 28, 743-745, 2003.

[[20]] M. LELEK, F. LOURADOUR, A. BARTHELEMY, C. FROEHLI, *Time resolved spectral interferometry for single shot femtosecond characterization*, Opt. Commun. 261, 124-129, 2006.